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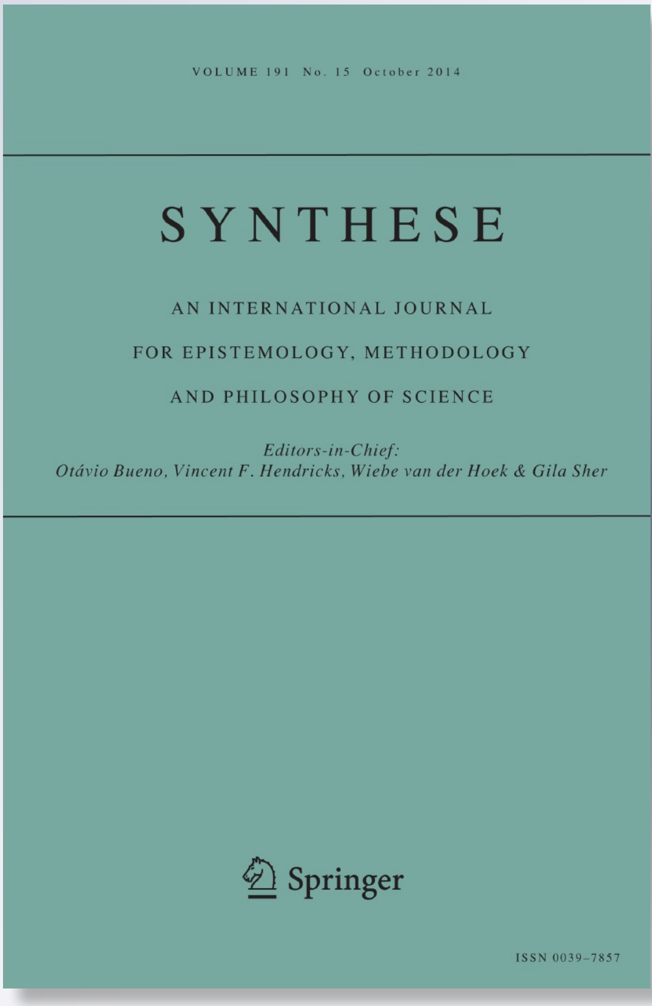

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The holistic presumptions of the indispensability argument

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Abstract The indispensability argument is sometimes seen as weakened by its reliance on a controversial premise of confirmation holism. Recently, some philosophers working on the indispensability argument have developed versions of the argument which, they claim, do not rely on holism. Some of these writers even claim to have strengthened the argument by eliminating the controversial premise. I argue that the apparent removal of holism leaves a lacuna in the argument. Without the holistic premise, or some other premise which facilitates the transfer of evidence to mathematical portions of scientific theories, the argument is implausible.

Keywords Indispensability · Holism · Philosophy of mathematics · Platonism · Explanation

1 The indispensability argument and its central premises

Much recent work in the philosophy of mathematics has focused on the indispensability argument. Roughly, the indispensability argument says that our mathematical beliefs are justified by the uses of mathematics in scientific theories or explanations. The provenance of the argument is a matter of some dispute.[1] Its most important early proponents, Quine and Putnam, never formulated the argument precisely. There is no canonical version. Still, while philosophers formulate it with idiosyncratic attributes,

[1] Mark Colyvan credits Frege and Gödel; see Colyvan (2001): §1.2.1. Garavaso (2005) properly disputes the attribution to Frege. Gödel's indispensability argument is at most an intra-theoretic argument, rather than a standard inter-theoretic argument. (For the difference, see Marcus 2010: § 8).

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there is some agreement about its central theses. Mark Colyvan presents an influential version focused on the core claims.

CIA 1. We ought to have ontological commitment to all and only those entities that are indispensable to our best scientific theories.
2. Mathematical entities are indispensable to our best scientific theories.
Therefore:
3. We ought to have ontological commitment to mathematical entities (Colyvan 2001, p. 11).

In the first premise of CIA, two central presumptions of the argument, holism and naturalism, are implicit: holism is the 'all' portion and naturalism is the 'only' portion. 'Holism' and 'naturalism' are liable to various interpretations. But there is again some consensus about their references, though a greater measure for the former than the latter. Michael Resnik characterizes the theses in a version of the argument explicitly dependent on them:

> *Confirmation Holism*: The observational evidence for a scientific theory bears upon the theoretical apparatus as a whole rather than upon individual component hypotheses.
> *Naturalism*: Natural science is our ultimate arbiter of truth and existence...
> Mathematics is an indispensable component of natural science; so, by holism, whatever evidence we have for science is just as much evidence for the mathematical objects and mathematical principles it presupposes as it is for the rest of its theoretical apparatus; whence, by naturalism, this mathematics is true, and the existence of mathematical objects is as well-grounded as that of the other entities posited by science (Resnik 1997, p. 45).

Some recent writers have attempted to eliminate holism and naturalism from the indispensability argument. Since the rejection of holism in CIA and Resnik's argument renders those arguments unsound, new versions of the argument have to be developed. The result has been a proliferation of various incompatible versions of the indispensability argument.

2 Indispensability without naturalism?

Given the plasticity of 'naturalism', it is possible to formulate indispensability arguments without some forms of it. But some version of naturalism seems essential to motivate the argument or at least to debar autonomous justifications of mathematical beliefs which render the indispensability argument otiose.

At least two interpretations of 'naturalism' conflict precisely at the indispensability argument. On one interpretation, 'naturalism' refers to the preeminence of scientific methods in answering philosophical questions: there is no first philosophy. On another interpretation, 'naturalism' refers to the denigration of any non-natural (i.e. non-physical) elements of one's ontology: all that exists is the spatio-temporal continuum and its denizens. On the first interpretation, we defer ontological questions to scientists. But scientists invoke mathematics ubiquitously and so seem committed to real numbers and Hilbert spaces and such. On the second interpretation, references to abstract objects should be forbidden. Something has to give.

Penelope Maddy notices a different tension among interpretations of 'naturalism' at the indispensability argument.[2] On what she calls Quinean naturalism, the scientific enterprise is preeminent and questions about mathematical truth and existence are answered by looking at uses of mathematics in empirical science. On her own naturalism, in contrast, mathematical practice itself is the ultimate arbiter of mathematical claims. Maddy proposes a reconciliation which need not concern us here, except to say that the result is, perhaps inevitably, something that one could reasonably call naturalistic. Lacking all naturalist constraints, those who wish to justify mathematical beliefs can appeal to non-naturalistic (i.e. traditional rationalistic) justifications which make the indispensability argument unneeded. The argument is supposed to leverage those who are suspicious of abstract entities but who welcome the commitments of science. A non-naturalistic version of the argument is, at best, besides the point.

I will say no more about attempts to develop a version of the argument without appealing to naturalism. Instead, I will focus on the increasingly common but no less challenging task of attempting to develop non-holistic versions of the argument.

3 Indispensability without holism

A brief survey of the growing consensus about the promise of non-holistic versions of the argument is in order.

> Reflecting on [the attitudes of scientists] leads one to another indispensability argument - one that is not subject to the objection [to holism] Maddy and Sober raise and that supports mathematical realism independently of scientific realism (Resnik 1997, p. 46).
>
> As a matter of fact, I think that the argument can be made to stand without confirmational holism: it's just that it is more secure *with* holism (Colyvan 2001, p. 37).
>
> Not all platonists are holists, and it would be useful to have a version of the Indispensability Argument that did not rely so crucially on holism (Baker 2005, p. 224).
>
> Confirmational holism is dispensable to the Quine–Putnam Indispensability Argument; and eliminating this superfluous premise, in addition to strengthening the argument, provides important insight into indispensability arguments in general (Dieveney 2007, p. 106).
>
> Unlike the standard 'Quine–Putnam' argument...these arguments do not invoke confirmational holism. This is an advantage, because some of the most important attacks on the indispensability argument target this premiss (Sober 1993; Maddy 2005). These attacks are no threat to the two arguments just given... The advocate of platonism may find these easier to defend than the confirmational holism the standard argument invokes (Liggins 2008, p. 125).

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[2] See Maddy (1997): §III.4.

> We can still have a proper version of IA and yet consider the part of the Quinean heritage consisting of [naturalism] and [confirmation holism] as dispensable from the argument (Busch and Sereni 2012, p. 351).

While 'naturalism' can refer to a variety of contrasting views, the different interpretations of 'holism' invoked by the indispensability argument lack such tension.[3] The core claim is that justification spreads throughout our belief set so that confirmation or disconfirmation of a particular claim can only be achieved by considering a whole theory: the claim, all background claims, and our inferential apparatus.

Given recent attempts, especially by Elliott Sober, to undermine the indispensability argument by rejecting holism, some proponents of the argument hope to develop a version independent of the controversial premise. But any version of the argument which does not rely on holism is either easily refutable or implicitly relying on holism (or some other premise like it). I'll make a *prima facie* case for that claim and then proceed to the details for each of five proposals.

4 The *Prima Facie* argument

The indispensability argument is a reluctant platonist's argument. The enthusiastic platonist has no need of it because she can appeal directly to mathematical methods as justifying mathematical beliefs: intuitions, say, or Hume's principle, or a brute ability to recognize consistency. The reluctant platonist rightly worries that such methods might themselves require justification.

The reluctant platonist thus looks to empirical science. Our beliefs grounded in empirical scientific theories are our best. If we can use the same methods to justify our mathematical beliefs, then mathematics will be held on grounds as good as our most successful endeavors.

Once our reluctant platonist begins to examine whether our mathematical beliefs are confirmed by the evidence used to support scientific theories, she quickly notices a problem. Mathematical objects are causally inert. They have no spatio-temporal location and no effects on anything that does. They are, in Hartry Field's words, absolutely insular, both brute and barren.[4] The evidence for our scientific theories is essentially sensory. We have theories of physical objects insofar as we have sense experiences of physical objects. Our theories extend to distant and small objects of which we have no (direct) sense experience. But we have explanations of our distance which do not entail that, say, quarks and dark matter are insular. Indeed, we posit such objects in order to explain their constitutive or causal relations with the objects we do experience.

Whatever mathematics we use in our scientific theories, it is difficult to see how evidence can extend to absolutely insular mathematical objects. *Pace* the odd

[3] See Morrison (2010) for recent relevant discussion.

[4] "Let us call a claim brute if its obtaining or not obtaining doesn't depend on anything else; barren if no phenomena from a different domain depend on it; and absolutely insular if it is both brute and barren: (Field 1993, p. 296). Alternatively, see Mark Balaguer's principle of causal isolation which denies causal interactions between mathematical and physical objects: (Balaguer 1998, p. 110).

Pythagorean, trees are not made of numbers or sets. Numbers and sets are not causally efficacious, even in deep space. It is much more likely that the mathematics is a second-class artefact of our formulations of those theories, unworthy of sincere belief.

Moreover, it is not typical practice to think of empirical evidence as extending to mathematical claims, except in one trivial sense. Scientists seeking a mathematical framework for a physical theory, a proper set of differential equations, say, will ordinarily test and discard many mathematical hypotheses. They struggle to find the correct constant (e.g. the fine-structure constant) for an equation or formula. We do not ordinarily understand their rejected hypotheses as evidence against any mathematical theory. Conversely, the adoption of a mathematical theory by scientists is not naturally seen as providing mathematical evidence for that theory, except insofar as (this is the trivial case) mathematicians might seek a model for a theory and the physical world can be a model, given appropriate bridge principles.

Mathematical realism, the claim that our mathematical beliefs are justified, is sometimes framed as object realism, the claim that there are numbers and sets and such, and sometimes as sentence (or truth-value) realism, the claim that some mathematical sentences are non-trivially true. So on either object realism or sentence realism, scientific evidence, the results of experiment and observation and theory construction, seems irrelevant to our mathematical beliefs.

But not if we're holists. Once holism is invoked, it is no longer puzzling how empirical evidence can extend to mathematical claims: all evidence for any portion of our theory is evidence for every portion of our theory. Our evidence for our scientific theories is evidence for their mathematical theorems and objects as much as it is for their physical theorems and objects.

That's the *prima facie* case for the essential inclusion of holism in the indispensability argument. Holism bridges science and mathematics. Unless we introduce holism, or some other premise which does that bridging work, into the argument, empirical evidence does not transfer to mathematical claims.

In the next five sections of this paper, I show in detail how the *prima facie* argument applies to recent attempts to formulate non-holistic versions of the indispensability argument. I show that those arguments are implausible without holism and that they are more plausible with holism. It may be that they can be rescued with another premise, weaker than holism, which similarly facilitates the transfer of evidence from science to mathematics. Given the holistic provenance of the indispensability argument, it might be natural to see these so-called non-holistic arguments as smuggling-in their holism. But one can see them, less contentiously, as merely containing some lacuna.

5 Resnik's pragmatic indispensability argument

Resnik makes an early case for a non-holistic version of the indispensability argument, the pragmatic indispensability argument.

RP RP1. In stating its laws and conducting its derivations, science assumes the existence of many mathematical objects and the truth of much mathematics.

RP2. These assumptions are indispensable to the pursuit of science; moreover, many of the important conclusions drawn from and within science could not be drawn without taking mathematical claims to be true.
RP3. So we are justified in drawing conclusions from and within science only if we are justified in taking the mathematics used in science to be true.
RP4. We are justified in using science to explain and predict.
RP5. The only way we know of using science thus involves drawing conclusions from and within it.
RPC. So we are justified in taking mathematics to be true. (Resnik 1997, pp. 46–48)

The potential benefits of RP are twofold. First, on the standard indispensability argument, our mathematical beliefs are justified only insofar as our scientific beliefs are. Some philosophers, like Nancy Cartwright and Bas van Fraassen, have argued that science, or much of it, is false or idealized. If the justification of our mathematical beliefs is based on the uses of mathematics in scientific theories and scientific theories are not strictly true, then mathematics requires an auxiliary defense. RP might avoid this potential problem.[5] Even if our scientific theories are false, their practical utility still justifies their use. RP states that we should presume the truth of mathematics even if science is merely useful.

Resnik presents RP as a response to criticisms of the indispensability argument by Maddy and Sober. My concern in this paper is not to evaluate holism, so an extended detour into the motivations for the non-holistic argument is unnecessary. But since Sober's opposition to holism motivates all of the arguments I will discuss, a brief digression will be useful.

Sober argues that we subject mathematical claims and empirical claims to different kinds of tests and thus, *pace* holism, do not hold mathematical claims open to confirmation or refutation by empirical evidence. We evaluate a scientific hypothesis against other hypotheses, but we are only able to do this when other hypotheses are available. For example, Sober considers Y1–Y3.

Y1 Space-time is curved.
Y2 Space-time is flat.
Y3 Space-time is not curved, although all evidence will make it appear that it is.[6]

Empirical evidence will discriminate between Y1 and Y2, but no evidence will discriminate between Y1 and Y3. Similarly, no discrimination problem can help us to confirm mathematical claims, since they are assumed by every experiment or observation. The holist's allegation that it is always (in principle) possible to cede any beliefs in light of recalcitrant experience, is (in practice) contradicted by the ways we test our hypotheses; we do not give up our mathematical beliefs when an observation contravenes a theory. "If the mathematical statements M are part of every competing

[5] Resnik's defense of RP precedes Robert Batterman's recent work on asymptotic reasoning (see Batterman 2003) but can also be seen as a way of attempting to avoid concerns which arise from considering the idealizations that Batterman convincingly argues are central to scientific discourse.

[6] Similarly, see Sober (1999, pp. 52–53).

hypothesis, then, no matter which hypothesis comes out best in the light of the observations, M will be part of that best hypothesis. M is not tested by this exercise, but is simply a background assumption common to the hypotheses under test" (Sober 1993, p. 45).

More recently, Sober calls holism bizarre for its consequence that evidence for an empirical theory is supposed to extend to all background beliefs, not just those in mathematics. "If I believe relativity theory, and this theory is confirmed by some observation that I make, then *everything* I believe is also confirmed. To say otherwise is to say that the observation impinges only on part of what I believe; my total system of beliefs then would not have confronted the tribunal of experience as a corporate body (Sober 2005, p. 266).

Sober's argument does not refute holism. The practical isolation of mathematics is consistent with its in-principle interconnectedness with our other beliefs. But if all examples in which we would cede mathematical beliefs are unavoidably abstruse, holism appears suspect.

Let's put Sober's criticisms aside. The question here is whether RP really eschews reliance on holism. To see that it does not, consider RP1, which says that science assumes the truth of much mathematics. Resnik appears to invoke the uses of mathematics in science without the holistic claim that evidence for a scientific theory transfers to an embedded mathematical theory. He is correct that scientists invoke mathematical machinery for inferences, modeling, and measurement. The question remains whether we should believe that the applied mathematics is true. The mere inclusion of mathematical axioms within a scientific theory does not entail that those who use the scientific theory are assuming the truth of the existential quantifications within that theory. The utility of mathematics is not by itself an argument for its truth. We need, further, reason to believe that the inclusion of mathematical axioms within a scientific theory is serious enough to support beliefs in those axioms, their entailments, and (perhaps) the objects to which those axioms refer. Otherwise, we are free to take the mathematics as merely instrumental, for representation and modeling, but not indicative of our most sincere, austere beliefs about truth and ontology. Scientists make idealized assumptions all the time, especially in simplifying deductions and calculations. Such assumptions are made without any commitment to their truth.

The same problem appears in RP2. The scientist may work comfortably without taking mathematical theorems to be true. We need reason to believe that the uses of mathematics are supposed to be serious, entailing justifications of our beliefs in those axioms.

In both cases, holism can neatly supply that reason: our evidence for the scientific theory (or for the utility of the practice of science) transfers to the mathematics used in the theory because evidence for any portion of a theory is evidence for every portion of the theory. But RP was intended as a non-holistic version of the argument and so its proponents can not appeal to holism to support RP1 or RP2. It is possible that a premise weaker than holism, one which focuses specifically on the evidential relations between science and mathematics, could suffice while avoiding Sober's criticisms. Resnik does not provide such a proposal and I do not think that one would be promising in light of the *prima facie* argument. Without some such premise, RP is too weak to support its conclusion. To do their work, scientists merely need mathematics to be useful.

They need not presuppose the truth of any mathematical claims or the existence of mathematical objects.

Resnik rightly notices the compelling questions about why mathematics is useful in science. “[The pragmatic argument] has the fairly limited aim of defending mathematical realism by pointing out that any philosophy of mathematics that does not recognize the truth of classical mathematics must then face the apparently very difficult problem of explaining how mathematics, on their view of it, can be used in science” (Resnik 1997, p. 47).

One appropriate response is the tidy explanation of Field 1980: mathematics is useful because it is a convenient shorthand for complicated statements about physical quantities. An option for the non-indispensabilist realist uses Balaguer’s plenitudinous platonism (or ‘FBP’, from its original name, ‘full-blooded platonism’).[7] FBP takes any consistent mathematical theory to be true and solves the problem of application merely by noting that for all physical situations there is a mathematical theory which applies to it. The problem of application can be solved without justifying mathematical beliefs on the basis of their utility to science.

Colyvan defends Resnik’s invocation of a non-holistic argument despite skepticism about whether a non-holistic version is strong enough to do the indispensabilist’s work.

> This argument has some rather attractive features. For instance, since it doesn’t rely on confirmational holism, it doesn’t require confirmation of any scientific theories in order for belief in mathematical objects to be justified. Indeed, even if all scientific theories were disconfirmed, we would (presumably) still need mathematics to do science, and since doing science is justified we would be justified in believing in mathematical objects. This is clearly a very powerful argument and one with which I have considerable sympathy (Colyvan 2001, p. 15).

Again, it is implausible that mathematical beliefs can be justified by scientific evidence without holism or something like it facilitating the transfer of evidence from science to mathematics. Colyvan sees the virtue of RP as showing that we need not think of evidence as transferring from individual sentences of a scientific theory to other individual sentences. Instead, evidence is supposed to transfer from the practice of science to our mathematical beliefs. Resnik and Colyvan claim that scientific practice is justified, whatever the truth values of its theories, so even if all scientific theories were disconfirmed, our uses of mathematics in scientific practice would still justify our mathematical beliefs.

But if scientific practice were no more justifiable an intellectual pursuit than, say, playing *Call of Duty*, and mathematics were needed only in order to play the game, we would see mathematics as a merely instrumental practice for a merely instrumental pursuit. It is only because we are so thoroughly convinced that our scientific practice is justifiable (even if the laws are not precisely true) that the indispensabilist can suppose that we do not need a premise like holism to transfer justification. If the indispensability argument were the only reason to believe mathematical claims and

[7] See Balaguer (1998, Chap. 7).

the practice of science had the value of, say, astrology, we would have no reason to believe them.

6 Baker's enhanced indispensability argument

Alan Baker introduces his influential explanatory (or enhanced) indispensability argument in part as an attempt to develop a version of the argument that does not rely on holism.[8]

EI (1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
(2) Mathematical objects play an indispensable explanatory role in science.
(3) Hence, we ought rationally to believe in the existence of mathematical objects (Baker 2009, p. 613).

Baker argues robustly for the second premise by describing an explanation of the life-cycle of a variety of cicada: that prime-numbered life-cycles minimize the intersection of cicada life-cycles with those of both predators and other species of cicadas explains why three species of cicadas share a life cycle of thirteen or seventeen years, depending on the environment. Other philosophers provide further examples of mathematical explanations of empirical phenomena.[9] I have no quarrel with that premise. But if the argument is going to meet Baker's goal of providing an indispensability argument which does not rely crucially on holism, the first premise had better not hide holistic presumptions.

But it does, or, again, it contains another implicit premise which does the same work. The viability of EI depends on whether we should take the mathematical references in scientific explanations seriously. We would need to know more about scientific explanation and how and when uses of mathematics improve explanations to evaluate such a claim. Scientific explanation is complicated and there may be no good unary theory of explanation. Still, we can see the problem given any account. On a covering-law account, for example, explanations are performed by austere theories. On such an account, one likely presumed by standard indispensability arguments, we ought to be committed to the theoretical posits made by our explanations because evidence for our theory extends, holistically, to any aspect of the explanation. Such an account, with its holistic presumption, is not available here.[10]

The explanatory indispensability argument is sometimes taken as an improvement on the standard, Quinean argument because it replaces appeals to the holistic confirmation of theories with an appeal to an inference to the best explanation (IBE). On this

[8] Baker has other purposes, including appealing to the explanatory role of mathematics in addition to its representational role. Some critics of the indispensability argument (e.g. Melia 2000; Leng 2010) claim that the merely representational role of mathematics in science is insufficient to justify our mathematical beliefs.

[9] See Lyon and Colyvan (2007) on the honeycomb conjecture; Mancosu (2011) on the twisting tennis racket theorem (among other explanations); and Bangu (2012) on uses of mathematics in economics.

[10] Other accounts of explanation fare worse. As I argue in Marcus (2014), the version of 'explanation' on which Baker must rely in order to distinguish EI from the standard indispensability argument is too weak to support Baker's first premise, whether or not holism is presumed.

interpretation, the argument does not say that our mathematical beliefs are justified by the confirming evidence that supports our beliefs in scientific explanations. Instead, our mathematical beliefs are justified by their appearances in our best explanations.[11] Still, whether we take the basic principles of scientific reasoning to be inductive or abductive is independent of whether we are holists. Explanations, presumably, invoke theories. We require evidence for those theories, reasons which can support the theory more or less strongly. For a holist, such evidence spreads through a theory. For a non-holist, such evidence has more limited scope.

Whatever the proponent of the explanatory indispensability argument says about explanation, and whether we see the structure of scientific reasoning as inductive or IBE, EI must overcome the *prima facie* argument of Sect. 3 of this paper. Without holism or something like it in the background, transferring empirical evidence to mathematical claims, it is utterly implausible that we would take literally a theory including mathematical axioms as a best explanation. If we had an empirical theory entangled with a completely implausible theory, one which invoked angels or demons or magic, we would not see it as a best explainer. We would rewrite the theory to avoid reference to the odd entities, or weasel away our commitments to them, or re-think our opposition to them.

The traditional indispensability argument is buttressed by the claim that we can settle on a best theory and see that mathematical axioms are essential to it. The explanatory argument must rely on a claim that we could settle on a best explanation which includes mathematical content. The lack of a determinate theory of explanation makes this claim less plausible. The proponent of EI must show that mathematical portions of scientific explanations play the same kind of role as the empirical portions. The holist can do so but the non-holist's prospects are dim. For EI to succeed as a non-holistic version of the indispensability argument, Baker needs a non-holistic reason to believe his first premise, to block an instrumentalist interpretation of the mathematics in our explanations, one which has not been provided.

7 Dieveney

While Baker presents EI as a non-holistic argument, it may best be understood holistically. Providing a non-holistic version of the indispensability argument is only one of Baker's goals and EI may retain its interest even if I am right about its relation to holism. In contrast, Patrick Dieveney argues that the indispensability argument is better without holism. "[T]here is a stronger version of the argument...that does not include confirmational holism as a premise" (Dieveney 2007, p. 126).

Unlike many writers on the indispensability argument, Dieveney correctly and explicitly emphasizes the role of Quine's criterion for determining the ontological commitments of a theory. Indeed, Dieveney argues that the invocation of Quine's criterion is what obviates any need for appeals to holism.

11 See Bangu (2012, Chap. 8) and Morrison (2012, p. 276). Morrison finds a lacuna similar to the one I describe in this paper.

> Quine's Criterion is a crucial premise in the Quine-Putnam Indispensability Argument... This combination [of naturalism and Quine's criterion] provides the inferential link between The Indispensability Thesis, i.e., that sentences involving existential quantification over mathematical objects are indispensable to our scientific theories, and our being committed to the existence of mathematical objects. Notice that the above explanation of the role of Quine's Criterion in the indispensability argument eliminates the need for confirmational holism. The Indispensability Thesis together with both Theory Naturalism and Quine's Criterion generate the conclusion that we are committed to the existence of the mathematical objects required to satisfy the existential sentences of our best scientific theories (Dieveney 2007, pp. 108–109).

Dieveney's move is smart. Whether holist or not, once we accept a theory, we have to figure out exactly what it entails for our ontology. Quine's criterion for determining the commitments of a theory, which says that reference is to be found in a theory's existential quantifications, is not the only one available. One alternative is Jody Azzouni's claim that we are only committed to the objects to which we have thick or thin access.[12] A related eleatic principle says that only those objects which are causally effective or those with space-time properties exist. On either of these alternatives, quantification over mathematical entities is no evidence that we should believe that they exist. But, once we adopt a theory which includes mathematical axioms and we invoke Quine's criterion, it seems that we are stuck with commitments to mathematical objects without any holistic presumptions.

But Dieveney's argument is too quick. To see why, we have to unpack the arguments which remain in Dieveney's version of the argument. In particular, we have to ask why we should believe in the existence of all the objects over which a theory quantifies (i.e. the mathematical as well as the concrete). Once we look toward the arguments for his criterion, we find that the link between existential quantification and ontological commitment is not independent of Quine's arguments for holism.[13]

Even as early as 1939, in "Designation and Existence," in which Quine was mainly focused on shifting the ontological burden of a theory from names to quantification, we see discussions of the commitments of a whole language, of expressing all of our commitments in a theory, and of distinguishing between realism and nominalism according to the ways in which languages are formulated. "As a thesis in the philosophy of science, nominalism can be formulated thus: it is possible to set up a nominalistic language in which all of natural science can be expressed. The nominalist, so interpreted, claims that a language adequate to all scientific purposes can be framed in such a way that its variables admit only concrete objects, individuals, as values..." (Quine 1939, pp. 50–51).

Quine's arguments for using first-order quantifiers to express commitments are related to his larger project of finding a canonical language for our best theory. He relies on technical virtues like completeness, definability of logical truth, compactness,

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12 See Azzouni (2004), Chap. 6.

13 Morrison argues that some defenses of naturalism in indispensability arguments depend on holism; see Morrison (2012): § 2.

and the Löwenheim-Skolem theorems. He favors unifying the referential apparatus of a language, rather than having nouns and pronouns do the work. In Quine's view, choosing a language and its referential apparatus is getting at what exists. "The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality" (Quine 1960, p. 161).

Quine's use of, "To be is to be the value of a variable," is connected securely to his holism. It is possible to apply his criterion to various independent theories, given disciplinary boundaries which are incompatible with holism. But then we have to ask which theories we believe. Do we really believe mathematical theories, independent of physical ones? Are there zoological categories or mental states in addition to the objects of physics? Quine debars approaching these questions independently of scientific theorizing. His implementation of the criterion is essentially linked to his holism.

Here is another way to see the point: Quine's criterion, without holism, is compatible with Carnap's earlier views about theory choice. For Carnap, it is a pragmatic, non-factual question whether to adopt mathematical language. From an internal view (whether we rely on existential quantification or not) mathematical theories refer to mathematical objects and we should take them to be so committed. From an external view, we can choose whether or not to adopt mathematical language. Quine denied that Carnap can make such a distinction: we either are committed to the existence of mathematical objects or not, but we can not maintain two separate theories, speaking equivocally. Non-holistic applications of Quine's criterion lead to illicit double-talk: we can affirm the mathematical quantifications within our mathematical theory and deny them from the external perspective.

Dieveney misrepresents Quine by denying that holism is assumed by Quine in the arguments for his criterion. "The justifications for accepting both Quine's Criterion and the Indispensability Thesis do not require any appeal to confirmational holism. Quine's justification for his criterion of ontological commitment offered in his "On What There Is" consists primarily of appeals to ontological parsimony and linguistic intuition" (Dieveney 2007, p. 116, fn 31).

I believe that Dieveney is wrong about Quine's justifications for his criterion. First, while Quine encouraged parsimony in much of his work, his criterion itself is agnostic toward our commitments. It is a tool for unifying our referential apparatus, often in light of prior commitments: we existentially quantify where we believe in objects. We can construct both profligate theories and austere theories with the apparatus of existential quantification.

Second, Dieveney's allusion to linguistic intuition may be a reference to Quine's appeals to the analogy between existential quantification and the natural language 'there is'. That relationship may be intuitive or not. But if we are going to use a formal language as canonical, as Quine insists we do, we should have at least some rough ideas of how to move between artificial languages and natural ones. Our intuitions about these matters, whatever they are, are not an argument for Quine's criterion.

In contrast to Dieveney's interpretation, Quine's real arguments for his criterion are, first, that we need a canonical language to debate what there is and, second, that first-order languages are clear and univocal for the purpose. Quine assumes holism in the first argument by presuming that the theory we use to express our commitments is unary, with evidence spreading through it. Without the holistic transfer of evidence from empirical theories to mathematical ones, the empiricist has no reason to believe the existential quantifications of an isolated, unapplied mathematical theory.

Thus, Dieveney's mere appeal to Quine's criterion combined with his naturalism does not provide us with a non-holistic version of the indispensability argument, let alone a stronger version.

8 Liggins

At the end of his defense of a Harvard-realist interpretation of the indispensability argument[14], David Liggins presents two arguments which he claims eschew appeals to confirmation holism.

L1 (1a) We should believe the measurement claims made by well confirmed scientific theories—for instance, astronomy's claim: 'Saturn has surface area 1.08×10^{12} km^2'.
(2a) If these measurement claims are true, then there are abstract mathematical entities.
(3a) So we should believe that there are abstract mathematical entities.

L2 (1b) We should believe the law-statements that figure in well confirmed scientific theories.
(2b) If these law-statements are true, then there are abstract mathematical entities.
(3b) So we should believe that there are abstract mathematical entities (Liggins 2008, p. 125).

Liggins believes that L1 and L2 are immune to problems of the standard argument's reliance on holism. Again, we have a clear instance of suppression of the holistic premise or something else to do holism's work. In Liggins' cases, premises 2a and 2b must be defended. It is both difficult to see how they can be defended without holism and easy to see how they can be defended with holism.

Consider, as an example of the implausibility of Liggins' inferences, what Joseph Melia calls the nominalist's weaseling strategy against the indispensability argument. Melia claims that we differentiate among different aspects of our discourse. "It is quite common for both scientists and mathematicians to think that their everyday, working theories are only partially true" (Melia 2000, p. 457). Even if mathematical claims are ineliminable from scientific theory, we need not believe the mathematical portions of a theory. The weasel asserts that Saturn has a surface area of 1.08×10^{12} km^2 but denies that the assertion carries with it any commitment to the existence of mathematical objects or to the truth of mathematical claims. In other words, Melia denies the indispensabilist's claim that our ontological commitments are (all of) those objects over which we first-order quantify in our best theories. We can believe our

[14] Liggins' Harvard realism is what I earlier called sentence realism. The Harvard realist takes existentially quantified mathematical sentences to be true without accepting the existence of abstract mathematical objects. Alternative semantics for such sentences include taking mathematical terms to refer to modal properties or to arrangements of physical objects (or to possible such arrangements).

theories, including their measurements (L1) and their law-statements (L2), without believing in the mathematics used to express those measurements or law statements.

Melia argues that scientists use mathematics in order to represent or express facts that are not representable without mathematics. When constructing theories of the physical world, it is sometimes necessary to invoke mathematics. We should not be misled by such invocations into beliefs in mathematical objects. Such representations are not supposed to be ontologically serious. "The mathematics is the necessary scaffolding upon which the bridge must be built. But once the bridge has been built, the scaffolding can be removed" (Melia 2000, p. 469).

Without the implicit holistic premise, the proponent of L1 or L2 has no good response to the weasel who denies premises 2a and 2b. The indispensabilist needs an argument for those premises, a way of caging the weasel. That argument is likely to rely on the claim that evidence for the empirical portions of a measurement or law-statement extends to the mathematical machinery used to express it, i.e. holism. Without such support, L1 and L2 are implausible.

9 Busch and Sereni

Most recently, Jacob Busch and Andrea Sereni propose what they call a minimal indispensability argument, similar to those of Liggins.

BSM (i) We are justified in believing some scientific theories to be true;
(ii) Among them, some are such that some mathematical theories are indispensable to them;
(iii) We are justified in believing true these scientific theories only if we are justified in believing true the mathematical theories that are indispensable to them;
(iv) We are justified in believing true the mathematical theories indispensable to these scientific theories.
(v) We are justified in believing true a mathematical theory only if we are justified in believing the objects it is about to exist;
(vi) We are justified in believing the objects which the indispensable mathematical theories are about to exist (Busch and Sereni 2012, p. 349).

BSM contains no explicit reference to holism or anything like it.[15] Is it a suppressed premise? Consider BSM iii: How could one argue that the evidence for our scientific theory extends to its mathematical elements without holism?

Busch and Sereni propose to avoid appeals to the controversial premise by arguing that we can replace confirmation holism (which they label '[CH]') with an appeal to a concept they call theory contribution: mathematical and theoretical entities enter our

[15] An exegetical aside: Busch and Sereni support their claim that holism is inessential by referring to Putnam's elision of holism: "So far I have been developing an argument for realism roughly along the following lines: quantification over mathematical entities is indispensable for science, both formal and physical, therefore we should accept such quantification; but this commits us to accepting the existence of

theories in the same way.[16] Instead of invoking holism, Busch and Sereni invoke an inference to the best explanation. "Appeal to theory contribution is supposed to make [CH] redundant: if mathematical theory M contributes appropriately to a scientific theory T that counts as a best explainer, we thereby have a justification for M, and there is thus no need to adopt [CH]" (Busch and Sereni 2012, pp. 352–353).

Unfortunately, this approach does not yield a new and non-holistic indispensability argument. As we saw in Sect. 6, we only accept a scientific theory which includes mathematical axioms as a best explainer if evidence for the empirical claims can extend to the mathematical ones. According to the indispensabilist, we should believe all of the aspects of our best theory, all of the various theorems and all of the objects to which they refer, because evidence extends through the theory. Whether we state that argument explicitly, or leave the argument, like BSM, enthymemic, the underlying appeal to holism is indispensable.

Busch and Sereni do not attribute the foregoing argument to Quine, but they develop an argument which they do ascribe to Quine and which they believe also eschews reliance on holism.[17]

> So quite independently of consideration about [CH], Quine produced an argument for believing that mathematical entities exist... Quine's argument works by first pointing out the parity of evidence for believing that ordinary sized objects – posited in our commonsense 'theory' of the world exist, and for believing that molecules – posited in some of our scientific theories about the world – exist. Then it is pointed out that the evidential grounds we have for believing that molecules exist are similar to those for believing that mathematical entities exist. In each case, posits are postulated because of pragmatic and purpose-oriented reasons (Busch and Sereni 2012, p. 356).

Footnote 15 continued
the mathematical entities in question" (Putnam 1971, p. 347; cited at Busch and Sereni 2012, p. 350). Busch and Sereni go on to say: "No mention is made here of naturalism nor holism."
Busch and Sereni are correct that Putnam hints at a non-holistic version of the indispensability argument, one like some of the non-holistic versions here. But the fact that Putnam does not mention holism is no evidence that it is not lurking in the background. Putnam's argument is really a forebear of Resnik's RP and similarly presumes transfer of evidence from science to mathematics.
Similarly, Busch and Sereni find support for their non-holistic version from a thin reading of Quine's work. First, they quote Quine: "Ordinary interpreted scientific discourse is as irredeemably committed to abstract objects – to nations, species, numbers, functions, sets – as it is to apples and other bodies. All these things figure as values of the variables in our overall system of the world. The numbers and functions contribute just as genuinely to physical theory as do hypothetical particles" (Quine 1981, pp. 149–150). Then they claim that holism is absent from the argument: "Even though [holism] might have been a working hypothesis of Quine's throughout his works, there is no explicit mention of it in the quotation above" (Busch and Sereni 2012, p. 351).

16 They ascribe the appeals to theory contribution to Colyvan (2001) and Baker (2009). I see no evidence for the ascription in Colyvan's work, which Busch and Sereni misquote. (Compare Colyvan 2001, p. 37 with Busch and Sereni 2012, p. 348, fn 6).

17 The goal of their paper is to clarify the Quinean roots of the indispensability argument and to argue that what we ordinarily ascribe to Quine is not really his argument. The exegetical point is beyond the range of this paper.

This argument is not independent of holism. Without holism, we lack parity of evidence for our beliefs in ordinary objects, theoretical posits, and abstract mathematical objects. On a non-holistic (or atomistic) view, mathematics is a discipline independent of empirical science. Our evidence for mathematics is independent from our evidence for empirical theories, as independent and dissimilar as we can imagine. Holism unites the evidential grounds.

10 The weasel/separatist

A consistent theme of my responses to non-holistic versions of the indispensability argument is that without holism, the argument is liable to weaseling, which Dieveney calls separation. The weasel/ separatist argues that we can divide our commitments within scientific theory, believing the empirical consequences and disbelieving the purely mathematical ones. Dieveney believes that the indispensabilist can block the weasel with a premise weaker than holism.

First Dieveney argues that the conjunction of naturalism and Quine's criterion for ontological commitment suffice to debar weaseling. "[O]ne cannot consistently accept both Quine's Criterion and Theory Naturalism while still maintaining that our ontological commitments are determined by only part of our scientific theories" (Dieveney 2007, p. 115).

Dieveney's claim seems false unless one uses versions of those premises which hide a holistic assumption. Quine's criterion is merely a method for determining the commitments of a theory; it does not tell us which theories to accept. If holism is not assumed, we can separate our beliefs about mathematical content of our theories from our beliefs about the physical content. There is some difficulty in writing attractive physical theories while not quantifying over mathematical objects. But it is easy enough to produce theories without mathematical quantifications, especially if one does not mind oddly gerrymandered theories.

Quine's naturalism does not block the weasel either. Some versions of 'naturalism' entail that only spatio-temporal or causally active entities exist; on that version of naturalism, any theory which quantifies over mathematical objects is immediately ruled out as one to whose existential quantifications we look for our commitments. From such versions of 'naturalism', weaseling follows quickly.

Even on weaker versions of naturalism, ones on which scientific theories are the ultimate arbiters of ontology, conjoining them to the denial of holism yields disciplinary boundaries and an open question whether to accept mathematical theories as legitimate sources of ontological commitments. We can have naturalism and Quine's criterion and still be open to the falsity of mathematics.

Second, Dieveney accuses the weasel of intellectually dishonest double-talk.

> If the mathematical parts of our scientific theories really are indispensable as The Indispensability Thesis claims, then it is intellectually dishonest to claim they are merely useful fictions.... [P]roponents of the Quine-Putnam Indispensability Argument have missed the fact that this charge alone constitutes a response to the Separation Objection. There is no need to appeal to confirmational holism to charge the Separatist with intellectual dishonesty (Dieveney 2007, p. 115).

The double-talk criticism appears throughout Quine's work, and Putnam's.[18] It is clearly a denial of weaseling. But it is not, by itself, an argument. The weasel recognizes that she is denying a portion of her discourse, taking back at one moment what she affirmed in the prior one. The weasel claims that such discourse is often acceptable and not intellectually dishonest. Holism provides support for the claim that double-talk is intellectually dishonest; without holism, the case is harder to make.

Much has been written on whether weaseling is a legitimate response to the indispensability argument.[19] Certainly, the weasel is in a stronger position if holism turns out to be false and a weaker position if holism turns out to be true. Dieveney argues that one can block the weasel without appealing to holism by denying that we can separate the mathematical and non-mathematical portions of our scientific theories. "[C]ontemporary physics makes it doubtful that there is a principled means of separating scientific theories into these parts. Consequently, one might respond to the Separation Objection by simply denying this presupposition" (Dieveney 2007, p. 114).[20]

It is true that, especially from a holistic perspective, it is difficult to determine which elements of a scientific theory are strictly physical and which are mathematical. Space-time points, probability spaces, and even electromagnetic fields have such thin causal or spatio-temporal properties that one might be tempted to blur the line between physical and mathematical objects in such cases. But there is a quick and easy way for the weasel to deny her commitments to the mathematical portions of a physical theory without pursuing a difficult disentanglement. All the weasel need do is deny the existential quantifications of the purely mathematical portions of a physical theory. These are easy to isolate and may be as simple as the axioms of ZFC (or whatever mathematical theory used) and their entailments.

11 A stronger indispensability argument

The so-called non-holistic versions of the indispensability argument are not what they appear to be. There is a lacuna in all such arguments, one easily filled by holism. The denial of holism weakens the argument so thoroughly that it is ineffective in justifying our mathematical beliefs.

It is understandable that the proponent of the indispensability argument might want to eschew holism, especially considering Sober's objections. But the holism in question is not particularly problematic. As a logical matter, it is undeniable: in response to recalcitrant experience, any statement of a theory may be held true as long as the truth values of others are adjusted to compensate. Sober's argument against holism, that we hold some premises in the background whenever we perform an experiment or an observation, may be practically accurate. We don't bring all of our background beliefs to play in any given observation. Such a practice can be observed without holism being

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18 For examples, see Quine (1948, p. 13) and Putnam (1971, p. 356). Relatedly, see Field (1980, p. 2).

19 See especially Melia (2000), Colyvan (2002), Melia (2002), Leng (2002), Daly and Langford (2010), Melia (2010); and Colyvan (2010).

20 Dieveney puzzlingly cites Resnik (1997, pp. 101–110) in support. Resnik is working within a holistic framework, so his arguments are inapplicable to a non-holistic response to the weasel.

false. If holism is correct within science, the indispensabilist still has a case to make for the extension of evidence from science to mathematics.

A stronger version of the argument will appeal to both naturalism and holism, as QI does.

QI QI1. We should believe only the theory which best accounts for our sense experience (naturalism).
QI2. If we believe a theory, we must believe in all of its ontological commitments (holism).
QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QIC. We should believe that mathematical objects exist.[21]

Science is thoroughly imbued with mathematics: in the process of discovery, in measurement, in the representation of its most basic laws, in the derivation of inferences. Those who believe that a non-holistic indispensability argument can be successful are adopting an easy inference from the utility of mathematics in science to its justification. But there is a long tradition, dating at least to Carnap, of denying that mathematical beliefs are justified by their utility in science. From a pre-Quinean perspective, the idea that our mere uses of mathematics in science should justify our beliefs in abstract objects, or in the truth of mathematical claims, can seem far-fetched.

> Some contemporary nominalists label the admission of variables of abstract types as "Platonism". This is, to say the least, an extremely misleading terminology. It leads to the *absurd consequence*, that the position of everybody who accepts the language of physics with its real number variables (as a language of communication, not merely as a calculus) would be called Platonistic, even if he is a strict empiricist who rejects Platonic metaphysics (Carnap 1950, p. 215, emphasis added).

Carnap can see this easy inference as absurd because one needs a serious argument to support the transfer of evidence from whatever supports our scientific theories to the mathematics they invoke. That serious argument is supplied by whatever supports Quine's holism. Carnap saw what proponents of some recent so-called non-holistic indispensability arguments do not: without holism, or some other premise to facilitate the transfer of evidence, the indispensability argument is ineffectual.

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[21] QI is my version of Quine's argument, one which I think best captures Quine's central intent. See Quine (1939, 1948, 1951, 1955, 1958, 1960, 1978, 1986); also see Marcus (2010) and Marcus forthcoming.

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