A Jigsaw Lesson for First-Order Logic Translations Using Identity

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Abstract:

Jigsaw lessons and classrooms were initially developed in the 1970s by Elliot Aronson in the wake of school desegregation. This article presents instructions and materials for a jigsaw lesson that I have used successfully in symbolic logic classes for translations using the identity particle. The content is substantial, most students enjoy the class, and it is a satisfying and productive experience for the instructor.A Jigsaw Lesson for First-Order Logic Translations Using Identity

Jigsaw lessons and classrooms were initially developed in the 1970s by Elliot Aronson in Austin, Texas, in response to poor performance and low self-esteem of African-American children in the wake of school desegregation. The technique has been widely adapted, and can be used in a wide range of contexts, including philosophy courses. This article presents instructions and materials for a jigsaw lesson for symbolic logic classes. The topic for the lesson is translation using the identity particle. The content is substantial, most students enjoy the class, and it is a satisfying and productive experience for the instructor.[1]

In brief outline, jigsaw lessons are cooperative (or group) lessons in which each student is a member of two distinct groups: a base group and a work group.[2] Students start the lesson in base groups, which are assigned a project with several distinct tasks. Each student then chooses, or is assigned, a task, and moves to a distinct work group where s/he sutdies and tries to master the task. Students then return to their base groups and, in turn, teach all of the other base group members what they have learned.[3] At the end of the jigsaw lesson, each student in each base group has had the opportunity to learn each of the parts of the complete project.

The jigsaw's structure requires both a significant amount of preparation on the instructor's part, and, more importantly, trust on the students' parts that the moving parts will resolve appropriately.

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[1] See Choe and Drennan 2001: 330; and Morgan et al. for striking data on undergraduate and graduate students' enjoyment of jigsaw lessons. Morgan et al. describes some concerns, especially for stronger students. Aronson et al. 1978 reports that high-achieving elementary-school students in jigsaw classrooms suffer no reduction in performance, p 118, and that enjoyment of school is improved, p 120. Slavin 1995: 33-35 provides some data on achievement in jigsaw classrooms for younger students. See also Johnson et al. for data on the success of various cooperative learning techniques, including the jigsaw.

[2] Base groups are sometimes called home groups, jigsaw groups, or cooperative groups; work groups are sometimes called expert groups or counterpart groups.

[3] See Figure 1, which depicts the groups used in Example B, below. Also see Lucas 2000: 220; and [author's article] for diagrams of distributions of students in base and work groups. Kagan 1992, Chapter 8, details a variety of jigsaw structures, with diagrams.

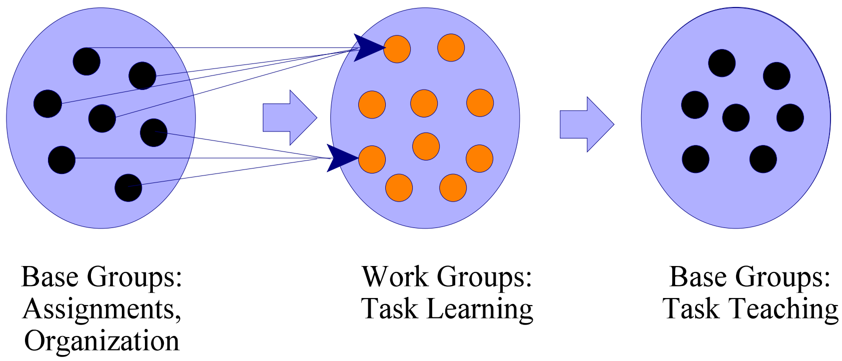

Figure 1: The three steps of the jigsaw lesson. Each member of each base group attends a work group with a different topic, and then returns to his/her original base group. The diagrams depict Example B, below.

Because it requires significant preparation and may demand quick-thinking, especially in cases of student absences, a jigsaw lesson can seem more theoretically interesting than practically useful. Additionally, while such lessons are ideal for small, content-delivery tasks, they are less obviously useful in an undergraduate setting.[4] Jigsaw lessons require classes with predictable attendance, and four or five distinct topics, roughly equal in difficulty. These topics must be crafted so that students can both learn them quickly in a small group and subsequently teach them to their peers in a different group. The topics must be substantial enough to justify the use of class time, yet not too difficult for the students to master quickly without extensive help from the instructor. The instructor must prepare materials to facilitate the students' learning the chosen topics and s/he must master the jigsaw's organizational structure, including group assignments.

I proceed to describe my logic lesson. Since group assignments can be daunting, I will discuss assignments for two different class sizes. In example A, a class with 25 students, the numbers work out

[4] Most of the research on jigsaw lessons focuses on elementary through high school classes. There is less evidence of its use at the undergraduate or graduate level, outside of education or psychology departments, as in Perkins and Saris 2001. Though, for discussions of its use in anthropology, biology, chemistry, geology, history, literature, and sociology classes, see: Choe and Drennan 2001; Resor 2008; and, especially, Mills and Cottell 1998.

neatly. Example B is a class with 37 students. After describing both examples, I discuss some potential difficulties that can arise in forming groups, and a neat trick for assigning students to groups.

In Example A, base groups and work groups all have five students. First, the class must be divided into five base groups. Each student in each base group chooses one of five different tasks of regimenting different kinds of natural-language sentences involving identity:

1. Sentences using ‘only’;
2. Sentences using ‘except’;
3 Superlatives;
4. ‘At most’ sentences; and
5. ‘At least’ sentences.[5]

In each base group, each person chooses a different topic, so that each topic will be learned by a member of the group. The students must also familiarize themselves with their base group members; once they have mastered their specific task, they will return to these groups and teach their task to the other four members of the base group.

To form work groups, we re-shuffle the class. All of the students who chose to work on sentences using ‘only’ form one work group, all of the students who chose to work on sentences using ‘except’ form a second work group, etc. The instructor should prepare, in advance, enough worksheets both for the members of the work groups and for all the members of the base groups to which they will return; i.e. enough for the whole class. Each work group gets copies of its task, a worksheet, and begins to work.[6]

The worksheets each have five sample English sentences and corresponding regimentations in

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[5] Topics 4 and 5 are the simplest topics, and may be easily combined if the instructor prefers to work with four work groups. Note that missing from this list are two key topics: translations using ‘exactly’ and Russell’s theory of definite descriptions. I discuss these further topics with my students either after the group part of the lesson is completed or in the next class.

[6] See Appendix 1 for copies of the task worksheets and Appendix 2 for solutions to the additional problems on each of the task sheets.

first-order logic. They also have three additional English sentences with no corresponding regimentations. Students in the work groups must learn from the samples and regiment the additional sentences. Ideally, and the instructor must emphasize this fact, each student learns his/her small task well enough to teach it to the other members of the base group to which he/she will return. As always with group work, the instructor should circulate among the groups, facilitating interactions and answering specific questions where appropriate. Work groups take ten to fifteen minutes to complete their tasks.

Once the work groups have finished their tasks, the students all return to their original base groups, taking enough worksheets for the rest of the members of the group. Each member of the base group has now become an expert on a different task. In turn, they teach their tasks to each of the other members of the group. After twenty-to-twenty-five minutes, each member of the group has had a chance to learn each of the five tasks, and has a copy of each of the worksheets.

Time permitting and instructor willing, the groups can dissolve and, as a whole, discuss the lesson.

In Example B, the structure of the lesson remains the same, but the group sizes and number of groups change. The sizes of base groups in jigsaw lessons are determined by the number of tasks. In this lesson, since there are five tasks, base groups ideally will have five members each. Since our class size of thirty-seven is not divisible by five, we will form five base groups of five and two base groups of six. In the base groups which have six members, two members will choose the same task for their work groups, and share responsibility for teaching their task when they return to base groups. It is preferable to have more members of the base group than numbers of tasks, rather than fewer, so that each base group will have at least one person responsible for each task.

The only other change from Example A to Example B is that we will have ten work groups, each with three or four students,[7] rather than five work groups of seven or eight members. That is, there will be two distinct work groups for each task. This change keeps the work groups small, which is generally preferable.[8]

While the instructor of a jigsaw lesson is focused on the content of the assigned tasks, and on facilitating the organization of the lesson, students in classes where a jigsaw lesson is used, as in any cooperative learning situation, are often also anxious about interpersonal social issues. As much as some of us hope that our classes may be immune to social hierarchies and cliques, that in our classes the students work together and focus entirely on the content, it may not be the case. Cooperative lessons often bring out social complexities, as students are required to interact directly and explicitly with each other. In fact, Aronson developed the jigsaw in order to improve social interactions in recently desegregated schools, attempting to replace a competitive atmosphere with a cooperative one, based on student interdependence. The organization and its social factors were the primary content of the jigsaw at its inception.

While long-term uses of the jigsaw classroom can improve relationships among students, allowing students to choose their own groups for individual lessons reinforces existing social structures. Random group assignments can minimize the deleterious effects of social hierarchies. Random assignments done transparently in class presume and display no preference among students; they level the playing field.

The simplest method for transparent random group assignments is to have the students count-off

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[7] Or five students, if either of the pairs of students from base groups with six members who work together choose work groups that would otherwise have four students.

[8] In general, and in this case, group sizes are best kept small, but not too small. In large groups, individual students are too easily lost or ignored. I would be extremely wary of the effectiveness of groups larger than five. In a group with n members, there are ${}_nC_2$ one-to-one interactions, a number which gets quite large even for small n (e.g. in a group of six students, there are fifteen one-to-one interactions). I would not recommend the jigsaw for fewer than four topics, though it could probably work with three. See Cooper 1990 for discussion of group sizes.

by the number of groups that will be formed. To engage the students with each other further, I have the students form groups using homemade jigsaw puzzles of pictures of logicians.[9] Pictures of many logicians are readily available on the web. For Example B, I printed seventeen pictures on card stock and laminated them. I cut five of the pictures into five pieces, and two of the pictures into six pieces, so that I had thirty-seven pieces. In class, I mixed the pieces into a basket which I had the students pass around, each drawing a piece, while I introduced the lesson. To form base groups, each student merely had to find the other students with matching pieces. Once the students formed base groups, the base group pictures were returned to me.

I used the remaining ten pictures to form the work groups. I cut five of the remaining pictures into three pieces. On the backs of each of the three pieces of each picture I wrote the name of one of the tasks: five pictures, five tasks. I cut the other five pictures into four pieces, and again wrote the name of one task on each piece of each picture. All the pieces of any one picture had the same task name. Re-organizing these thirty-five pieces, I made seven packets of five pieces. In each packet, each piece had a distinct task name. In class, I distributed one packet to each of the seven work groups while they were first meeting. Work groups were then easily formed by students finding the matching pieces from this second round of puzzle pieces.

In classes in which time is short, specific time limits for each task must be strictly enforced. I have completed this lesson in fifty-minute classes with five-to-ten minutes for an introduction in which I outline the jigsaw method, and briefly introduce the '='; five-to-ten minutes for the first base groups; ten-to-fifteen minutes for the work groups; twenty minutes for the second base groups; and perhaps five minutes for reflection at the end.

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[9] The two uses of 'jigsaw' in this lesson are coincidental. Aronson's use of the term indicated that the tasks pursued by each member of the base groups would fit together, forming the larger project.

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Appendix 1: The five work-group work sheets: only, except, superlatives, at least, and at most[10]

Identity Theory Jigsaw Lesson
Work Group: Only

I. Translation key:

a: Andy; d: Dwight; g: Angela; j: Jim; m: Michael; o: the Office; p: Pam; t: Toby
Ax: x is an accountant; Mx: x is a regional manager; Rx: x is a raise; Sx: x is a salesperson
Dxy: x despises y; Ixy: x is in y; Lxy: x loves y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Jim loves Pam.

   Ljp

2. Jim only loves Pam.

   $Ljp \bullet (x)(Ljx \supset x=p)$

3. Only Andy and Dwight love Angela.

   $Lag \bullet Ldg \bullet (x)[Lxa \supset (x=a \vee x=d)]$

4. There is only one accountant in the office.

   $(\exists x)\{Ax \bullet Ixo \bullet (y)[(Ay \bullet Iyo) \supset y=x]\}$

5. Only Michael would give Angela a raise.

   $(\exists x)(Rx \bullet Gmxa) \bullet (x)[Rx \supset (y)(Gyxa \supset y=m)]$

III. Try these, using the key in I.

6. Michael is the only regional manager.

7. There is only one salesperson who despises Toby.

8. Only Dwight and Jim are salespeople in the office.

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[10] The sentences used in the worksheets refer to characters in NBC's *The Office*.

Identity Theory Jigsaw Lesson
Work Group: Except

I. Translation key:

c: Creed; g: Angela; m: Michael; n: Jan; p: Pam; o: the Office; r: Scranton; s: Stanley; t: Toby
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Px: x is a person; Sx: x is a salesperson; Tx: x is a product
Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Pxy: x passed y; Rxy: x resides in y; Sxy: x sells y; Txy: x tolerates y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Everyone loves Pam.

   $(x)(Px \supset Lxp)$

2. Everyone except Angela loves Pam.

   $Pa \bullet \sim Lap \bullet (x)[(Px \bullet x \neq a) \supset Lxp]$

3. Someone likes all employees except Toby.

   $Et \bullet (\exists x)\{Px \bullet \sim Kxt \bullet (y)[(Ey \bullet y \neq t) \supset Kxy]\}$

4. Everyone in the office except Pam resides in Scranton.

   $Pp \bullet Ipo \bullet \sim Vps \bullet (x)[(Px \bullet Ixo \bullet x \neq p) \supset Vxs]$

5. Everyone but Creed passed a drug test.

   $Pc \bullet (x)(Dx \supset \sim Pcx) \bullet (x)[(Px \bullet x \neq c) \supset (\exists y)(Dy \bullet Pxy)]$

III. Try these, using the key in I.

6. All employees are happy except Stanley.

7. No one except Michael tolerates Jan.

8. Some products are sold by all employees except Michael.

Identity Theory Jigsaw Lesson
Work Group: Superlatives

I. Translation key:

c: Creed; d: Dwight; j: Jim; m: Michael; n: Jan; p: Pam; r: the Scranton branch; u: the Utica branch
Ax: x is an accountant; Bx: x is a branch; Ex: x is an employee; Ox: x is an office; Sx: x is a salesperson
Bxy: x is bigger than y; Hxy: x has y; Ixy: x is in y; Mxy: x is smaller than y; Nxy: x is nicer than y; Zxy: x is lazier than y
Nxyz: x is nearer than y to z.

II. Examine the translations below, which use the key in I.

1. Jim is a nicer salesperson than Dwight.

   Sj • Sd • Njd

2. Jim is the nicest salesperson.

   Sj • (x)[(Sx • x≠j) ⊃ Njx]

3. Utica is the smallest branch.

   Bu • (x)[(Bx • x≠u) ⊃ Mux]

4. Creed is the laziest employee in the office.

   Ec • Ico • (x)[(Ex • Ixo • x≠c) ⊃ Zcx]

5. Michael is the employee who has the biggest office.

   Em • (∃x){(Ox • Hmx) • (y){(Ey • y≠m) ⊃ (z)[(Oz • Hyz) ⊃ Bxz]}}

III. Try these, using the key in I.

6. Scranton is the biggest branch.

7. Utica is the nearest branch to the Scranton branch.

8. Some employee is the biggest accountant in the office.

Identity Theory Jigsaw Lesson
Work Group: At Least

I. Translation key:

j: Jim; o: the Office
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Ix: x is in the office
Bxy: x is bigger than y; Ixy: x is in y; Pxy: x passed y; Txy: x tolerates y

II. Examine the translations below, which use the key in I.

1. There is at least one accountant in the office.

$(\exists x)(Ax \bullet Ixo)$

2. There are at least two accountants in the office.

$(\exists x)(\exists y)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet x \neq y)$

3. There are at least three accountants in the office.

$(\exists x)(\exists y)(\exists z)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet Az \bullet Izo \bullet x \neq y \bullet x \neq z \bullet y \neq z)$

4. There are at least two happy employees who tolerate each other.

$(\exists x)(\exists y)(Hx \bullet Ex \bullet Hy \bullet Ey \bullet x \neq y \bullet Txy \bullet Tyx)$

5. At least three accountants passed their drug tests.

$(\exists x)(\exists y)(\exists z)[Ax \bullet Ay \bullet Az \bullet x \neq y \bullet x \neq z \neq y \neq z \bullet (\exists w)(Dw \bullet Pxw) \bullet (\exists w)(Dw \bullet Pyw) \bullet (\exists w)(Dw \bullet Pzw)]$

III. Try these, using the key in I.

6. There are at least two employees bigger than Jim.

7. There are at least three employees bigger than Jim.

8. There are at least four accountants in the office.

Identity Theory Jigsaw Lesson
Work Group: At Most

I. Translation key:

a: Andy; d: Dwight; g: Angela; m: Michael
Ax: x is an accountant; Ex: x is an employee; Mx: x is a regional manager; Px: x is a person
Axy: x is y's assistant; Bxy: x is bigger than y; Hxy: x has y; Ixy: x is in y; Kxy: x likes y

Note: 'At most' statements make no existential commitments.

II. Examine the translations below, which use the key in I.

1. At most one person is Michael's assistant.

$(x)(y)[(Px \bullet Axm \bullet Py \bullet Aym) \supset x=y]$

2. At most two employees are accountants.

$(x)(y)(z)[(Ex \bullet Ax \bullet Ey \bullet Ay \bullet Ez \bullet Az) \supset (x=y \vee x=z \vee y=z)]$

3. At most two people are Michael's assistants.

$(x)(y)(z)[(Px \bullet Axm \bullet Py \bullet Aym \bullet Pz \bullet Azm) \supset (x=y \vee x=z \vee y=z)]$

4. There is at most one accountant in the office bigger than Dwight.

$(x)(y)[(Ax \bullet Ixo \bullet Bxd \bullet Ay \bullet Iyo \bullet Byd) \supset x=y]$

5. At most two regional managers have employees bigger than Andy.

$(x)(y)(z)\{[Mx \bullet (\exists w)(Ew \bullet Hxw \bullet Bwa) \bullet My \bullet (\exists w)(Ew \bullet Hyw \bullet Bwa) \bullet Mz \bullet (\exists w)(Ew \bullet Hzw \bullet Bwa)] \supset (x=y \vee x=z \vee y=z)\}$

III. Try these, using the key in I.

6. There is at most one accountant in the office.

7. There are at most three accountants in the office.

8. Some people like Angela, but at most two.

Appendix 2: Solutions to the 'Try these' examples on each worksheet

Translation key for all problems on all five worksheets:

a: Andy; c: Creed; d: Dwight; g: Angela; j: Jim; m: Michael; n: Jan; o: the Office; p: Pam; r: the Scranton branch; s: Stanley; t: Toby; u: the Utica branch

Ax: x is an accountant; Bx: x is a branch; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Mx: x is a regional manager; Ox: x is an office; Px: x is a person; Rx: x is a raise; Sx: x is a salesperson; Tx: x is a product

Axy: x is y's assistant; Bxy: x is bigger than y; Dxy: x despises y; Fxy: x farms y; Hxy: x has y; Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Mxy: x is smaller than y; Nxy: x is nicer than y; Pxy: x passed y; Rxy: x resides in y; Sxy: x sells y; Txy: x tolerates y; Zxy: x is lazier than y

Gxyz: x would give y to z; Nxyz: x is nearer than y to z.

Only

6. $Mm \bullet (x)(Mx \supset x=m)$
7. $(\exists x)\{Sx \bullet Dxt \bullet (y)[(Sy \bullet Dyt) \supset y=x]\}$
8. $Sd \bullet Ido \bullet Sj \bullet Ijo \bullet (x)[(Sx \bullet Ixo) \supset (x=d \vee x=j)]$

Except

6. $Es \bullet {\sim}Hs \bullet (x)[(Ex \bullet x \neq s) \supset Hs]$
7. $Pm \bullet Tmn \bullet (x)[(Px \bullet x \neq m) \supset {\sim}Txn]$
8. $Em \bullet (\exists x)\{Tx \bullet {\sim}Smx \bullet (y)[(Ey \bullet y \neq m) \supset Syx]\}$

Superlatives

6. $Br \bullet (x)[(Bx \bullet x \neq r) \supset Brx]$
7. $Br \bullet Bu \bullet (x)[(Bx \bullet x \neq u) \supset Nuxs]$
8. $(\exists x)\{Ex \bullet Ixo \bullet Ax \bullet (y)[(Ay \bullet Iyo \bullet y \neq x) \supset Bxy]\}$

At least

6. $(\exists x)(\exists y)(Ex \bullet Ey \bullet x \neq y \bullet Bxj \bullet Byj)$
7. $(\exists x)(\exists y)(\exists z)(Ex \bullet Ey \bullet Ez \bullet Bxj \bullet Byj \bullet Bzj \bullet x \neq y \bullet x \neq z \bullet y \neq z)$
8. $(\exists x)(\exists y)(\exists z)(\exists w)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet Az \bullet Izo \bullet Aw \bullet Iwo \bullet x \neq y \bullet x \neq z \bullet x \neq w \bullet y \neq z \bullet y \neq w \bullet z \neq w)$

At most

6. $(x)(y)[(Ax \bullet Ixo \bullet Ay \bullet Iyo) \supset x=y]$
7. $(x)(y)(z)(w)[(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet Az \bullet Izo \bullet Aw \bullet Iwo) \supset (x=y \vee x=z \vee x=w \vee y=z \vee y=w \vee z=w)]$
8. $(\exists x)(Px \bullet Kxa) \bullet (x)(y)(z)[(Px \bullet Kxa \bullet Py \bullet Kya \bullet Pz \bullet Kza) \supset (x=y \vee x=z \vee y=z)]$