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Structuralism, Indispensability, and the Access Problem

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§1: Structuralism and the Problem of Mathematics

The central epistemological problem for mathematics is sometimes called the access problem. The access problem arises from a two-realm view, the supposition that the referents of mathematical singular terms inhabit a realm which is separate from us. The history of philosophy is littered with attempts to solve the access problem, which is just the old question of how concrete humans can have knowledge of abstract mathematical objects. Quine's approach in the philosophy of mathematics, his indispensability argument, dissolves the access problem, though his solution sometimes goes unrecognized, even by those who are sympathetic to his work. This paper shows that the proponent of the indispensability argument can, to a certain extent, deflect criticisms based on the access argument.

One way to try to solve the access problem involves positing a special human ability to learn about abstract objects. This rationalist approach is exemplified by Plato, Descartes, Gödel, and Jerrold Katz, among many others. The central complaints about rationalism involve accusations of mysticism and desperation, and failure of parsimony, both epistemological parsimony and parsimony of the resulting ontology.[1]

A second approach to the access problem denies the supposition that mathematical objects are separate from us. This approach is exemplified by Aristotle and Mill, and by Penelope Maddy's early work. The central complaints about the Aristotelian approach involve the insufficiency of concrete objects as models of mathematical theories.[2]

1 Gödel 1961 discusses a special faculty of mathematical insight. See Katz 1998 for a recent defense of rationalism. Field 1982 accuses the rationalist of mysticism and desperation; §B of Field 1989a is in the same spirit.

2 Maddy 1990 contains her early position; Balaguer 1994 criticizes that position.

Even more radically, one could deny, as Berkeley and Hartry Field do, that we have mathematical knowledge. Calling his position fictionalism, Field agrees that we have logical knowledge, but he denies that there are any mathematical objects to which we need access.[3] Against fictionalism, we seem to have mathematical knowledge, and we can not, with the fictionalist, just wish away the problem.

Quine's approach replaces questions of access with justifications of theories and their corresponding posits. As an example of how Quine's solution may be neglected, consider MacBride 2004's criticism of Michael Resnik's structuralist attempt to solve the access problem.[4] Resnik 1997 argues that our experience with concrete templates leads to knowledge of abstract patterns. He describes how our knowledge of patterns could have been posited by our intellectual ancestors in response to technological and intellectual needs. Resnik's account is a rational reconstruction of our ancestors' postulation of mathematical objects.

MacBride reasonably interprets Resnik's story as an attempt to solve the access problem. I will argue that MacBride is correct that Resnik's quasi-historical account is unhelpful in justifying our knowledge of mathematics. But Resnik's structuralism, depending on an indispensability argument, actually has no access problem. Such criticisms of Quine, and Quineans, based on access problems, are misdirected. I start with a discussion of Quine's argument, and then show that Resnik has no access problem to the objects of his mathematical ontology. At the end of the paper, I highlight a real problem with Quine's approach, which is deeper than either MacBride or Resnik indicates.

§2: Descriptive and Normative Epistemologies

There are at least two kinds of stories that one can tell which may be relevant to mathematical epistemology, indeed any epistemology. The first kind of story describes how we actually acquire our beliefs. The second kind of story attempts to justify or legitimate those beliefs. The

3 Field 1980 and Field 1982 discuss fictionalism. Field 1998 argues that we can account for the apparent objectivity of mathematics on the basis of the objectivity of logic.

4 Structuralism in the philosophy of mathematics is rooted in the argument of Benacerraf 1965 that the axioms of set theory are satisfiable by no unique set of objects. Different sets of objects, all in the same pattern, can satisfy the axioms. Thus, what the axioms define is really a pattern, or a structure.

second kind of story is prescriptive, or normative.[5] Why do I believe what I do, and should I?

Resnik's quasi-historical account of the acquisitions of mathematical beliefs describes how our ancestors might have come to posit abstract objects in response to practical problems. They might have recognized the advantages of idealization, and of recognizing commonalities.

> I hypothesize that using concretely written diagrams to represent and design patterned objects, such as temples, bounded fields, and carts, eventually led our mathematical ancestors to posit geometric objects as *sui generis*. With this giant step behind them it was and has been relatively easy for subsequent mathematicians to enlarge and enrich the structures they knew, and to postulate entirely new ones (Resnik 1997: 5).

Resnik's account is quasi-historical, and not really historical, since it is merely an attempt to describe the ways in which our ancestors could have come to believe in mathematical objects, and not an attempt to describe the ways in which they actually did come to believe in them. Resnik calls his approach postulational, and it is so in two ways. He makes ontological posits, of abstract objects. More relevantly here, Resnik makes epistemological posits of how our ancestors could have come to make those ontological posits. Resnik sees his task as defending these quasi-historical posits by their utility.

Resnik's quasi-historical account describes the ways in which one might come to acquire mathematical beliefs. It is not a prescriptive account. Exactly the same kind of story could be told about the utility of beliefs in Santa Claus or the Olympic gods, but we would not take these stories as justifying beliefs in Santa Claus or Zeus.

Evaluated on its descriptive merits alone, Resnik's quasi-historical story is quite odd. We do not really learn mathematics in history classes, by examining the thoughts or beliefs of those who come before us. Even if we tried to learn what our ancestors believed, we lack sufficient historical evidence of the beliefs of those who originally posited abstracta. Instead, a good descriptive account would rely on stories about our experiences in mathematics classes, or alone in studies with textbooks, failed attempts to solve problems, and late-night epiphanies.

Descriptive stories need not be irrelevant to justification. If you ask how I know that the man over there has three coins in his pocket, I might describe how I saw him place those coins in his pocket. Still, you may ask

5 A third kind of epistemic story involves questions of our ability to know whether claims are true. A fourth might refer to any other conditions we might place on a belief in order to consider it knowledge. These two further kinds of accounts need not concern us here.

further questions, not merely about chocolate coins and holes in pockets (i. e. about whether your perceptions are accurate in this particular case) but about the reliability of sense experience generally, and whether I know I am not now in a dream state. These latter questions concern whether reference to the genesis of a belief can suffice to justify that belief. They are prescriptive questions, and should lead us to the conclusion that descriptive stories are never sufficient for justification, by themselves. To justify any belief, I must supplement the description of how I acquire that belief with an account of why the belief-forming processes to which my descriptive account refers are in fact reliable, or conducive to truth, or whatever standard you may prefer. The insufficiency of descriptive stories is not limited to sense perception. If I believe a mathematical theorem on the basis of an a priori proof, I still must account for the reliability of the proof procedure. Indeed, taking a descriptive story as a sufficient account of knowledge is a kind of genetic fallacy.

Quine, as is well known, urged us to relegate epistemology, naturalized epistemology, to the description of how stimulus leads to science. Epistemology, Quine urges, should be a form of empirical psychology. Jaegwon Kim criticized Quine for eliminating epistemology's normative element. The naturalized epistemologist has a story to tell, but if that story omits a prescriptive justificatory account, it will have to be incomplete. "For epistemology to go out of the business of justification is for it to go out of business" (Kim 1988: 393).

Kim, I think, is correct that we need a normative account, but wrong that Quine fails to provide one. To see that Quine at least implicitly presents a normative epistemology, consider the following well-known facts. First, Quine preferred, where possible, desert landscapes; he would have liked to do without abstract objects if he could. Second, he did not think that he could, in fact, do without mathematical objects and so he admitted extensional ones, i.e. sets, on the basis of what has become known as the indispensability argument.

§3: Indispensability and Access

The indispensability argument says that our knowledge of the abstract objects of mathematics is justified by their ineliminable uses in empirical science.[6]

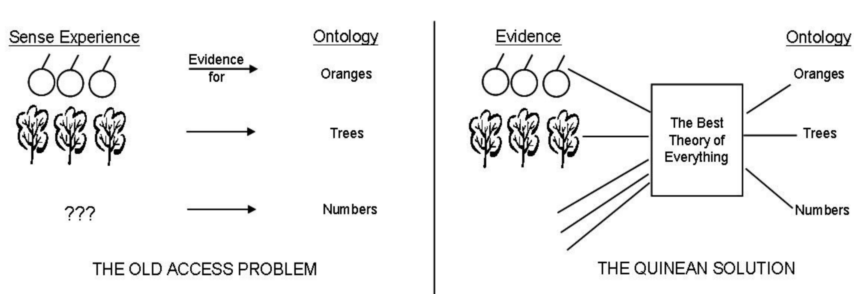
6 Quine nowhere presents a detailed indispensability argument, though he alludes to one in many places. Among them: Quine 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986.

(QI) QI.1: We should believe the (single, holistic) theory which best accounts for our sense experience.
QI.2: If we believe a theory, we must believe in its ontic commitments.
QI.3: The ontic commitments of any theory are the objects over which that theory first-order quantifies.
QI.4: The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QI.C: We should believe that mathematical objects exist.

For the indispensabilist, no independent mathematical epistemology, beyond that for empirical science, is necessary. The indispensability argument dissolves the traditional access problem. According to the confirmation holism which underwrites Quine's indispensability argument, evidence for any claim is spread throughout the entirety of the theory which yields that claim. The indispensabilist needs no account of access to his so-called mathematical objects, since they are posited by a theory which takes only our sense experience as evidence. Even for ordinary objects the question of access is moot. All objects are posits.

To demand access is to demand that a perceiver be able to correlate the objects he or she believes exist with particular perceptions. The traditional empiricist requires lines of access from, say, the tree to my eyes, to my brain, to my beliefs. He or she draws roughly parallel lines to account for beliefs about all objects. The sense-data reductionist demands, as did Hume, a connection to sense experience for every legitimate claim.

Quine denies that a satisfactory account of piecemeal access, like that of the sense-data reductionist, is available. His methodology is a response to the difficulties of describing our access to all objects. Instead, Quine isolates evidence, on one side, and ontology on the other. Between them stands a theory which, as a whole, must be consistent with the evidence. The indispensability argument is merely a corollary of this procedure.



The indispensabilist argues that we posit sets as values of variables bound by the quantifiers of NF, for example, not to account for mathematical facts such as '2 + 2 = 4'. We posit them because we need them to account for the stimulations of our nerve endings by ordinary objects. Instead of an access problem, the indispensabilist has a problem of sufficiency of evidence for the theory.

One might wonder, still, given the two-realm view, if the indispensabilist has solved the access problem. In fact, implicitly, the indispensabilist has discarded the mathematical realm. All posits are equal, in the relevant sense. The mathematical objects in the indispensabilist's ontology need not be deemed apart from us. Since the indispensabilist's so-called mathematical objects are posits of a theory designed to account for our knowledge of ordinary, concrete objects, there is really no need to consider them as abstract. Indeed, the terms 'abstract' and 'concrete' become rather meaningless for the holist, vulgar terms in which the learned may only lightly indulge.

> In the case of abstract entities, certain protests against Platonism become irrelevant. There is no mysterious 'realm' of, say, sets in the sense that they need to have anything akin to location, and our knowledge of them is not based on any mysterious kind of 'seeing' into such a realm. This 'demythologizing' of the existence of abstract entities is one of Quine's important contributions to philosophy… (Parsons 1986: 377–78)

Though he is critical of Quine's method, Stewart Shapiro also relies on indispensability's dissolution of the mathematical realm. "My view is that, extensionally speaking, there is no…philosophically illuminating difference [between mathematical structures and other kinds of structures]." (Shapiro 1983: 542)

One might think that the fact that the indispensability argument dissolves rather than solves the access problem is a problem for the indispensabilist, and I do think this. But others, e.g. Resnik, take Quine's method as something like cutting the Gordian knot. We can, they say, have an abstract ontology with only an empiricist epistemology.

§4: Structuralism and Indispensability

Which brings us back to Resnik and MacBride. Resnik's structuralism relies on Quine's indispensability argument, and he makes no secret of it. In fact, the first third or so of Resnik 1997 is dedicated to a defense of the indispensability argument, which provides Resnik a normative epistemic story. Resnik appeals to the indispensability argument, and its holism, to dissolve the access problem for mathematical objects. The indispensabilist need not describe any access to any particular objects, including those of the purported second (abstract) world.

It is difficult to understand why Resnik presents his quasi-historical account in connection with our posits of structures, since it answers neither the normative nor the descriptive questions. The indispensability argument alone suffices to justify knowledge of its posits, so we do not need the quasi-historical story to provide the normative account. Further, we can not use it to answer the descriptive question because Resnik's postulation inaccurately describes our actual belief-forming processes. Perhaps Resnik thought that the indispensabilist's dissolution is inadequate. It easy to see why MacBride would make the mistake of thinking that the structuralist needs a solution to the access problem, since Resnik seems to think so too.

§5: A Real Access Problem

Lastly, there is another way in which the structuralist does have an access problem. According to the indispensabilist, there is no abstract realm into which one must see in order to know about mathematical objects. Still, the indispensabilist claims justification of our knowledge of mathematical objects.[7] One might think that it is essential to mathematical objects that they have certain properties traditionally ascribed to them. Mathematical objects are traditionally understood to be abstract, to lack spatio-temporal position, to be knowable a priori, and to exist necessarily. If we require that mathematical objects have these traditional properties, then the one-world picture is unsatisfactory. For, none of these properties follow on the indispensabilist's argument.

For example, consider necessity. Mathematical objects are traditionally thought to exist necessarily, perhaps since their existence does not appear to be contingent on any features of the concrete world. While some philosophers, like Quine, have impugned all modalities, necessity has a stubborn resilience. Even if we are convinced that there are deep problems with modalities, my existence seems contingent in a way that the existence of mathematical objects does not.

In contrast, according to the indispensabilist, mathematical objects are posited to account for our experience of a contingent world. If the world were different, it would require different objects. Suppose, for example, that charge really is a continuous property of real particles in

7 Quine, for example, never presented the indispensability argument as dissolving the two-realm picture in the way that Parsons does.

this world. The indispensabilist alleges that the world thus contains continuous functions. Further, suppose that in a different world, or if this world were different, there were no continuous properties. In that alternative world, says the indispensabilist, there are no continuous functions. Similar arguments can easily be constructed for other traditional mathematical properties.

One might argue that the indispensabilist's so-called mathematical objects, lacking so many traditional properties, are not really mathematical objects. They are empirical posits, made to account for the stimulations of our senses, which are merely less tractable than ordinary objects. The indispensabilist's mathematical objects are to electrons, say, as electrons are to tables. Though these so-called mathematical objects are not themselves sensed, they are no different in kind from other posits. Thus, goes the objection, the structuralist who relies on an indispensability argument has no basis on which to claim knowledge of mathematical objects. This is a real access problem, and nothing in structuralism, or the indispensability argument, seems likely to solve, or dissolve, it.

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