International Journal of Philosophical Studies, 2013
Vol. 21, No. 2, 163–183, http://dx.doi.org/10.1080/09672559.2012.727011

Intrinsic Explanation and Field's Dispensabilist Strategy

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Abstract

Hartry Field defended the importance of his nominalist reformulation of Newtonian Gravitational Theory, as a response to the indispensability argument, on the basis of a general principle of intrinsic explanation. In this paper, I argue that this principle is not sufficiently defensible, and can not do the work for which Field uses it. I argue first that the model for Field's reformulation, Hilbert's axiomatization of Euclidean geometry, can be understood without appealing to the principle. Second, I argue that our desires to unify our theories and explanations undermines Field's principle. Third, the claim that extrinsic theories seem like magic is, in this case, really just a demand for an account of the applications of mathematics in science. Finally, even if we were to accept the principle, it would not favor the fictionalism that motivates Field's argument, since the indispensabilist's mathematical objects are actually intrinsic to scientific theory.

Keywords: philosophy of mathematics; indispensability argument; Hartry Field; intrinsic explanation

1. Overview

Quine argued that we should believe that mathematical objects exist because of their indispensable uses in scientific theory. Hartry Field rejects Quine's argument, arguing that we can reformulate science without referring to mathematical objects. Field provided a precedental reformulation of Newtonian Gravitational Theory (NGT) which has been refined, improved, and extended in the years since his original monograph. In this paper, I argue that Field's impressive construction and its extension do not impugn Quine's argument in the way that Field alleges that they do. I do not defend the indispensability argument. I merely attempt to undermine Field's influential line of criticism.

Field's reformulation of NGT is simply a formal construction. Field argues for its relevance in a defense of nominalism on the basis of a principle of intrinsic explanation. I argue that this principle is not sufficiently motivated or defensible, and that it can not do the work for which Field uses it. I start with the relevant background, in §2, and a discussion of Field's principle, in §3. In §4–§6, I present and reject Field's arguments for that principle. In §7, I show that even accepting Field's principle would not lead to his nominalist, or fictionalist, conclusion.

2. Quine's Indispensability Argument and the Dispensabilist Response

Quine never presented a detailed indispensability argument, though he alluded to one in many places. I interpret Quine's argument, QI, as follows.

QI1. We should believe the single, holistic theory which best accounts for our sense experience.

QI2. If we believe a theory, we must believe in its ontological commitments.

QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.

QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.

QIC. We should believe that mathematical objects exist.[1]

An instrumentalist who believes that our uses of mathematics in science do not commit us to the existence of mathematical objects, may deny either QI1 or QI2, or both.[2] Regarding QI1, there is some debate over whether we should believe our best theories. Regarding QI2, one might interpret some of a theory's references fictionally. I return briefly to instrumentalist responses to QI in §7 of this paper.

Quine's procedure for determining the ontological commitments of theories, QI3, is less controversial than QI1–2, and may be taken as definitional. Still, instrumentalists who dismiss QI should be prepared to defend alternative criteria for determining their ontological commitments. One alternative to QI3 would be to adopt an eleatic principle on which the ontological commitments of any theory are, approximately, those objects in the causal realm.[3]

Debate over QI has focused mostly on QI4. To oppose QI4, Field provided two synthetic reformulations of NGT, replacing the standard analytic version of the theory, which relies on real numbers and their relations, with theories based on physical geometry. A second-order reformulation replaced quantification over mathematical objects with quantification over space-time points. A first-order reformulation referred instead to space-time regions. There are technical questions about whether Field's reformulations are adequate for NGT. Field mostly ceded the second-order reformulation, due to problems involving incompleteness.[4] The first-order version, using Quine's canonical language, is a more appropriate response to QI anyway. There are also questions about whether analogous strategies are available for other current and future theories.[5] I put these questions aside, for this paper, and suppose that reformulations in the spirit of Field's construction are available for our best theories.

My concern in this paper is whether such reformulations are better theories than standard ones, for the purposes of QI. The superiority of dispensabilist reformulations is important because the indispensability argument relies on the claim, at QI1–2, that we find our ontological commitments in our best theory. Field defends his reformulation on the basis of a principle of intrinsic explanation. I argue that this principle is false, and that the standard theory is preferable to its dispensabilist counterpart. Thus, I reject Field's claim that QI4 is false, not because reference to mathematical objects is ineliminable from science, but because the reformulated theories which eliminate quantification over mathematical objects are not our best theories.

The value of dispensabilist reformulations has been questioned before. Pincock (2007) argues that the standard theory is better confirmed. To construct representation theorems which demonstrate that a reformulation is adequate, the dispensabilist adopts axioms about the physical world and its properties. For example, to measure mass or temperature, Field assumes the existence of spatio-temporal regions or points, and orderings among them, to do the work that connected sets of real numbers do in the standard theory.[6] But, Pincock argues, those assumptions about the physical world are not as well confirmed as the corresponding mathematical axioms and mappings between physical and mathematical structures.

Against Pincock's claim, even if the dispensabilist's axioms are less well confirmed than the mathematical axioms they replace, they may derive some measure of confirmation from their adequacy. Furthermore, the dispensabilist reformulation, eschewing mathematical objects, makes fewer commitments. It is not clear how to balance the virtue of having fewer commitments with the benefit of having a greater degree of confirmation.

Burgess and Rosen (1997) argue that a better theory should be publishable in scientific journals, and adopted by working scientists; since dispensabilist reformulations are not preferred by practicing scientists, they are no better. This is a wrong way to measure the value of a theory. The practicing scientist wants a useful theory to produce and replicate empirical results. The scientist is mainly unconcerned with ontological commitments. While scientists do seek parsimony among the concrete elements of their theories, an alternative formulation of a given theory whose only advantage is the removal of abstract objects is unlikely to be of interest to the readers of a scientific journal. Field's defense of his reformulation correctly emphasizes concerns about ontological commitments. Field shows that it is reasonable, even preferable, to continue using our standard theories, even if we do not really believe that the mathematical objects to which they refer exist. Dismissing concerns about the ontological commitments of our theories, as Burgess and Rosen do, ignores the questions raised by QI about whether scientific theories must quantify over mathematical objects.[7]

Much of the debate over whether dispensabilist reformulations are better than their standard counterparts has focused on their attractiveness. Field uses attractiveness as a criterion for acceptable reformulations in his original work.[8] But, attractiveness is a vague and malleable criterion. One might find a theory attractive based on its strength, simplicity, or explanatory power. It is unclear how to balance such considerations. 'Of course, it is a deep and difficult question how the various attributes that contribute towards a theory's attractiveness ought to be spelled out, and how these attributes are to be independently measured and weighed against each other' (Melia 2000: p. 472).

Mark Colyvan, arguing that the standard theory is more attractive than Field's reformulation, mentions the unification achieved by the standard theory, and its boldness, simplicity, and predictive powers.[9] I believe that Colyvan's argument can be made more precise. This paper pursues and extends Colyvan's argument, criticizing Field's own criterion for attractiveness, his principle of intrinsic explanation. I present a specific explanation of why Field's reformulation is not a better or more attractive theory than the standard one.

3. Intrinsic and Extrinsic Explanations and Theories

Field defends his reformulation by appealing to a general preference for intrinsic explanations over extrinsic ones.

> If in explaining the behaviour of a physical system, one formulates one's explanation in terms of relations between physical things and numbers, then the explanation is what I would call an *extrinsic* one. It is extrinsic because the role of the numbers is simply to serve as labels for some of the features of the physical system: there is no pretence that the properties of the numbers influence the physical system whose behaviour is being explained. (The explanation would be equally extrinsic if it referred to *non-mathematical* entities that served merely as labels...) (Field 1989b: pp. 192–3)

Field uses 'intrinsic' and 'extrinsic' to apply to entities, theories, and explanations. The application to entities is basic, and he classifies explanations and theories depending on the types of objects used. Explanations and theories are intrinsic if they make no demand for extrinsic objects.

According to Field, numbers are extrinsic to physics, while physical objects and space-time regions are intrinsic.[10] Field also applies the intrinsic/extrinsic distinction within mathematics. Numbers are extrinsic to geometry, while line segments and their properties are intrinsic.

> The fact that geometric laws, when formulated in terms of distance, are invariant under multiplication of all distances by a positive constant, but are not invariant under any other transformation of scale, receives a satisfying *explanation* [in Hilbert's geometry]; it is explained by the *intrinsic facts* about physical space, i.e. by the facts about physical space which are laid down without reference to numbers in Hilbert's axioms. (Field 1980: p. 27)

The application of 'intrinsic' within mathematics proper raises several questions about the relationships among mathematical theories. Are real numbers intrinsic or extrinsic to the theory of natural numbers? Are sets extrinsic to category theory? Are topological spaces extrinsic to Euclidean geometry?

Similar questions can be asked purely within empirical science. Are biological or psychological predicates extrinsic to physics? The objects of physics could be considered extrinsic to the special sciences, especially if there are emergent properties in those sciences. In fact, the classification of objects or theories as intrinsic or extrinsic seems suspiciously flexible. Consider how an Aristotelian would deem terrestrial objects as objects extrinsic to theories about planets and stars.

These questions about Field's principle within either mathematics or empirical science should make us wary of the commonsense intrinsic/extrinsic distinction. But, since my goal at this point is just to illustrate Field's distinction, and since my concern in this paper is with the relationship of mathematics to physical theories, I shall put them aside.

Field presumes that mathematical objects do not influence physical systems to classify them as extrinsic to physical theory.

> If, as at first blush appears to be the case, we need to invoke some real numbers … in our explanation of why the moon follows the path that it does, it isn't because we think that the real number plays a role as a *cause* of the moon's moving that way… (Field 1980: p. 43)

Field's preference for intrinsic explanations is a broad methodological principle.

> Extrinsic explanations are often quite useful. But it seems to me that whenever one has an extrinsic explanation, one wants an intrinsic explanation that underlies it; one wants to be able to explain the behavior of the physical system *in terms of the intrinsic features of that system*, without invoking extrinsic entities (whether mathematical or non-mathematical) whose properties are irrelevant to the behavior of the system being explained. If one cannot do this, then it seems rather like magic that the extrinsic explanation works. (Field 1989b: p. 193; see also Field 1980: p. 44 and Field 1989a: pp 18–9)

Call this principle PIE: we should prefer intrinsic explanations over extrinsic ones, when they are available. PIE is supposed to account for Field's preference for synthetic physical theories over analytic ones. PIE also supports Field's argument for substantivalist space-time (since relationalist theories require references to extrinsic real numbers) and explains his hostility to modal reformulations of science (since modal properties are extrinsic to physics).

Field's focus on intrinsic explanations, rather than intrinsic theories, might seem a bit puzzling. His project is clearly a response to Quine's indispensability argument, which is formulated in terms of theories because of Quine's demand that we find our ontological commitments in our best theory. To reformulate the indispensability argument in terms of explanations would force the indispensabilist to argue that we determine our commitments by consulting our best explanations. Though recent work by Colyvan and Alan Baker develops an explanatory indispensability argument, that argument is not Quine's argument, nor is it the argument to which Field is responding. I shall not pursue it here.[11]

If one thinks that scientific explanations are exhausted by the applications of our most austere scientific theories to sets of initial conditions, then there is no significant difference between appeals to explanations and theories. For traditional covering-law analyses of explanation, we need not worry whether Field's principle is made in terms of explanations or theories. Indeed, Field seems to have such a model of explanation in mind.

> What we must do is make a bet on how best to achieve a satisfactory overall view of the place of mathematics in the world … My tentative bet is that we would do better to try to show that the explanatory role of mathematical entities is not what is superficially appears to be; and the most convincing way to do that would be to

> show that there are some fairly general strategies that can be employed to purge theories of all reference to mathematical entities. (Field 1989a: p. 18; see also fn15)

Furthermore, Field says that an explanation is, 'A relatively simple non-*ad hoc* body of principles from which [the phenomena] follow' (Field 1989a: p. 15).

In contrast, one might believe that criteria for good explanations are different from criteria for good theories, especially when theories are used for revealing ontological commitments. One might, say, wish that explanations be perspicuous. If so, one could not prefer Field's reformulation of NGT to the standard theory. Indeed Field's reformulation is imperspicuous, and hardly recognizable as NGT. It would be impossible to use, which is why he attempts to establish that mathematical theories are conservative over nominalist physical ones.[12] If we were to adjust QI to focus on explanation in this sense, a preference for intrinsic explanations could not support a dispensabilist reformulation. I will focus on PIE in the sense that I believe Field intended. In this standard sense, explanatory power is an important theoretical virtue.

4. Field's Motivation for PIE: Hilbert's Intrinsic Geometry

I discern three arguments for PIE in Field's work. There is the 'magic' argument mentioned in the previous section, which I assess in §6. Field also relies on an implicit Okhamist argument, which I consider in §5. Lastly, Field argues that the importance of Hilbert's 1899 reformulation of Euclidean geometry, which inspired Field's project, can be explained by PIE: Hilbert's axiomatization is superior because it is an intrinsic theory.[13] In this section, I argue that we can understand the success of Hilbert's axiomatization without adopting PIE as a general principle which supports Field's reformulation as a best theory for the purposes of QI.

The projects of axiomatizing mathematics in the late nineteenth century were motivated by diverse factors, two of which stand out: the oddities of transfinite set theory, and the development of non-Euclidean geometries.[14] In both cases, traditional mathematical ontology was contentiously extended without obvious inconsistency. Rigor in the form of axiomatic foundations was sought to put the controversial new theories on firm ground.

Since the development of analysis in the seventeenth century, formulations of Euclidean geometry had used real numbers to represent lengths of line segments and triples of real numbers to represent points. Hilbert's new axiomatization referred to regions of geometric space in lieu of real numbers, and used the geometric properties of betweenness, segment congruence, and angle congruence in the way that real numbers, and their ordering, were used in analytic versions. Hilbert constructed representation and uniqueness theorems which assured the adequacy of his so-called synthetic theory.

We can understand why Hilbert would prefer a synthetic geometry over analytic versions without appealing to PIE. Here is what Hilbert says about his motivation:

> I wanted to make it possible to understand those geometrical propositions that I regard as the most important results of geometric inquiries: that the parallel axiom is not a consequence of the other axioms, and similarly Archimedes' axiom, etc. I wanted to answer the question whether it is possible to prove the proposition that in two identical rectangles with an identical base line the sides must also be identical, or whether as in Euclid this proposition is a new postulate. I wanted to make it possible to understand and answer such questions as why the sum of the angles in a triangle is equal to two right angles and how this fact is connected with the parallel axiom … (Frege 1980: pp. 38–9)

Hilbert's account of his motivations make it clear that he wanted to clarify relations within geometry. By relying on geometric relationships to explain geometric phenomena, he avoided worries about the consistency of analysis in addition to the worries about geometry. Hilbert thought that his axiomatization better explained geometric entailments.

We can best interpret Hilbert's motivation as purely mathematical, rather than ontological. He makes no suggestion that his new theory is better for the purposes of revealing ontology, which is how Field uses his formulation of NGT. Hilbert devised his axiomatization well before Hempel's work on scientific explanation, which linked explanation with formal theories, and well before Quine's work on ontological commitment, which linked formal theories with commitments. Furthermore, there are no benefits of parsimony arising from Hilbert's work. He only shows that real numbers are avoidable in the axiomatization of geometry. His project was not intended to eliminate commitments to numbers.

Regarding more general ontological questions, our main worry within mathematics is antinomy, not parsimony. Worries about antinomy within analysis motivated the arithmetization project of Cauchy, Weierstrass, Dedekind, and others, as well as Hilbert's axiomatization of geometry. But, contradictions may be more easily discovered in larger, more comprehensive theories than in smaller, more isolated ones. The superiority of Hilbert's axiomatization for the purposes of revealing geometric relations does not entail its superiority in constructing proofs and discovering contradictions. For a recently well-worn example, consider that

Fermat's theorem, a number-theoretic claim, was proved by mapping formulas to topological spaces, after hundreds of years of more direct, intrinsic attempts to prove it.

The notion of intrinsic explanation within mathematics proper is closely related to what is called purity. 'A pure proof or solution is one which uses only such means as are in some sense *intrinsic to* (a proper understanding of) a theorem proved or a problem solved' (Detlefsen and Arena 2011: p. 1). Many mathematicians prefer pure proofs, in this sense. But impure, extrinsic proofs are not universally denigrated. Detlefsen and Arana describe a controversy between Descartes and Wallis, on one side, and Newton and MacLaurin, on the other, concerning the uses of analytic (algebraic) methods in geometry. Pure, intrinsic methods did not prevail. 'Despite Newton's reservations concerning algebraic methods, mathematicians of the eighteenth century and later generally followed Descartes and Wallis in sanctioning the relatively free use of "impure" algebraic methods in geometry' (Detlefsen and Arena 2011: p. 4).

Purity, or intrinsicness, is a mathematical value, but one among many. Different axiomatizations serve different purposes. For ontological purposes, we are interested in the conjunction of Hilbert's construction with analysis, which maps geometric structures onto those of number theory. We need not invoke a general principle, PIE, to explain the utility of Hilbert's reformulation, and insist that his intrinsic, synthetic theory is superior to the analytic formulation. While Hilbert's axioms emphasize narrow geometric relations, they omit broader, edifying relations between analysis and geometry.

5. Unification and Parsimony

Since we need not appeal to PIE to see the virtues of Hilbert's reformulation, Field's response to QI loses some of its motivation. More importantly, PIE, which seeks to isolate theories and explanations according to their intrinsic elements, conflicts with our general preference to unify theories, revealing connections among diverse disciplines. A comprehensive theory simplifies, by showing how different commitments cohere. As Michael Devitt argues, ontology is not to be found in isolated theories. 'The best ontology will be that of the best unified science' (Devitt 1984: §4.9; see also §7.8).

For examples of the virtues of extrinsic theories within mathematics, consider how the fundamental theorem of calculus bridges geometry and algebra. Algebra could be seen as extrinsic to geometry, but uniting the two theories yields a more comprehensive, and more fruitful, theory.

Also, we can prove more in an extrinsic second-order theory than we can in an incomplete first-order theory, like first-order arithmetic, itself.

The many precedents for unification in science include Newtonian Gravitational Theory unifying Kepler's celestial mechanics with Galileo's terrestrial mechanics, and Maxwell's electrodynamics unifying electrical and magnetic theories with optics. Consider how welcome bridge laws between physics and chemistry or biology would be. Indeed, unification may be central to our notions of scientific explanation. All of the most promising accounts of scientific explanation emphasize unification. Covering laws are preferred if they are broader. Causal accounts seek fewer, more unifying causes. On Kitcher's unificationist account, it is essential to an explanation that it unify a range of disparate phenomena. Kitcher even argues that unification is the underlying principle which the covering-law model was intended to capture.[15]

Of course, many good explanations appeal only to isolated portions of our theories. For example, even if we presume that mental-state predicates are somehow reducible to physical ones, a psychological explanation may not need to appeal to any physical principles. But, such limited explanations do not conflict with our broader presumption toward unification. Even a dualist would have to appreciate the development of bridge laws between cognitive and physical sciences. It would be implausible for Field to reject the general desire for unification, and there is no evidence that he does. But granted that we seek unifying explanations and theories, Field's general principle of intrinsic explanation seems difficult to defend.

One might wonder if the unification of mathematics and physics is a special case which resists a general preference for unification. Such resistance might be supported, say, by the observation that mathematical objects are causally independent of the physical world. Does our preference for unification prevail? Or, do the differences between mathematics and physical science entail that we should prefer scientific theories which eschew extrinsic mathematical objects? In the latter case, our preference for intrinsically isolating physics from mathematics is a specific case, not a corollary of a general principle. Even if one agreed with Field that his reformulation of NGT were a preferable, more attractive theory, that preference would not derive from a general principle of intrinsic explanation.

In the former case, we are left to wonder whether a limited principle of intrinsic explanation, in this particular case, supports a preference for Field's reformulation. Colyvan directly tackled the question of whether Field's reformulation is more attractive than standard NGT. He argued that unifying mathematics and science leads to a preferable theory. 'Mathematics contributes to the unification and boldness of the physical theory in question, and therefore *is* supported by well-recognised principles of scientific theory choice' (Colyvan 2001: p. 81).

Colyvan provides three examples. First, the introduction of complex numbers as missing solutions to quadratic differential equations simplifies mathematics, since we need not wonder why some quadratic equations have only one, or even no, root. It unifies exponential and trigonometric functions, and any scientific theory which uses such functions. Second, Dirac predicted the existence of positrons by relying on the mathematical solutions to his eponymous equation in relativistic quantum mechanics; positrons were not experimentally verified for another five years. The unification of mathematics and physics allowed for faster scientific progress. Lastly, the Lorentz transformations were initially derived as an account of the failure of the Michelson-Morley experiment intended to provide evidence of the ether. Lorentz, who was in the grip of a false scientific theory, nevertheless developed equations which were later derived from a better scientific theory, viz. special relativity. Without the underlying extrinsic mathematics, it is difficult to see how Lorentz could have developed his equations.

Colyvan's examples illustrate how, in the absence of an over-riding principle, being intrinsic is just one among many characteristics to be weighed when evaluating the attractiveness of a theory. The principle of intrinsic explanation seems especially disfavored when applied specifically to the mathematics used in science, i.e. in the specific case on which QI depends. The unification of mathematics with physics yields a simpler and more powerful theory, a point which Field grants by arguing for conservativeness. And, the isolation of scientific theory from mathematics, especially on the basis of a dispensabilist reformulation, denies important relations among mathematical and physical objects. For a simple example, it is a mathematical property of a three-membered set that it has exactly three two-membered subsets. Applying this property, we can account for why we can, with a red marble, a blue marble and a green marble, form exactly three different-looking pairs of marbles.

The ability of a theory to unify disparate phenomena is only one factor among several that we use to evaluate theories. Others include strength, simplicity, fruitfulness, perspicuity, and parsimony. Complete lists are difficult to formulate. Field's argument is that we should include on such a list whether a theory is intrinsic, and I have argued that our desire to unify theories is more important. Still, the specific case at hand, the one for which Field invokes PIE, is whether standard (extrinsic) science is preferable to Field's nominalized (intrinsic) theory. The only obvious theoretical virtue of the nominalized theory is parsimony.[16]

Desire for parsimony proceeds from a principle applicable to the concrete objects posited by our best theories: do not multiply physical entities without good reason. When constructing scientific theories, it is important not to posit more in the world than that which accounts for the phenomena.

It is an open question whether principles of parsimony should apply to the mathematical objects used in standard formulations of scientific theories. In mathematics proper, parsimony is not the most important theoretical virtue. In contrast to the natural scientist, the mathematician explores her/his universe with a desire to multiply entities. In mathematics, it is often a virtue to be plenitudinous, as long as we avoid antimony. Once we have admitted abstracta into our ontology, we do not run out of room. Maddy makes this point especially in regard to set theory.

> If mathematics is to be allowed to expand freely in this way, and if set theory is to play the hoped-for foundational rule, then set theory should not impose any limitations of its own: the set theoretic arena in which mathematics is to be modelled should be as generous as possible; the set theoretic axioms from which mathematical theorems are to be proved should be as powerful and fruitful as possible. Thus, the goal of founding mathematics without encumbering it generates the methodological admonition to MAXIMIZE. (Maddy 1997: pp. 210–11)

Set theorists proudly present discoveries of distinct new cardinals. For another example, Kripke models for modal logic have ameliorated mathematical worries about modality without resolving persistent philosophical worries about possible worlds. The belief that principles of parsimony are applied differently in mathematics is also a basis for Mark Balaguer's plenitudinous platonism on which every consistent set of mathematical axioms truly describes a mathematical universe. Worries about the introduction of new mathematical entities, as with complex numbers, or transfinites, tend to focus mainly on their consistency, or the rigor with which they are introduced.

PIE would reduce the ontology of scientific theories at the expense of perspicuity, explanatory power, fruitfulness, and coherence with other theories. And it is not even clear that the reduced ontology is preferable. Burgess and Rosen argue against reduced mathematical ontology as a theoretic virtue. 'It is at least very difficult to find any unequivocal historical or other evidence of the importance of economy of abstract ontology as a scientific standard for the evaluation of theories' (Burgess and Rosen 1997: p. 206).

The indispensability argument is alluring since it seems to provide a framework on which mathematical nominalists and platonists can agree.[17] Like QI, PIE was intended as a non-question-begging approach to the nominalist/platonist debate. But, the old debate remains.

6. The 'Magic' Argument

I have argued that two of Field's three arguments for PIE (its ability to explain the value of Hilbert's geometry and a general preference for parsimony over unification) are unsuccessful. Lastly, in defending his general principle, Field granted the utility of extrinsic explanations, but argued that they seem like magic if there is no underlying intrinsic explanation presupposed. Field's claim is that explanations of physical phenomena should be possible which refer only to entities which are active in producing those phenomena.

The obvious defense of the general demand for intrinsic theories comes from linking theories with ontological commitment, as Quine does. We want our theories to refer only to relevant objects in order to avoid errant commitments. But if we have the mathematical commitments already, the extrinsic theory involves us in nothing untoward, and simplifies and unifies our theory. We do not want mistakenly to impute causal powers to mathematical objects by using the extrinsic mathematical theory within physics. But, merely noting that mathematical objects are non-spatio-temporal blocks any such confusion.

Thus, the strength of the magic argument depends on whether we have a prior commitment to mathematical objects. The nominalist sees the uses of mathematics in science as magical, since s/he denies the existence of mathematical objects. The platonist sees no magic, only a reasonable demand for an account of the application of mathematics.

Several accounts of the application of mathematics in science, compatible with platonism, have been developed since Field's original monograph. Mark Balaguer (1998) argues that there is nothing magical about the utility of mathematics in physical science, since mathematics provides a theoretical apparatus for all possible physical states of affairs. Pincock (2004) provides an explanation of the applications of mathematics which, though ontologically neutral, is compatible with platonism and so can also undermine Field's magic argument.

Dissent remains among philosophers of mathematics over whether these particular platonist accounts of the applicability of mathematics are successful.[18] Anti-platonist philosophers may see the challenge to provide a platonist account of the applications of mathematics to be insuperable. If they are correct, then Field's magic argument may hold. But, the existence of recent platonist attempts to account for mathematical applications can give the platonist hope. The resolution of this debate is beyond the range of this paper.

More importantly, Field's magic argument is undermined by the very nature of the epistemology supporting QI. Quine's holism entails that mathematical objects are intrinsic to physical systems. The indispensabilist and the dispensabilist are both committed to the intrinsicness of mathematics, as I will discuss in the next section.

7. PIE Does Not Favor Fictionalism

I have presented considerations which favor extrinsic theories over intrinsic ones, and which undermine PIE, and thus deflect Field's criticism of QI. In addition to Field's negative argument against QI4, he presents a positive account of mathematics, which he calls fictionalism. According to fictionalism, mathematical existence claims are false, and mathematical conditionals are vacuously true, if true. In this section, I argue that even if we accept PIE, it does not favor fictionalism. The indispensabilist can accept PIE since s/he should deny that mathematical objects are extrinsic to physical theory.

It is well-known that the indispensabilist has trouble accepting a distinction between abstract and concrete objects. As Charles Parsons notes,

> Although Quine makes some use of very general divisions among objects, such as between 'abstract' and 'concrete', these divisions do not amount to any division of *senses* either of the quantifier or the word 'object'; the latter sort of division would indeed call for a many-sorted quantificational logic rather than the standard one. Moreover, Quine does not distinguish between objects and any more general or different category of 'entities' (such as Frege's *functions*). (Parsons 1986: p. 377)

Furthermore, Quine himself wonders if such distinctions are sustainable.

> [O]dd findings [in quantum mechanics] suggest that the notion of a particle was only a rough conceptual aid, and that nature is better conceived as a distribution of local states over space-time. The points of space-time may be taken as quadruples of numbers, relative to some system of coordinates... We are down to an ontology of pure sets. The state functors remain as irreducibly physical vocabulary, but their arguments and values are pure sets. The ontological contrast between mathematics and nature lapses. (Quine 1986: p. 402; see also Quine 1978; 1960: p. 234; Quine, 1974: p. 88; and Quine, 1969: p. 98)

The indispensabilist's theory is constructed to explain or represent phenomena involving ordinary objects. 'Bodies are assumed, yes; they are the things, first and foremost. Beyond them there is a succession of dwindling analogies' (Quine 1976: p. 9).

As these analogies dwindle, the traditional abstract/concrete distinction blurs, and so does the intrinsic/extrinsic distinction. For Quine, these distinctions must be made within science. But Quine's preferred theory does not support them. All of the indispensabilist's objects are posits of the same monolithic theory, made in the same way, for the same purposes of explaining our sense experience. There is no basis for discrete differences in type, no basis for either an intrinsic/extrinsic distinction or a related abstract/concrete distinction. Call this facet of indispensabilism ontic blur.

Field classified mathematical objects as extrinsic to NGT in part because of their causal isolation from physical ones. Field, though, is clearly thinking of traditional mathematical objects: abstract objects that exist in all possible worlds, and are knowable *a priori*, for example. Mark Balaguer defends a principle of causal isolation (PCI) governing the traditional separation of mathematical and physical objects. But, PCI is off limits to the indispensabilist. In fact, Balaguer notes that ontic blur is definitive of QI. 'The Quine-Putnam argument should be construed as an argument not for platonism or the truth of mathematics but, rather, for the falsity of PCI' (Balaguer 1998: p. 110).

Just as the indispensabilist does not countenance traditional abstract objects, discretely distinct from ordinary objects, any dispensabilist must accept ontic blur. For, the dispensabilist accepts the indispensabilist's terms of debate, including QI1-3, which are the source of the blur. Given blur, Field can not call mathematical elements extrinsic to physical theory. The indispensabilist's mathematical objects are actually intrinsic to the one, holistic best theory.

Field tries to establish that the posits of space-time points differ from posits of mathematical objects in order to admit space-time points as intrinsic to physics. He claims that mathematical objects are supposed to be known *a priori*, while physical space is not (Field 1980: p. 31). But, for the indispensabilist, mathematical objects, like all objects, are known *a posteriori*. A defense of the apriority of our knowledge of mathematical objects would undermine both the dispensabilist and the indispensabilist, making Field's reformulation moot. An apriorist could argue for mathematical knowledge more forcefully, independently of QI.

Field also argues for the difference between the posits of mathematical objects and space-time points on the basis of the richer mathematical ideology (Field 1980: p. 32). But Resnik (1985) develops an impressive amount of mathematics within Field's space-time, the geometry of which corresponds to second-order analysis. Not only do we get addition and multiplication over the reals and the natural numbers, but we can set up a coordinate system, and define ordered n–tuples. We can even avoid the arbitrary choice of points to serve as 0 and 1 by substituting individual variables.

It is difficult to see how Field could deny that numbers are intrinsic to physical theories without turning PIE into some version of an eleatic principle, appealing to the causal isolation of mathematical objects. But, if he is presuming an eleatic principle, it is difficult to see why indispensability holds any sway. The eleatic can just deny Quine's argument in favor of a causal criterion for ontological commitment. The eleatic can be an instrumentalist about a theory's references, and need not reformulate physical theory to avoid commitments to mathematical objects.

Field's dispensabilist ideology and the indispensabilist's quasi-mathematical ideology both apply to intrinsic objects. The traditional platonist can make the extrinsic/intrinsic distinction. But by definition, the traditional platonist has an independent epistemology for mathematics. The dispensabilist reformulation of standard science does not denigrate our beliefs about mathematical objects if they are independently justified.

8. Conclusion

It is difficult to see any value in PIE, as a general principle guiding theory choice. Resnik, reviewing Field's monograph, argues that we can see it at work in economics.

> The Expected Utility Theorem, which underwrites the use of utility functions, establishes that if an agent's preference ordering satisfies certain conditions then it can be represented by a real valued function which is unique up to positive linear transformations. From this it is usually argued that there is no need to presuppose ill understood utilities in accounting for behavior which maximizes expected utility because an account can be given directly in terms of preferences. (Resnik 1983: p. 515)

Resnik says that an intrinsic account, in terms of preferences, is desirable because utilities are ill understood. But, if they were better understood than preferences, then the account would go the other way. If we could order utilities uniquely, while remaining confused about inter- and intra-personal comparisons of preferences, we would seek to explain preferences in terms of utilities. One principle underlying Resnik's preference is that we should explain things we do not understand in terms of things we do understand. Appropriate Ockhamist principles also guide the avoidance of utilities. It is ironic that Resnik uses an example which employs mathematics to characterize the elements we understand. If utilities were as well understood as mathematical theories, then accounts in terms of them would be welcome. PIE is doing no work, here.

I have argued that our preference for unification of theories undermines PIE. A proponent of PIE might complain that once we introduce bridge principles which unify two distinct theories, they are no longer extrinsic to each other, and thus that PIE is not in conflict with unification. Before unification, we have separate theories, and explanations of the principles of one theory in terms of principles of the other would be disfavored. After unification, such explanations would be welcome.

To be slightly more precise, consider (the conjunction of axioms of) two completely independent theories, T_1 and T_2. We could take T_1 to be biology and T_2 to be quantum mechanics; or, we could take T_1 to be ZFC and T_2 to be general relativity. But, assume that T_1 and T_2 are indisputably extrinsic to each other. The theory $T_1 + T_2$ which merely conjoins two sets of axioms is thus an extrinsic theory. Explanation of phenomena governed by the axioms of T_1 in terms of the principles embodied in T_2 would be extrinsic explanations.

Now, consider a set of mapping principles, M, which bridge T_1 and T_2. We can see that Field thinks that $T_1 + T_2 + M$ is also an extrinsic theory by noting that standard (mathematized) physics includes physical axioms, mathematical axioms, and mappings between the two. These mapping principles are precisely at work when we measure the length of a wire in meters, or when we discuss the Hilbert space of an atom.

The defender of PIE who wishes to embrace unification claims that $T_1 + T_2 + M$ is an intrinsic theory, since the bridge principles connect the objects posited by T_1 with the objects posited by T_2. This approach would save PIE. We could all agree that extrinsic explanations, in the sense of explanations that used $T_1 + T_2$ (without M), were magical, and to be disfavored. But, this interpretation of PIE deprives it of all application. For, on this view there would be no extrinsic explanations. We would never appeal to mathematics in physics, or to quantum mechanics in biology, unless we had bridge principles in hand. Any plausible explanation would have to be intrinsic.[19] Unification really is opposed to intrinsic explanation.

We can appreciate both intrinsic and extrinsic theories. The situation is analogous to the relation between classical mathematicians and intuitionists, from a classical perspective. The classical mathematician can appreciate the distinction between constructive and non-constructive proofs, without concluding that only constructive proofs tell us what exists. Similarly, we can appreciate the technical acuity of Field's construction without inferring that there are no mathematical objects.

Philosophers with nominalist predispositions may see PIE as a commonsense principle, and so may have neglected to recognize a gap in Field's argument against QI. There also may be other reasons to reject

QI, or merely to prefer a theory which does not quantify over, or otherwise refer to, mathematical objects. But the principle of intrinsic explanations can not do this work.

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Notes

1 See Quine 1939, 1955, 1958, 1960, 1978, 1980a, 1980b, and 1986. For other versions of the indispensability argument, see Putnam 1975 (the success argument); Resnik 1997: §3.3 (the pragmatic indispensability argument); and Mancosu 2008: §3.2 (the explanatory indispensability argument). I focus on Quine's argument because Field's response is directed at it.
2 See Carnap 1950. For more recent defenses of instrumentalism in response to QI, see Melia 2000; Azzouni 2004; and Leng 2005a.
3 Contemporary discussions of the eleatic principle trace mainly to David Armstrong's work. Armstrong sometimes focuses on causation (see Armstrong 1978b: p. 46), at other times on spatio-temporal location (see Armstrong 1978a: p. 126). Other formulations are found in Oddie 1982: 286; Azzouni 2004: p. 150; and Field 1989a: p. 68.
4 See Shapiro 1983; Field 1989c, 1990.
5 Burgess and Rosen (1997) elegantly collects the slew of reformulation strategies published in the wake of Field's monograph. See especially the construction at §IIA in the spirit of Field's original work. Most reformulations replace mathematical references with modal ones.
6 See Field 1980, Chapter 7. See Field 1989b for his arguments for a substantivalist interpretation of space-time.
7 On Burgess and Rosen's suggestion: 'While entertaining as rhetorical flourishes, such demands leave a serious explanatory gap...' (Pincock 2007: p. 255).
8 See Field 1980: pp. viii, 8 and 41.
9 See Colyvan 1999 and Colyvan 2001: §4.3.
10 Joseph Melia argues that space-time points are actually extrinsic to physical theories; see Melia 1998: pp. 65–7.
11 See Mancosu 2008, §3.2, for a formulation of the explanatory argument, and Baker 2005; Colyvan 2001; Colyvan 2010; and Lyon and Colyvan 2007 for defenses of the argument.
12 If a mathematical theory is conservative over a nominalist physical theory, then we can use the mathematics to facilitate derivations in the physical theory with assurance that we will not derive any unacceptable empirical consequences. The conservativeness of mathematics would assure us that Field's reformulation need have no consequences for working scientists.
13 See Hilbert 1971. On Hilbert's influence, see Field 1980, Chapter 3.
14 Hilbert mentions both in a letter to Frege on his motivation for axiomatizing geometry (Frege 1980: Letter IV/4).
15 See Kitcher 1981: p. 508.
16 The nominalist theory may also be used in an account of the applicability of mathematics in empirical science. But, though the platonist can use any nominalist account just as well, so this does not serve to distinguish the nominalist from the platonist.
17 'On the one hand, the indispensability argument sides with nominalists in avoiding any presupposition that mathematical statements are intrinsically

privileged. On the other hand, the argument sides with Platonists in taking mathematical statements at face value, as making genuine ontological claims … This evenhandedness is an important strength of the indispensability argument …' (Baker 2003: p. 50).

18 See Leng 2005b and Yablo 2005.

19 As an anonymous reviewer notes, Field could object that there is a difference between the kinds of bridge principles involved when T1 and T2 are both scientific theories and when one is scientific and one is mathematical: such principles are causal in the former case and acausal in the latter. In such a response, PIE is doing no work. The difference would be based on an eleatic principle. Such a move, then, would be consistent with the claim of this paper that PIE does not support Field's reformulation of NGT.

References

Armstrong, David (1978a) *Nominalism and Realism: Universals and Scientific Realism*, Volume I, Cambridge: Cambridge University Press.

Armstrong, David (1978b) *A Theory of Universals: Universals and Scientific Realism*, Volume II, Cambridge: Cambridge University Press.

Azzouni, Jody (2004) *Deflating Existential Consequence. A Case for Nominalism*, New York: Oxford University Press.

Baker, Alan (2003) 'The Indispensability Argument and Multiple Foundations for Mathematics', *The Philosophical Quarterly* 53(210): 49–67.

Baker, Alan (2005) 'Are there Genuine Mathematical Explanations of Physical Phenomena?', *Mind* 114: 223–38.

Balaguer, Mark (1998) *Platonism and Anti-Platonism in Mathematics*, New York: Oxford University Press.

Burgess, John and Gideon Rosen (1997) *A Subject with No Object*, Oxford: New York.

Carnap, Rudolph (1950) 'Empiricism, Semantics, and Ontology', in *Meaning and Necessity: A Study in Semantics and Modal Logic*, 2nd ed., Chicago: University of Chicago Press, 1956; Midway reprint, 1988.

Colyvan, Mark (1999) 'Confirmation Theory and Indispensability', *Philosophical Studies* 96: 1–19.

Colyvan, Mark (2001) *The Indispensability of Mathematics*, Oxford University Press.

Colyvan, Mark (2010) 'There's No Easy Road to Nominalism', *Mind* 119(474): 285–306.

Detlefsen, Michael, and Andrew Arana (2011) 'Purity of Methods', *Philosophers' Imprint* 11(2): 1-20.

Devitt, Michael (1984) *Realism and Truth*, 2nd ed. with Afterword, Princeton: Princeton University Press.

Field, Hartry (1980) *Science Without Numbers*, Princeton: Princeton University Press.

Field, Hartry (1989a) *Realism, Mathematics, and Modality*, Oxford: Basil Blackwell.

Field, Hartry (1989b) 'Can We Dispense with Space-Time?', in *Realism, Mathematics, and Modality*, Oxford: Basil Blackwell.

Field, Hartry (1989c) 'On Conservativeness and Incompleteness', in *Realism, Mathematics, and Modality*, Oxford: Basil Blackwell.

Field, Hartry (1990) 'Mathematics Without Truth (A Reply to Maddy)', *Pacific Philosophical Quarterly* 71: 206–22.

Frege, Gottlob (1980) *Philosophical and Mathematical Correspondence*, Chicago: University of Chicago Press.

Hilbert, David (1971) *Foundations of Geometry*, La Salle: The Open Court Press.

Kitcher, Philip (1981) 'Explanatory Unification', *Philosophy of Science* 48(4): 507–31.

Leng, Mary (2005a) 'Mathematical Explanation', in Carlo Cellucci and Donald Gillies (eds) *Mathematical Reasoning and Heuristics*, London: King's College, pp. 167–89.

Leng, Mary (2005b) 'Platonism and Anti-Platonism: Why Worry?', *International Studies in the Philosophy of Science* 19(1): 65–84.

Lyon, Aidan and Mark Colyvan (2007) 'The Explanatory Power of Phase Spaces', *Philosophia Mathematica* 16(2): 227–43.

Maddy, Penelope (1997) *Naturalism in Mathematics*, Oxford: Clarendon Press.

Mancosu, Paolo (2008) 'Explanation in Mathematics', *The Stanford Encyclopedia of Philosophy* (Fall 2008 Edition), Edward N. Zalta (ed.) <http://plato.stanford.edu/archives/fall2008/entries/mathematics-explanation/>.

Melia, Joseph (2000) 'Weaseling Away the Indispensability Argument', *Mind* 109: 455–79.

Oddie, Graham (1982) 'Armstrong on the Eleatic Principle and Abstract Entities', *Philosophical Studies* 41: 285–95.

Parsons, Charles (1986) 'Quine on the Philosophy of Mathematics', in Lewis Edwin Hahn and Paul Arthur Schilpp (eds) *The Philosophy of W.V. Quine*, La Salle: Open Court.

Pincock, Christopher (2004) 'A New Perspective on the Problem of Applying Mathematics', *Philosophia Mathematica* 12(2): 135–61.

Pincock, Christopher (2007) 'A Role for Mathematics in the Physical Sciences', *Nous* 41: 253–75.

Putnam, Hilary (1975) 'Philosophy of Logic', in *Mathematics, Matter, and Method: Philosophical Papers*, Vol. I, Cambridge: Cambridge University Press.

Quine, W. V. (1939) 'A Logistical Approach to the Ontological Problem', in *The Ways of Paradox*, Cambridge: Harvard University Press.

Quine, W. V. (1955) 'Posits and Reality', in *The Ways of Paradox*, Cambridge: Harvard University.

Quine, W. V. (1958) 'Speaking of Objects', in *Ontological Relativity and Other Essays*, New York: Columbia University Press.

Quine, W. V. (1960) *Word & Object*, Cambridge: The MIT Press.

Quine, W. V. (1969) 'Existence and Quantification', in *Ontological Relativity and Other Essays*, New York: Columbia University Press.

Quine, W. V. (1974) *The Roots of Reference*, La Salle: The Open Court Press.

Quine, W. V. (1976) 'Things and Their Place in Theories', in *In Theories and Things*, Cambridge: Harvard University Press.

Quine, W. V. (1978) 'Success and the Limits of Mathematization', in *Theories and Things*, Cambridge: Harvard University Press.

Quine, W. V. (1980a) 'On What There Is', in *From a Logical Point of View*, Cambridge: Harvard University Press.

Quine, W. V. (1980b) 'Two Dogmas of Empiricism', in *From a Logical Point of View*, Cambridge: Harvard University Press.

Quine, W. V. (1986) 'Reply to Charles Parsons', in Lewis Edwin Hahn and Paul Arthur Schilpp (eds) *The Philosophy of W.V. Quine*, La Salle: Open Court.

Resnik, Michael D. (1983) 'Review of Hartry Field's Science without Numbers', *Nous* 17: 514–19.

Resnik Michael, D. (1985) 'Ontology and Logic: Remarks on Hartry Field's Anti-platonist Philosophy of Mathematics', *History and Philosophy of Logic* 6: 191–209.

Resnik, Michael D. (1997) *Mathematics as a Science of Patterns*, Oxford: Oxford University Press.

Shapiro, Stewart (1983) 'Conservativeness and Incompleteness', *The Journal of Philosophy* 80(9): 521–31.

Yablo, Stephen (2005) 'The Myth of the Seven', in M. Kalderon (ed.) *Fictionalism in Metaphysics*, New York: Oxford University Press.