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A Jigsaw Lesson for Symbolic Logic

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Jigsaw lessons were initially developed in the 1970s by Elliot Aronson in response to poor performance and low self-esteem of Black children in the wake of school desegregation. They are perfect for active learning in philosophy classrooms, fostering collaboration and interdependence. The core idea of this cooperative structure is to emphasize the importance of each student's contributions to a classroom community. In a jigsaw structure, students work collaboratively on a complex task with several (ordinarily three to five) distinct aspects. Each student focuses on one aspect. These distinct aspects combine like puzzle pieces into the larger task and students learn about all of them. Jigsaws can be used for a single lesson or for long-term projects. Jigsaw lessons are especially effective in philosophy classrooms because they promote the active and social learning and conversation that are characteristic of our discipline, historically and globally.[1] This chapter presents an overview of the jigsaw structure and instructions for a sample jigsaw lesson for translation using identity in first-order logic.

Each student in a jigsaw structure is a member of two distinct groups: a base group and a work group. (Base groups are sometimes called home groups, jigsaw groups, or cooperative groups; work groups are sometimes called expert groups or counterpart groups.) Students ordinarily begin in base groups, in which they choose, or are assigned, one aspect of the larger project. Next, we reshuffle the class, with all students moving to distinct work groups where they attempt to master their assigned aspect of the larger task. In work groups, students collaborate with members of different base groups

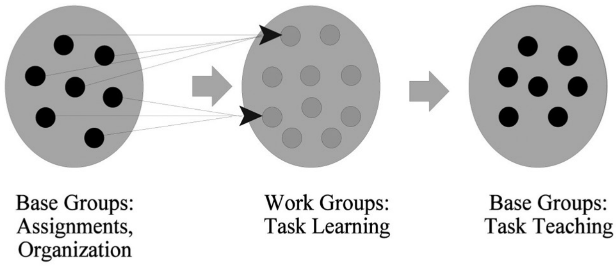

Figure 27.1 The three steps of the jigsaw lesson. Each member of each base group attends a work group with a different topic, and then returns to their original base group. © Russell Marcus.

to learn their aspect of the larger project well enough to teach it. Finally, all students return to their base groups and each one in turn teaches the other base group members what they have learned, combining the pieces of the puzzle. At the end of the lesson, each student in each base group has had the opportunity and responsibility both to teach their aspect of the larger task to the other base-group members and to learn each part of the complete project from the others in their group. Because of the distributions of tasks, there are no free riders in a jigsaw: everyone contributes.

Instructors can develop jigsaw lessons from any activity that can be divided into distinct parts. For example, an ethics instructor could construct a jigsaw out of a case study, forming, say, four work groups that each focus on how different moral theories assess the case: utilitarianism, deontology, virtue ethics, and feminist ethics. An instructor teaching a unit on free will in an introductory philosophy survey could design a jigsaw around three different responses to a Frankfurt case: the libertarian, the hard determinist, and the compatibilist. Below, I describe in detail a jigsaw with five work groups in a logic class.

For greater efficiency in single class meetings, the first base-group step may be omitted and students can start in work groups and then form base groups out of members from different work groups. For longer jigsaw activities, group projects that last over weeks or even whole semesters, students may convene their base groups or their work groups repeatedly. The same base groups can even be used for distinct activities, with different work groups. In a philosophy of education course, I dedicate two weeks at the end of the term to a jigsaw project in which students in base groups design an ideal school. First, we convene five work groups to review material for the five antecedent units of the course: knowledge, human nature, learning, teaching, and community. Later, a different set of four work groups focuses on aspects of the schools designed by the base groups: curriculum, pedagogy, physical space, and connection to community. Students get to collaborate with lots of others in the class, sharing collaborative research and developing greater expertise on areas of particular interest.

Students are typically and naturally actively engaged during jigsaw lessons. Instructors can judge whether their students need motivation in addition to the specific learning goals of the lesson, for example a grade on the complex task. Students mainly need to trust that the moving parts, which can seem complex at first, will resolve appropriately. Instructors must communicate confidence in both the effectiveness of the activity and how to manage it.

During class, the instructor may roam through different groups, providing support, but the instructor's work is mainly behind the scenes, in preparation. To foster interdependence, it is useful to use jigsaws to introduce new concepts so that the members of the base groups really need to be taught by each of the experts coming out of the work groups. Thus, work groups need materials to help them to develop that expertise, one for each aspect of the larger project, preferably roughly equal in difficulty. These materials should be crafted so that students can learn them quickly enough to be able to teach them to their base groups. The topics should be substantial enough to justify the use of class time, yet not too difficult for the students to master without extensive help from the instructor.

While the instructor of a jigsaw lesson is focused on facilitating the organization of the lesson and on the content of the assigned tasks, students in jigsaw lessons, as in any cooperative learning situation, are often also anxious about interpersonal social issues. We may hope that our classes are immune to hierarchies and cliques and that our students collaborate eagerly and productively, focusing entirely on philosophical content. That may not be the case. Cooperative lessons often bring out social complexities, as students are required to interact directly and explicitly.

Still, jigsaw lessons, because of their structured interdependence, can mitigate some social problems present in other cooperative structures. Aronson developed the jigsaw specifically to improve social interactions in recently desegregated schools, attempting to replace a competitive atmosphere with a cooperative one. The interdependent structure and its social factors were the primary content of the jigsaw at its inception. Especially long-term uses of jigsaws can improve relationships among students.

Random group assignments, done transparently in class, can also minimize the deleterious effects of social hierarchies. Allowing students to choose their own base groups can reinforce existing social structures. Random assignments presume and display no preference among students, leveling the playing field.[2] The simplest method for transparent random base-group assignments is to have the students count-off by the number of groups that will be formed. To make forming base groups a bit more fun, I sometimes have students use homemade jigsaw puzzles of pictures of philosophers printed on card stock and laminated. This technique is tricky, since you never know precisely how many you need, given potential class absences, so preparing various options can be useful.

The remainder of this chapter describes a specific jigsaw lesson in a formal logic class.

A Jigsaw Lesson in Formal Logic

Group assignments can be daunting, especially when you don't know who will show up to class on a given day. I'll sketch the lesson for an ideal class size, twenty-five students, and then make some suggestions for adjusting to less elegant numbers.

The main goal for this lesson is to have students learn how to translate English sentences into first-order logic with identity, focusing on five specific tasks:

1. "At least" sentences;
2. "At most" sentences;
3. "Except" sentences;
4. "Only" sentences; and
5. Superlatives.

In our ideal example, base groups and work groups all have five students. First, I divide the class into five base groups. In this first meeting of the base groups, students perform two small administrative tasks: distributing the five tasks and familiarizing themselves with each other. Once they have gone to their work groups and mastered their specific task, they will return to these groups and teach their tasks to the other members of their base groups.

To form work groups, all of the students who chose to work on "at least" sentences, one from each base group, form one work group. All of the students who chose to work on "at most" sentences form a second work group, and so on for the other three tasks.

Once the work groups are formed and students are re-settled, I distribute worksheets, prepared in advance, enough for all students. The worksheets each have five sample English sentences and corresponding model regimentations in first-order logic, along with additional English sentences with no translations to logic. Students in the work groups learn from the samples and translate the additional sentences. I emphasize that each student should learn their small task quickly and well enough to teach it to the other members of the base group to which they will return. Work groups ordinarily take ten-to-fifteen minutes to complete their tasks. A sample worksheet for "only" is below.

Once the work groups have finished, students return to their original base groups. Members of the base group have now all become experts on distinct tasks. In turn, they teach the other members of the group, distributing the relevant sheets, taking about five minutes each. After about twenty-five minutes, each member of the group has had a chance to learn each of the five tasks. Ta-da.

Rarely do classes have such neat numbers, five groups of five students in both base and work groups. Still, it is not too difficult to adjust to odder numbers. The sizes of base groups in jigsaw lessons are ideally determined by the number of tasks and it's important to have at least that many students in each base group, so that no tasks are omitted. If there are more people than tasks in a base group, two students can choose the same task (and work group) and can share responsibility for teaching their task when they return to base groups.

Any number of base groups can work. In large classes, you can have multiple work groups for each task. Imagine a class with sixty students and five tasks. You could have twelve five-person base groups. Five twelve-person work groups, though, would be unwieldy. Groups function best with three to seven students. In larger groups, individual students are too easily lost or ignored. Moreover, the number of individual social interactions that students must navigate increases exponentially with the size of the group. In a group with n members, there are $n\mathrm{C}_2$ one-to-one interactions, a number which gets quite large even for small n (e.g. in a group of six students, there are fifteen one-to-one interactions).[3] Instead, you can have multiple parallel work groups for each task. Rather than one twelve-person work group, consider two sixes or even three fours.

Final Thoughts

Jigsaw lessons can be intense. Students are typically highly engaged and there are lots of moving pieces. During class time, the instructor's job is mainly to direct traffic with confidence and then support the groups as they work. In classes in which time is short, specific time limits for each task must be strictly enforced. I ordinarily complete the logic lesson in fifty-minute classes: five minutes for an introduction in which I outline the jigsaw method and briefly introduce the "="; five minutes for the first base groups; fifteen minutes for the work groups; and twenty-five minutes for the second base groups. That last stage takes the most time because each student has to teach their newly mastered skill.

Lastly, remember that students can be unnerved by surprise changes in class structures and expectations. Because jigsaws require so much moving around and interacting, it is useful to alert students in advance. Or, better still, accustom your students to various creative cooperative structures early and often.

Appendix: Sample Worksheet for Work Group "Only"

I. Translation key

b: Berkeley; d: Descartes; h: Hume; k: Kant; l: Locke; n: Nietzsche; s: Spinoza;
Ex: x is an empiricist; Px: x is a philosopher; Rx: x is a rationalist
Lxy: x likes y; Mxy: x is read more widely than y; Pxy: x plays billiards with y; Rxy: x respects y

II. Use these five examples with the translation key above to teach yourself and your work group how to work with "only" sentences.

1 Nietzsche respects Spinoza
Rns

2 Nietzsche respects only Spinoza
$Rns \bullet (\forall x)(Rnx \supset x{=}s)$

3 Only Nietzsche doesn't like Nietzsche.

$\sim$Lnn • $(\forall x)(\sim Lxn \supset x{=}n)$

4 Only Locke plays billiards with some rationalist who is read more widely than Descartes.

$(\exists x)(Rx \bullet Mxd \bullet Plx) \bullet (\forall x)[(Rx \bullet Mxd) \supset (\forall y)(Pyx \supset y{=}l)]$

5 Only Kant is read more widely than Descartes and Hume.

$Mkd \bullet Mkh \bullet (\forall x)[(Mxd \bullet Mxh) \supset x{=}k]$

III. While still in your work groups, determine how to translate these two sentences. After moving back toyour base groups, you can use these, along with the five in II, to teach the rest of your group.

6 Nietzsche is the only philosopher read more widely than Descartes.

7 Kant is the only empiricist who is also a rationalist.

IV. If you have time in class, or after class if you do not, make sure that you can translate this sentencesuccessfully.

8 Only Locke and Berkeley are empiricist philosophers respected by some rationalist philosopher.

Solutions to 6–8 are shared with students after class.

6 $Pn \bullet Mnd \bullet (\forall x)[(Px \bullet Mxd) \supset x{=}n]\}$

7 $Ek \bullet Rk \bullet (\forall x)[(Ex \bullet Rx) \supset x{=}k]$

8 $El \bullet Pl \bullet (\exists x)(Rx \bullet Px \bullet Rxl) \bullet Eb \bullet Pb \bullet (\exists x)(Rx \bullet Px \bullet Rxb) \bullet (\forall x)$
$\{[Ex \bullet Px \bullet (\exists y)(Ry \bullet Py \bullet Ryx)] \supset (x{=}l \vee x{=}b)\}$

Notes

1. For evidence of the effectiveness of active learning strategies, see Johnson, Johnson, and Smith 1998 and Freeman et al 2014. For the importance of interdependence, see Johnson, Johnson, and Smith 2014. See Choe and Drennan 2001: 330; and Morgan et al. 2008 for striking data on undergraduate and graduate students' enjoyment of jigsaw lessons. Morgan et al. 2008 describe some concerns, especially for stronger students, though Aronson et al. 1978 report that high-achieving elementary-school students in jigsaw classrooms suffer no reduction in performance, p 118,

and that enjoyment of school is improved, p 120. Most research on jigsaw lessons focuses on elementary through high school classes. Slavin 1995: 33–5 provides some data on achievement in jigsaw classrooms for younger students. There is less evidence of its use at the undergraduate or graduate level, outside of education or psychology departments, as in Perkins and Saris 2001. See Honeychurch 2012 for a report of successful uses in philosophy. For jigsaws in anthropology, biology, chemistry, geology, history, literature, and sociology classes, see: Choe and Drennan 2001; Resor 2008; and, especially, Mills and Cottell 1998. See also Johnson, Johnson, and Stanne 2000 for data on the success of various cooperative learning techniques, including the jigsaw.

2. For more on cooperative learning group assignments, see Marcus 2010.
3. See Cooper 1990 for discussion of group sizes.

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