A dialogue on naturalized platonism, the differences between physical and mathematical objects, and indispensability

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M: Numbers exist.
B: Really? How do you know?
M: They must. Consider the following two sentences: 'There are two middle infielders between first and third base.' 'There are two prime numbers between 6 and 12.' You accept the existence of the second baseman and the shortstop, don't you? The existence of the two prime numbers is just as certain.
B: No, that's too quick. You're relying on the grammar of the sentences to lead your metaphysics. But, not every noun refers to a definite object. Remember Quine's arguments concerning sakes. [W&O] 'Do it for Joey's sake' doesn't entail the existence of anything called someone's sake. We just have to re-analyze the sentence according to our metaphysical commitments. Similarly, we can re-analyze 'There are two prime numbers between 6 and 12' in order that we don't commit ourselves to anything such as a number.
M: But, why would we do that? I can see, in the case of sake's, that that sentence is a shorthand for another. Perhaps, 'Do it in order that Joey benefits,' which is much less pleasing a sentence. But why would we re-analyze in the numbers case?
B: We shouldn't let our grammar lead our metaphysics; we should only admit as existing those objects for which we can account.
M: Exactly. So, we should admit numbers, as we can learn all sorts of wonderful facts about them. We can prove many things about the world of numbers, and other abstract objects, as well. We can prove theorems about rhombuses and hyperbolic spheroids and n-dimensional spaces.
B: That's exactly the issue. How can we be sure that we've proved these things? We can derive them from axioms, but that doesn't mean we've proved them.
M: Well, this is a difficult question, the nature of proof in mathematics. But, consider this, don't we know that 2+2=4?
B: Maybe.
M: And isn't that true?
B: Perhaps.
M: And isn't it's truth necessary?
B: Possibly.
M: Why are you being so stubborn? Aren't the answers to these questions obviously 'yes'? On what possible interpretation of any of these notions could '2+2=4' not be necessarily true?
B: Well, '2+2=4' could just be a useful fiction.
M: But then there's no more truth to it than to '2+2=5'.
B: Exactly.
M: Does that really cohere with your intuitions?
B: I don't want to rely on those things. They've led me astray so many times before. How many times have I been convinced that I was sitting here in front of the fire in this dressing gown...
M: But don't we have the ability to reason out these truths of mathematics?
B: Reason them out?! What kind of occult process might that be? This is exactly what I was worrying about, before. I like numbers, you know. They're intriguing and beautiful. I'm especially fond of 37. But I can't see how I can know about them. I don't see them in the world. I look around and there are desks and trees and people and ponies, but no numbers. And then people like you come along and tell me that I can see them with rational insight?! That kind of talk can really polarize an agnostic.
M: You do accept that we have the ability to reason, don't you? For example, we can apply rules of inference in our daily lives. We carry an umbrella when it looks like rain. We know from the frown on

our lovers' faces that they are feeling low. We make empirical generalizations based on limited experimental data. We reason.
B: Yes, we reason about the physical world, but that's entirely different from the reasoning we do about mathematics, on your story. My knowledge of the physical world is, if not uncontentious, at least a starting point. Whatever else we learn, in the absence of any good reason for a Berkeleyan-type skepticism about the external world, we need to account for our knowledge of the external, physical world. But your world of abstract objects is different. It's not nearly as obvious that we need to account for this.
M: But 2+2=4!
B: Let me put it this way. Our metaphysics would be much more simple if we could account for our knowledge of mathematics in a way similar to the way we account for our knowledge of the world. Let's call 'science' anything we do in order to understand our physical world. So, whether it's building theories or performing experiments, or just looking out the window and seeing the gladiolus in bloom, we are doing science. Shouldn't we try to see mathematics as a science, in that way? In fact, I heard my friend R was thinking about just this sort of project. Hi-Ho! Here he is, with his friend David Armstrong! David Armstrong lives in his hat, you know. He takes him out when he wants some austere company. What luck running into you two today.

R: Salutations.
M: B, here, was just telling me that you've been thinking about a naturalized epistemology for a platonist ontology.
R: Why, yes. David Armstrong and I were just discussing this. David Armstrong believes, as you know, that there are no such things as numbers. He's a committed naturalist and empiricist, and a deep despiser of abstractions.
David Armstrong: Grrr. I hate numbers.
R: Down boy. Have a concrete biscuit. David Armstrong is an epistemologist, at heart, but I'm sympathetic to his empiricism. We disagree because I think it can have broader metaphysical ramifications. I've been working on a project to justify platonism with a naturalist epistemology.
M: That's quite interesting. What is your project?
R: I call it 'Mathematics as the Science of Patterns'.
M: But mathematics is about mathematical objects, like numbers, not patterns!
R: If we know about the patterns, like chessboards and pyramids, we can derive the objects, as positions in the patterns. Numbers are derivable as points in the structures. We can eschew a direct metaphysical commitment to numbers, this way.
M: Look, if you're so opposed to accepting mathematical objects into your metaphysics, why accept them at all? Why not deny their existence entirely, and make do without them?
R: Some of the best and brightest have tried that. It doesn't work.
M: So, you think that you have to accept them.
R: They're indispensable to our science.
M: Ah. I think I've heard this story before, While Visiting Our Quintessence and Happily Puttering. But doesn't that make mathematics subject to all the epistemic problems for science? Aren't you increasing the scope of those epistemic issues?
R: Yes, but we have those already. Surely, we need an epistemology for concrete objects, as those patterns are. This way, we have only one set of epistemic problems, instead of two.
M: So, what exactly is your position?
R: Mathematical objects play irreducible roles in physical explanations, in science. Whatever science says exists, exists. If science tells us that gladiolus exist, then gladiolus exist. If science tells us that atoms exist, then atoms exist. If science tells us that quantum particles exist, then quantum particles exist. And if science tells us that numbers exist...
M: ...then numbers exist. I get it.

M: This is very strange, to me, this connection between science and mathematics. Aren't the two quite different disciplines?
R: Why do you think so?
M: Well, mathematics deals with abstract objects and the truths of mathematics seem to be paradigmatically a priori and even necessary. Science on the other hand, deals with paradigmatically contingent facts, empirically. I can see how you might want to connect conceptually physics with biology, say. But mathematics and physical science?
R: We need mathematics to do science. That's the indispensability claim.
M: Why do you think that science needs math?
R: Look at our best physical theory. Doesn't it use numbers? Look at our best scientific explanations. ["Mathematical facts and properties of mathematical objects play essential roles in physical explanations themselves." (NE, 42-3)] Consider a ball traveling upward, having been thrown straight up in the air. At some point it turns around and starts to fall downward. Why?
M: Well, gravity exerts a force, I guess, that decelerates the ball's upward velocity until it stops moving upward and starts moving downward.
R: Exactly. More simply put, the ball changes direction when the vector sum of the upward and downward velocities equals zero. That's a number! The mathematical properties of the ball are essential to our scientific explanation.
M: Are those mathematical properties necessary parts of our explanation?
R: They're parts of it.
M: But in order to establish the indispensability of mathematics, you have to show not only that mathematics plays a role in our scientific explanations, but that it has to do so.

David Armstrong: Exactly right, M. You know I don't like agreeing with you, but R, here, has gone too far. I agree with R completely that our primary commitments are to scientific theory, but his examples just don't show any essential commitments to numbers.
R: But we use numbers in our scientific explanations. Look at the example!
David Armstrong: Your explanation is a mere heuristic device. It doesn't show anything about our metaphysical commitments. There's a great difference between constructing a formal scientific theory and explaining physical events. The theory may explain, or it may not, depending on what we want out of our explanation. But you can't believe that anything that serves as an explanation is theory building!
M: I think that what David Armstrong is saying is that the role of numbers in an explanation might be merely epistemic. We use numbers because that's how we do our explanations. That seems like a weak foundation for a metaphysically loaded position like your naturalistically acceptable platonism. It might turn out that certain elements of the explanation are dispensable
David Armstrong: Viva el Nominalismo!
M and R: Hush!
M: Let me ask you this, R. Couldn't some one argue that those properties are nominalistically describable? Couldn't we just nominalize our physics and dispense with the mathematics, here?
R: Go ahead. Make my day.
M: I seem to have heard of such a thing. I remember reading a book, sitting by a Hard Tree in a Field.
R: That project is a failure!
M: But what if it worked?
R: Then we would have no need for mathematics in our science, and so we would have no reason to believe that numbers exist.
M: Your commitment to the existence of numbers seems somewhat tenuous, then.
R: No, I'm firmly committed to my prediction that our best scientific theory will need mathematics.

M: How can you be so sure that science needs math? Isn't science about the physical world? There aren't even any mathematical objects in the physical world!

R: I think you have too narrow a view of the scope of mathematics. Remember our example of the ball thrown straight up in the air? Not only the vector sum, but the velocity itself is a mathematical object. It's a function. What could be more mathematical than that? And surely we will need functions in our best scientific theory.
M: I agree that we use functions to characterize the ball's velocity, but that doesn't mean that the velocity itself is a function. It's a property of the ball!
R: A function is a mathematical object. Velocity is a function. Therefore, velocity is a mathematical object. In fact, I don't believe that there is a sharp distinction between physical and mathematical objects.
M: What?! Chairs and chickens are physical objects; circles and numbers are mathematical objects. What could be more obvious than that distinction?
R: But there are odd cases between those. Velocity, for one. And, also, quantum particles, like electrons. And fields, are those mathematical or physical?
M: I see. So physical objects and mathematical ones are ontically undifferentiable.
R: Exactly.
M: So our epistemology for concrete objects extends to abstract ones as well.
R: Precisely.

M: But why should I buy your argument that there is no ontological distinction between mathematical objects and physical objects?
R: Actually, it's not one argument, but many.
M: Yes, I would think you would need quite a few arguments for such a shocking conclusion, both to refute the traditional bases for that distinction, as well as to establish positively their continuity.
R: Do you doubt that I possess such arguments?
M: Not at all, not at all. I am eager to hear them.

R: Why do you think there is a distinction between physical objects and mathematical objects? What differences do those objects have?
M: For one, physical objects are contingent. Whatever properties they have, they have contingently. They can change. Heraclitus. Plato. The river, if there even is a substance that we can call a river, changes its properties constantly. And so too, with all physical objects. On the other hand, mathematical objects are immutable. They have their properties necessarily. 2 is, was and always will be prime. A circle can never contain a radius not equal in length to all the other radii. These are eternal truths, the properties of mathematical objects.
R: Well, what if I told you that mathematical objects can actually change their properties?
M: I would be quite surprised, indeed.
R: I will now present to you a property of a number that can change. Before this week, 2 had the property of being the number of perfect games thrown by Yankee pitchers. Now it doesn't have that property anymore, and 3 does!
M: I would rather say that the set of perfect games used to have an extension of two, and now has an extension of three. The extension of the concept changed, not the number!
R: Both have changed. You're enamored by surface grammar. Why oppose this property?
M: It's pathological.
R: I agree, this property is an odd one. But how are you going to draw a distinction between the properties you call pathological and the one's you deem acceptable.
M: Let me try this: the property of being the number of perfect games thrown by a Yankee pitcher is a relational property of the number 2, and extrinsic property. Whereas the property of being an even prime is intrinsic.
R: But what about electrons, and other sub-atomic particles? All properties electrons have are relational.
M: Really?

R: Sure. We only know about them through their effects on observatory apparati. We never perceive any of their properties directly. So all their properties are known through their relations to other objects.
M: You're confusing our epistemic position toward an object whose properties are independent of our apprehension of that object, with those properties themselves. You are entitled to conclude that our knowledge arises relationally, but let's not then conclude that their nature is somehow relational. We may abandon our belief in their existence through refinement of our scientific theories. But if our theories are correct, then they have their properties non-relationally; if our theories are incorrect, and reject their existence, then they have no properties whatsoever, being non-existent.
R: Perhaps my example was not excellent, but still, such a distinction is difficult to make.
M: Even if I grant you that such pathological properties are somehow legitimate, it doesn't necessarily follow that numbers change their properties. Two still has the property of 'the number of perfect games thrown by Yankee pitchers before July 1999'. And it always will be. And three will always be the number of perfect games thrown by Yankee pitchers after July 1999, and until the time when another perfect game is thrown by a Yankee.
R: I still believe that any attempt to make a distinction between good properties and bad ones is doomed to failure.
M: Difficulty precisely making a distinction doesn't entail the lack of a distinction. Do you think all properties are legitimate properties?
R: Why not?
M: If we know anything about properties, Leibniz' law seems as uncontentious an assertion as we can make. But on it, , for any objects a and b, we can find a property that a has that b does not: 'being necessarily identical to a'. This would prove that no object is identical with any other, unless the two are necessarily identical. That would include, say, Bill Clinton and the husband of Hillary Clinton. Isn't this a reductio on the notion that all properties are equally coherent, and acceptable?
R: So it would seem.
M: Do you have a theory of properties on which your pathological properties are actually legitimate?
R: Well, I have other arguments for the lack of a good distinction between the physical and the mathematical.
M: Perhaps we should proceed to them, then!

R: Mathematical objects can participate in events.
M: Whoa! Really?
R: Yep.
M: Lines and numbers can have some kind of causal efficacy in the physical world.
R: No, not those kinds of mathematical objects. But functions, such as velocity, can. And so can subatomic particles and fields. Those are mathematical objects, too.
M: I don't think so. But let's examine your assertions one at a time.

R: Functions, like velocity, are mathematical, and they participate in events.
David Armstrong: I'd like interrupt for a moment.
R and M: Please.
David Armstrong: The mathematical aspects of our best scientific theory are just shorthand for nominalistically acceptable descriptions of the relations among physical objects. These aren't mathematical at all!
R: Nominalizing projects suck.
David Armstrong: But surely you admit that the jury is still out on such projects?
R: I'm skeptical of the possibility of their success.
M: It does seem a weak case on which to rest your assertion. Aren't their better ways to understand velocity? I would say that velocity isn't really a pure mathematical object. It's an applied mathematical object: the velocity of a particular object at a particular time.

R: I'll concede this much to David Armstrong: the nominalizing project is much more plausible with functions like velocity than it is with subatomic particles. My other examples are stronger.

M: I'd like to make a general comment at this point. If subatomic particles and fields are mathematical objects and if they participate in events, then your project might work. On the other hand, if it is impossible for mathematical objects to exist in space-time, then it follows, a fortiori, that they can not participate in events, and thus we have no reason to think that the distinction between mathematical objects and physical objects is at all blurry.
R: I agree. One man's modus ponens is another's modus tollens.
M: Why do you think that a nominalizing project is less likely to be successful on the subatomic level?
R: "We have no reason to think that [such a nominalizing project] can be done with events involving subatomic particles, whose basic features, such as charge, spin and energy level, correspond to no commonsense ideas." (Resnik, NE, 45)
M: Are you referring to the notion that we need mathematics in order to explain physical events?
R: No, I'm saying that those objects are mathematical. They participate in events. And so mathematical objects participate in events,
M: But why do you think that those objects are mathematical?
R: They're just theoretical elements.
M: But couldn't they be theoretical physical ones?
R: We don't even know what electrons really are. They might just be manifestations of fields at particular points! "Add to that the reflection that in quantum mechanics, talk of intensity at a **point** (or in a region) is really talk of the probability of an interaction taking place there, and you see how mathematical is the quantum field theoretic conception of particles." (NE, 44-5)
M: You correctly insist that mathematics is relevant in describing fields. You may even be right that mathematics is indispensable for describing the behavior of fields. But that does not make a field a mathematical object. It's unlikely that we can do away with mathematics for lots of physical descriptions and explanations. For example, we our best scientific explanation of this table in front of me will likely appeal to notions like size and mass, which are expressed in terms of their mathematical properties. But that doesn't prove that the table is a mathematical object, nor that any of the elements of matter are.
R: Fields aren't like tables. You can point to a table.
M: So your criterion for calling an object mathematical is our inability to point to it?
R: Why not?
M: We can't point to the planet Vulcan. Nor can we point to giant masses in the nether regions of space. But that doesn't make them mathematical. This criterion seems hardly worthy of pursuit.
R: Whatever. I continue to hold to my assertion that our concepts of fields and subatomic particles is inherently mathematical. Its only their mathematical properties that have any importance to us.
M: I still think you're failing to make an important distinction between an object and its properties. Look at how you used the term 'point', earlier. I think you blur the distinction between a space-time point and a mathematical point. If there were no difference, then I could see how you could call quantum particles and fields 'mathematical'. The probability distribution over a three dimensional field of abstract points is certainly a mathematical structure. But there must be a difference between mathematical, purely geometrical points and space-time points. Think about the divergence of Euclidean space, that described by the (axioms of) 3-dimensional planar geometry, and our actual space, which may be Riemannian, or elsewise. It's hard to see how one could even understand alternate geometries if there wasn't an important difference between points in space and geometric points. If geometry were just the study of our physical space, then there would be just one correct geometry, instead of several different ones.
R: There are different kinds of spaces in our physical world.
M: But geometry isn't the study of our physical world, it's the study of abstract spaces, some of which are nearly physically manifested; others which may not be, and some of which are impossible to manifest

physically. Mathematics is just not about the world. Godel once said something like that. "...[T]he certainty of mathematics is to be secured not by proving certain properties by projection onto material systems... but rather by cultivating (deepening) knowledge of the abstract concepts themselves which lead to the setting up of these mechanical systems..." (CG, 383) The world may in fact work as a way to check certain ramifications of instantiations of our mathematical theories, but the world can never tell us whether a theory is true or false. It can only tell us whether the instantiation of the mathematical theory is appropriate or not.

R: I agree with you, when it comes to tables. They have mathematical properties, shape and size, for example, but also non-mathematical properties, like brown-ness and hardness. On the other hand, all we know about subatomic particles, on the field view, and about fields, is mathematical.

M: Aren't the color and solidity of the table also mathematically describable? Brown has to do with the wavelength of light that's reflected off of the table, and we use math to describe that. The solidity of the table is due to its sub-atomic structure, which is also mathematically describable. It may be that all objects are describable in all the ways important to scientific theory using the language of mathematics. The book of nature is written in the language of mathematics, after all. But that doesn't make all objects themselves 'mathematical' objects!

R: It seems to me that we're arguing over what an object is. I think that the nature of an object is completely determined by the properties of that object. And since some objects have only mathematical properties, they must be mathematical objects.

M: Your premise works against your argument, my friend. In order to show that an object is a mathematical object, you must show that it has not merely mathematical properties, but that it is mathematically constituted; that is, you have to show that it only has mathematical properties! What do you think of the following syllogistic representation of your argument?

> There are objects which exist in space and time yet are mathematical.
> They can, therefore, participate in events.
> So there is no good way to draw the line between the mathematical and the physical.

R: Exactly right.

M: So, you think that a mathematical object can be located in space-time, but still be mathematical.

R: That's what I just said.

M: If an object is located in space-time, then I would think it would have to have some non-mathematical properties.

R: "But what is it to be in space-time? To be located in it? To be part of it? To be either? Are space-time points in space-time? Is all of space-time in itself? These are not idle questions. The ontic status of the universal gravitational and electromagnetic fields, prima facie physical entities, as well as that of space-time points, prima facie mathematical entities, turns on how we answer them... Moreover, even quantum particles, such as electrons, widely regarded as paradigm physical objects, pose difficulties for a locationally grounded division between the mathematical and the physical. Where are these particles when they are not interacting with each other? On one interpretation of quantum theory, under some circumstances, these particles are not even located within a finite region of space-time. Then, are they everywhere or nowhere?" (Resnik, NE, 44)

M: Wow! You seem to have some problems with the space-time.

R: Not with space-time itself, but just using it to make a distinction between the abstract and the concrete.

M: But I would think that that's a very good way to make this distinction. In fact, I've heard of someone who does just this. And here he is in this cage. I have to keep him here because he gets kind of grouchy and dangerous.

Jerry Katz: Greetings! I heard what you were saying, R. You're wrong. What you're saying, hardly even makes sense! Are you speaking English?!

M: Whoa, slow down. Would you remind us both of the right way to make the distinction?
Jerry Katz: "An object is abstract just in case it lacks both spatial and temporal location and is homogeneous in this respect. An object is concrete just in case it has spatial or temporal location and is homogeneous in this respect." (Katz, RR, 124)
M: I see, so chairs and donkeys are concrete, since they have spatial and temporal location, while numbers and circles are abstract, since they don't.
Jerry Katz: Exactly.
R: What about persons?
Jerry Katz: Well, like selves, they may not have a particular spatial location, but they have at least temporal location. So, they're concrete objects.
R: Fair enough. I understand the distinction. But it won't help M's argument, here.

M: Why not?
R: The criterion itself is based on the spatio-temporal properties of objects. That is, we first have to know whether an object is located in space and/or time in order to determine whether the object is abstract or concrete. But that's exactly what we don't agree upon.
M: I think we might still be able to use that distinction, but in reverse.
R: How do you mean?
M: Well, you presented me with a list of problems for making the distinction between mathematical and physical objects according to the space-time criterion. You mentioned:

a) space-time points
b) space-time itself
c) gravitational and electromagnetic fields
d) quantum particles

If the criterion can handle those putative problems, then your argument is unsupported, and we can be confident that mathematical objects can not be located within space-time.
R: Fine, if you like that kind of logic!
M: Now, the last two, fields and particles, are clearly concrete objects. They're located in space and time.
R: But we can't see them. We can't touch them or point to them. We can't even determine, sometimes, their location in space and time!
M: We can draw spatial and/or temporal boundaries around them. That's sufficient, according to the criterion, for concreteness. Our ignorance of precise location is no evidence for non-spatio-temporality!
R: "The tendency for physicists to seek structural explanations of the fundamental features of physical reality also undermines the idea that a fundamental ontic division obtains between the physical and mathematical... [P]hysicists have proposed that all of physical reality is an eleven-dimensional space, whose geometric properties give rise to all of the known physical forces." (Resnik, NE, 46)
M: Their geometric properties do this?
R: Certainly.
M: If reality is an 11-dimensional space, or any other geometrically describable space, then it follows the parts of that reality, the parts of the physical world, as we commonly describe it, are in space. If so, then those parts are concrete, according to the criterion. If you want to call the properties that give rise to physical forces 'geometric' you might get away with it, as long as we realize that that description doesn't completely do justice to their nature. They may be geometric, but they are also located in space and time, and, thus, are concrete! As for space-time points, those aren't "prima-facie mathematical objects" at all. They're certainly concrete according to the criterion.
R: What could be more mathematical than a point?
M: A mathematical point is surely purely mathematical. But a space-time point is different. Space-time itself may be modeled by a particular geometry. In such a model, the space-time points would be modeled by the points of that pure geometric system. But the space-time points and their models would

not be the same things. Space-time point might have a temporal location, for example, as on a substantivalist view. They might come into location at, say, the Big Bang, and exist only as long as the universe itself exists, but no longer. But substantivalism doesn't entail temporal location for pure geometric points; in fact, it has no ramifications for pure mathematics whatsoever. Similarly, relationalist nihilism about space-time points is independent of any geometric system.
R: The model is the object! The medium is the message! The chickens are at the gates! Cluck the ramparts!
M: Huh?
R: Is space-time located within space-time?
M: Ah, the remaining putative problem for the space-time criterion.
R: Well?!
M: This seems like an awfully weak case to make your argument. I don't know whether space-time contains itself, nor whether any object contains itself. My suspicion is that whatever the answer, this is a much more general issue, and will not support your claim.
R: More arguments! I more argument have! Sss!
M: I can see you're tired. Perhaps we should take a small break.

The next morning, they all meet again.

M: Good morrow! I hope you're well-rested and prepared to convince me that there's no difference between mathematical and physical objects. I, myself, am ready to tell you why I think your strategy is misguided.
R: Howdy! I'd love to hear your general criticisms, but let me first explain a few reasons why I think there's no distinction.
M: Please.
R: A posit is a posit.
M: Are you positive?
R: The introduction of mathematical objects is on a par with the introduction of any theoretical element into a scientific framework.
Jerry Katz: Grrr! For God's sake, we don't posit mathematical objects, we discover them, through our ability to reason!
David Armstrong: There are no numbers! Only those objects that help to explain the behavior of physical things exist. Numbers don't do that. "If any entities outside the [spatio-temporal] system are postulated, but have no effect on the system, there is no compelling reason to postulate them. (NM, 154)
M: Let's move away from the cage, why don't we?
R: As I was saying, David Armstrong's methodology is fine, but he doesn't realize that numbers do help us understand physical systems. That's why a platonist can be a naturalist.
David Armstrong: Numbers lack a causal role!
R: They're necessary for descriptions of physical events. You can't explain what you can't describe!
David Armstrong: But they're just heuristic devices.
R: Just like quantum particles. These are all on a par, convenient posits without direct causal influence on human senses. They're epistemically equivalent.
David Armstrong: Don't listen to him, M. Of course there's a distinction between the two kinds of objects. Physical objects exist, mathematical objects don't. There's a difference for you.
M: It seems to me that you guys have to sort out your naturalisms. I don't think this is going to be a convincing reason to blur the distinction between the physical and the mathematical. In any case, I'm still a little unclear as to why you want to blur this distinction. How does this relate to your project?

R: Let me put on my hat, and tell you a story. Mathematical objects are positions in patterns; our mathematical knowledge is knowledge of patterns. Consider four types of entities, two abstract, and two concrete:

Abstract

1) patterns

2) template types

Concrete

3) templates

4) objects patterned after the templates

A long, long time ago...

M: Wait, stop. Aren't we philosophers? Philosophers don't care about what happened a long long time ago! Anything that happened then is wrong! I learned that in grad school.

R: Understanding the ontogenesis of our community's knowledge of mathematics is essential to understanding our current knowledge. Our knowledge is transmitted through an educational process. Looking at our current knowledge can only mislead us.

M: But isn't any contemporary account going to be a rational reconstruction?

R: Just shut the fuck up and listen to my story!

M: Ok, Ok.

R: A long, long time ago, our ancestors only knew about physical objects, many of which had certain patterns. Our ancestors were able to create concrete representations of those patterned objects, to stand for them and others like them.

M: Like a drawing of a dress pattern which can be instantiated in many different dresses?

R: Yes, or like blueprints or musical scores. There are no abstract objects in the story yet, just objects and concrete representations. These allowed our ancestors to do quite a bit of work. They could talk about shapes and sizes, and about how things are arranged and designed. At some point, though, someone started thinking about limit entities.

M: Like points and circles?

R: Exactly, or like infinite numbers, and infinitesimals, too! So they posited patterns as abstract types, and, in contemporary parlance, template types as abstract tokens of those abstract pattern types. The concrete templates, then, may be seen as tokens of the abstract token types.

M: Is this a true story?

R: Actually, that question is irrelevant. I'm trying to present a naturalistic account of our knowledge of numbers, so all I need is to present a plausible explanation of how such objects might be apprehended by concrete beings like ourselves!

M: I agree that the historical accuracy is irrelevant, but I still have a few questions about your account. You claim that this provides an explanation of our knowledge of abstract, pl atonic mathematical entities.

R: Yes, I do.

M: By focusing on patterns as the source of our knowledge, you make arithmetic dependent on geometry.

R: So?

M: Doesn't that make our understanding of arithmetic before we knew about analysis puzzling? I mean, these were historically distinct fields until the seventeenth century.

R: Look, it's just a story.

M: Alright, let's put that aside. What about the problems that such a structuralism faces?

R: Like what?

M: We can't differentiate among isomorphic patterns, as those among various Peano systems, by appeal to the natures of the individual objects, as they have none except those they get from their positions in structures. For example, consider the following two (isomorphic) Peano sequences:

P_1: 1, 2, 3, 4, 5...

P_2: 1, 2, 4, 8, 16...

The first structure results from interpreting the axioms of Peano arithmetic in the standard way.

The second results from interpreting the notion of 'successor of a number' as 'twice that number' and interpreting 'number' as 'power of 2'. From within the structure of Peano arithmetic, the pattern defined by the axioms, there is nothing to differentiate the two structures. Thus, the third element of the first structure, holds the same position in the sequence as the third element of the second structure. It is the successor of the previous number, which is the successor of the initial number in the sequence. Yet the specific elements, 3 and 4, are not themselves equal. We have to go outside of the structure, to the interpretations of the meanings of terms like 'number' and 'successor' in order to disambiguate between the two sequences, and thus give the individual terms their specific properties. We deprive the numbers of their individual properties, and, thus, open up all sorts of difficulties in disambiguating them, when we only allow the individual objects to have their properties relative to a given structure, when we make our knowledge of mathematical objects depend on our knowledge of the patterns.
R: That's what we should expect, though, in a world of indeterminacy. I can bite the indeterminacy bullet; it comes with the naturalist epistemology.

M: How do we even know that we're positing the right objects?
R: What do you mean?
M: We can posit ghosts, the Ether, and phlogiston, can't we?
R: Yes.
M: But those things don't exist!
R: Exactly. "What distinguishes them? Primarily, truth and existence. Ghosts, the Ether, and phlogiston do not exist. Hypotheses that they do, are false." (Resnik, NE, 57)
M: How can you tell? Our knowledge of those and of numbers arises in the same way!
R: How do you make the distinction?
M: We discover facts about numbers and about the physical world. We know about mathematics through mathematical discovery, and about the non-existence of ghosts through our discovery of physical facts to better explain the phenomena which might impel us to posit the existence of ghosts.
R: "The line between positing and discovering often blurs." (Resnik, NE, 57)
M: When we posit, we have no independent basis for our semantic evaluation of claims involving that posit. When we discover, we do.
R: Discovery is just a metaphor, when you're talking about abstract objects.
M: Perhaps, but it highlights what your argument is missing. Your project might be naturalistic, but it isn't really platonist. You've missed some things.

R: How do you mean?
M: You're relying on an indispensability argument. Something of the form:

1. What there is is what our best theories about the world tell us there is.
2. Our best theories tell us that there are mathematical objects.

Therefore, mathematical objects exist.

R: So? It seems to me that indispensability arguments are the best arguments for the platonist. They allow us the theoretical benefits that the platonist gets, while committing ourselves only to a belief in the powers of natural science. I can have my cake and eat it too. [R bows, removing his hat]
David Armstrong [popping out of the hat]: I agree that indispensability arguments are the best for the platonist. And I'm not alone! [E.g. Field writes, "...[O]ne should shift to the fictionalist's practice unless one can argue that there are respects in which the platonist's practice serves our purposes better. *And the only way to argue that would be to appeal to considerations independent of initial plausibility, such as indispensability arguments.*" (RM, p. 12)]
R: But you're an anti-platonist!
David Armstrong: Exactly. Since those arguments are fallacious, we can see that the platonist position is clearly wrong.
M: What if David Armstrong is right about dispensability? What if we can formulate our best scientific

theory without appeal to mathematical objects? Then...
R: He's wrong, I tell you!
M: Then a naturalized platonist, like yourself, would have to give up his platonism. It seems you have a weak grasp on these objects. A dispensability argument shouldn't affect the ontological status of Mathematics. Even if it were true, that would only show that Mathematics is unnecessary for doing science, for a certain application. But that doesn't show that the discipline itself, seen autonomously, is any less valid. It only shows that we can dispense with applied Mathematics. It does not show that we can dispense with pure Mathematics. It only would show this if indispensability were the only possible argument for platonism.
R: Your conclusion follows, but your premise is false. We do need Mathematics to do science, so I don't really feel your worry about dispensability. It's not going to happen.

M: Well, let's put aside his quibble. I have another problem with your indispensability claim.
R: What's that?
M: You're confusing two kinds of claims which result in ontological commitments, theoretical posits and indispensability claims. Theoretical posits work as follows:

> (TP) A discipline, for reasons internal to its own structure and working, needs to posit the existence of an unobservable object. The ontological justification of the posted object comes from within the domain that posits that object.

Indispensability claims, on the other hand, bridge two independent disciplines.

> (IC) Given two independent disciplines, D_1 and D_2, there exists a link between them such that the objects to which D_2 is committed are justified by whatever justifies the objects of D_1.

R: I'm not sure that I agree with your distinctions between disciplines. Mathematics and physical science are both sciences. There aren't strict boundaries between them.
M: The non-existence of strict boundaries doesn't entail the lack of a distinction. Haven't we been here before?
R: Fair enough. On your scheme then, I urge that both types of claim are true. On the one hand, we posit mathematical objects abstract pattern types, to account for limit entities, like points and circles. [see Quine, W&O] That's a theoretical posit. And what justifies his claim is our need for such objects in order to do science. And that's an indispensability claim.
M: By Jove, I think he's got it! To be specific, given that D_2, in your (TP) is a platonist Mathematics, and given your overall intention of justifying that discipline through a naturalized epistemology, D_1 must be natural science.
R: Right.
M: Well, as we noted before, there are competing claims about how much Mathematics science can justify. Quine thinks you can have math, but that guy in your hat seems to think that science won't support anything mathematical.
R: Let's go with that Quine guy.
M: Even if we make the broadest, most favorable interpretations of natural science, we are still left without a full-fledged justification of platonist Mathematics. Any such justification renders extremely difficult to make the pure/ applied math distinction, and, moreover, leaves Mathematics with a weakened field, one that the practitioners of that discipline would hardly recognize as that discipline. An indispensability claim only justifies the introduction of such objects as are necessary for the workings of D_1; it does not establish the independent existence of a pure discipline D_2. For that, we would need independent justification.

R: Why would we need such a distinction?
M: We can be sure that the mathematicians working today in the farthest reaches of pure, abstract mathematics do so without the certainty that their work has any physical application. One may arise, or their work may lie fallow indefinitely. If the only justification for the existence of mathematics is in its application to scientific theory, then those mathematicians can not know that the work they are doing is at all justified.
R: Once we posit the existence of abstract mathematical objects, the mathematician is free to explore their nature, in whatever way he may.
M: Still, if you're hoping to defend those from ontological worries, don't you have to trace them back to some naturalistically acceptable source? I mean, you seem to be defending the existence of all the mathematical objects that the scientist needs, but leaving the mathematician hanging.
R: Our primary goal is to construct our best scientific theory.
M: Yes, that seems to be your goal. It links with Quine's notion that your ontology falls out of your formal scientific theory. But you make the work of the mathematician depend for its value on the results of the scientist. This seems at least counterintuitive.
R: Maybe to you.
M: But also to the mathematician.
R: Why does this matter? The mathematician is free to explore whatever mathematical theories he wants. But when it comes to the truth of his theory, we just have to wait until we discover which mathematical theory is the right one.
M: But mightn't there be many right ones? For instance, why should we pick one geometry over another, Lobachevskyan over Euclidean, as the right one? Aren't there more minimal conditions? Can't they all be right, or true, independently of being the right models for space-time?
R: What could you mean by calling such competing theories all true?
M: They're consistent. They model certain structures, which may or may not be found in real space. From the definition of the structures, certain consequences truly follow and others don't.
R: The ones that don't model the geometry of space could be fictions, then, couldn't they? We could reserve our notion of truth for the one that actually applies.
M: The problem with this line of thought is that there are certain mathematical theories, or parts of mathematical theories, which are not possibly linked to the world in any meaningful way. The objects of platonist Mathematics, for example, far outnumber the possible objects that natural science could posit.
R: "I have not been thinking of science as encompassing so-called pure Mathematics. It thus falls short of affirming the existence of many of the entities studied by the far reaches of contemporary Mathematics. I do not find this a drawback at this point..." (NE, 58)
M: But then you weaken Mathematics not only by eliminating pure Mathematics as an independent discipline, but also by rejecting transfinite cardinals, debates over the continuum hypothesis, and other interesting and substantial questions. Such a Mathematics is too weak to bear proudly the name 'platonist Mathematics'. It may be platonist, but it's not Mathematics.
R: Now we're quibbling over a title.
M: In fact, it may not even be really platonist.
R: Why not?
M: Isn't the platonist committed to objects which have their properties necessarily?
R: So?
M: The Mathematics generated by an indispensability argument aren't sufficient. They may even be contingent. There is no reason inherent to any of the relevant disciplines to suspect that the nature of the physical world is necessarily the way it actually is. The objects posited by the science that describes the world should be as contingent as the world described.
R: The world is contingently the way it is.
M: So, if the only justification of Mathematics is through their indispensability for doing natural science, and natural science is contingent, then the objects of Mathematics should be contingent as well.

R: But it's not just indispensability that gets us these numbers. We can posit the abstract structures, and derivatively numbers or whatever other objects. "[I]f we do take the plunge and countenance limit entities, then we could also countenance numbers, linguistic types, and a host of other abstract entities our templates represent more adequately than they represent limit entities." (NE, 56)
M: That's the claim of the form (TP).
R: Yes.
M: But I still don't understand how you can claim that we posit only the right objects, like 2, and never the wrong ones, like Casper the Friendly Ghost.
R: We posit only the right ones, the ones which we need for our science.
M: But of course mathematical objects are posits, in that sense.
R: What do you mean?
M: You're saying that we have some independent way of knowing whether the mathematical objects, or structures, which we posit are good posits or bad posits. That's on a par with what I call discovery. I think we discover mathematical objects, through our use of mathematical reasoning. You think we discover them by using science. If you want to call that a posit, be my guest.
R: But discovery seems like a causally loaded term. Aren't you implying something more literal, when you say that we discover mathematical objects?
M: No! You'd have to Mad (dy) to think we have literal perception of mathematical objects. Only those who believe that we have some sort of literal perception of mathematical objects could even think that the objects of Mathematics could be justified by anything other than claims of the form (TP).
R: So we're agreed, sort of.
M: Actually, we're not in agreement at all.
R: No? Why not?
M: I can rely on mathematical reason, rational epistemology, as an epistemic justification for the discipline of Mathematics, but you reject that.
R: I do.
M: So you're left without an epistemic justification for the mathematical project.
R: I have science.
M: [Sigh] I thought we agreed that your indispensability claims were fruitless. The problem is justifying the entire framework, the discipline of Mathematics. We have seen that indispensability claims are insufficient for doing just that. And claims of the form (TP) are irrelevant to the big question at hand. In fact, I'm ready to conclude that your project of a natural epistemology for a platonist ontology fails. The epistemology won't generate an appropriate ontology.

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