The Explanatory Indispensability Argument

Russell Marcus, Ph.D.

Department of Philosophy, Hamilton College

198 College Hill Road

Clinton NY 13323

rmarcus1@hamilton.edu

(315) 859-4056 (office)

(315) 381-3125 (home)

August 31, 2009

2835 words

Abstract:

A new strain of indispensability argument in the philosophy of mathematics, relying in part on recent work in mathematical explanation, alleges that our mathematical beliefs are justified by their indispensable appearances in scientific explanations. I first distinguish the explanatory argument from the traditional, Quinean indispensability argument, characterizing the sense of 'explanation' on which the new argument depends. I then argue that, given this sense of 'explanation', the premise of the new argument that has received most attention, the claim that there are mathematical explanations of physical phenomena, is actually uncontroversial. In contrast, the argument is weak at a premise which has not been sufficiently defended, the claim that our ontological commitments are found in our explanations. I conclude that the new explanatory indispensability argument is no improvement on the Quinean one.

The thirty-year debate over the indispensability argument, following the appearance of Hartry Field's seminal *Science without Numbers*, appears to be stuck in a deadlock. Field's attempt to rewrite Newtonian Gravitational Theory quantifying over space-time regions rather than real numbers, and John Burgess's later improvements, gave hope to dispensabilists.[1] Further advances, like Mark Balaguer's sketch of a dispensabilist project for quantum mechanics, supported the cause. Additionally, Burgess and Rosen's argument that the lack of dispensabilist projects currently available is weak evidence for their eventual non-existence provided the dispensabilist some solace in the face of difficulties.[2] While it is pretty clear that no neat, first-order theory which eschews all mathematical axioms will suffice for all of current and future science, the dispensabilist has reasonable hope of finding moderately attractive reformulations of large swaths of scientific theory.

Indispensabilists like Mark Colyvan have been both emboldened by the lack of convincing success on the side of the dispensabilists, and eager to fortify the original argument. According to a new explanatory indispensability argument, we should believe in mathematical objects because of their indispensable roles in our scientific explanations. Alan Baker defends the explanatory argument, and we can see versions of it in Colyvan's work.[3] Paolo Mancosu states the argument explicitly:

---

[1] Some terminology, as I am using it: a *platonist* believes in the existence of mathematical objects. An *indispensabilist* is a platonist who justifies his/her belief in mathematical objects by the ineliminable appearance of mathematical objects (in some discourse or other to be specified, e.g. scientific theory, metalogic, mathematical explanation). A *dispensabilist project* is a reformulation of some discourse which eliminates references to mathematical objects. A *dispensabilist* is someone who would believe in the existence of mathematical objects, if they were indispensable (from some discourse or other), but who believes that there can be successful dispensabilist projects (in that discourse). A *nominalist* is someone who does not believe in the existence of mathematical objects.

[2] See Burgess and Rosen 1997: 118.

[3] See Baker 2001: 211; Baker 2005: 224-5; Lyon and Colyvan 2008: 242; and Colyvan 2007: 119-122. Colyvan's work straddles the line between an explanatory indispensability argument, and a traditional indispensability argument which takes explanatory strength as one among several theoretical virtues. I distinguish these two arguments below.

EI EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question (Mancosu 2008: §3.2).

EI differs from the traditional indispensability argument by focusing on the role of mathematics in scientific explanations, rather than in scientific theories. Indeed, the original Quinean indispensability argument relies on the claim, distinct from EI2, that we find our ontological commitments in our best theories.

QI QI1. We should believe the theory which best accounts for our sense experience.
QI2. If we believe a theory, we must believe in its ontological commitments.
QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QIC. We should believe that mathematical objects exist.[4]

The differences between QI and EI may seem, at first, of little importance. If we take the instances of ‘explanations’ in EI to refer to scientific explanations in a standard sense, there is, in fact, no significant difference between a theory and an explanation. For example, a D-N explanation of a phenomenon P proceeds by proffering the laws of a serious theory, combined with appropriate initial conditions, from which a description of P is derived according to standard deductive rules. The theories to which such explanations appeal are ones in which we speak most strictly. Modifications of this traditional model, like Railton’s model of probabilistic explanation, or Kitcher’s unificationist account, work similarly. Kitcher, for example, relies on unifying argument patterns which also answer why-questions by proffering inferences made within a serious theory. Taking explanatory power as one of several theoretical virtues, like simplicity and parsimony, that scientists seek to optimize, Field, and others engaged in the debate, focused on whether our best theories can be reformulated to avoid

[4] See Quines 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986.

mathematical references. This is the question that has led to the current stalemate.

In contrast, proponents of EI think they have found a way to break the deadlock. Even if the nominalist can reformulate scientific theories to avoid mathematical commitments, they argue, the new theories do not have the same explanatory power. "Even if nominalisation via [a dispensabilist construction] is possible, the resulting theory is likely to be less explanatory; there is explanatory power in phase-space formulations of theories, and this explanatory power does not seem recoverable in alternative formulations" (Lyon and Colyvan 2008: 242).

Such claims in favor of EI are non-starters, if the sense of 'explanation' used in EI is the traditional one. First, for reasons I will not discuss, standard covering-law accounts of scientific explanation do not comfortably apply to mathematical explanation.[5] Second, EI must rely on a different notion of explanation if we are to take it as distinct from QI. It is a standard requirement of any dispensabilist project that the mathematized theory be able to derive no further conclusions than the nominalist reformulation.[6] That is the point of Field's attempts to construct representation theorems, and his more general arguments for the conservativeness of mathematics. One just could not have a successful nominalization of a scientific theory with less explanatory power, unless one is using a different sense of 'explanation'.

A plausible alternative sense of 'explanation' that could ground EI involves subjective understanding. Unlike standard scientific theories, dispensabilist reformulations will be imperspicuous, not useful to working scientists. The awkwardness of dispensabilist reformulations is granted by Field

---

[5] Briefly, mathematics contains too many entailments of the requisite type, some of which are clearly not explanatory. In response, some mathematicians and philosophers distinguish between explanatory and non-explanatory proofs. See Mancosu 2001, §1.1 and Mancosu 2008: §5-6.

[6] Or, anyway, no further *important* conclusions. See the exchange between Maddy 1990 and Field 1990, rooted in criticisms found in Shapiro 1983 and rumored to have originated in unpublished work by Saul Kripke. More recently, see Melia 2000 for concerns about the in-principle incompleteness of dispensabilist reformulations.

and other dispensabilists who generally do not suggest that scientists adopt the reformulations. The dispensabilist grants that standard theories are more comprehensible, more explanatory in this alternative sense. So, it is reasonable to expect that dispensabilist reformulations will lose explanatory power, in this sense, while retaining all (or most) of the inferential relations of their corresponding standard theories.

I will take the sense of 'explanation' in which dispensabilist projects, even if successful responses to QI, are not just as explanatory as their standard counterparts to involve subjective understanding, rather than deductive strength. I will call this an epistemic sense of 'explanation'.

The distinction between taking explanation to be a theoretical virtue, and thus working with a traditional Quinean argument, and looking at the indispensability of mathematics for epistemic explanations is subtle, but important. The availability of a dispensabilist reformulation of a standard scientific theory is essential to evaluating QI, but it is completely irrelevant to whether EI succeeds. A dispensabilist reformulation of a standard scientific theory which preserves deductive strength shows QI4 to be false; QI fails. In contrast, such reformulations do not show that we can eliminate mathematical objects from our mathematical explanations (if there are any) of physical phenomena, in the epistemic sense of explanation; EI remains open.[7]

The debate over EI has focused on EI1, on whether there are genuinely mathematical explanations of physical phenomenon. My first goal in this paper was to clarify the sense of

[7] Two qualifications. First, deductive strength is not the only criterion for success of a dispensabilist project. Still, (epistemic) explanatory merit has played an insignificant role in the debate over QI, since the central question has been whether one can eliminate quantification over numbers within a reasonable logical framework. Second, I am presenting EI as an additional option for the platonist, and thus an additional demand on the nominalist, who must show how we can eliminate mathematics from both theories and explanations. Alternatively, we could see the argument as an additional demand on the platonist such that even if dispensabilist constructions do not work, if there are no genuinely mathematical explanations we should withhold commitments to mathematical objects. See Bangu 2008, and Melia 1998: 70. Melia 2002 and Leng 2005: 179, though working with explanation as a theoretical virtue, can also be seen as taking this latter route. My ensuing criticisms of EI are neutral, I believe, between the two views.

'explanation' being used in EI. Having done so, I will argue that the lively discussions of EI1 are mainly irrelevant to the success of EI.

A wide range of interesting examples of purported mathematical explanations of physical phenomena have been discussed, many presented by Mark Colyvan.[8] The Borsuk-Ulam topological theorem explains the existence of two pressure/temperature antipodes in the Earth's atmosphere; that π is transcendent explains why we can not square the circle; Simpson's paradox may help explain the persistence of maladaptive traits like altruism. At face value, such examples provide compelling, if unsurprising, evidence for EI1, especially when 'explanation' is taken in the epistemic sense. In the standard sense, explanations are tied to our most austere, parsimonious theories. We are required to ask whether it is possible to re-describe some of the phenomena or explanations to eliminate their mathematical elements. In contrast, when our goal is an epistemic explanation, we are free to invoke fiction and metaphor. The question of whether such examples can be reformulated to eliminate mathematical objects is precisely what EI is designed to avoid.

Alan Baker's influential cicada example, used to support EI1, has been criticized by Sorin Bangu. Three species of cicadas of the genus Magicicada share a life cycle of either thirteen or seventeen years, depending on the environment. Baker claims that the phenomenon of having prime-numbered life-cycles may be explained thus:

CP
- CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily disadvantageous.
- CP2. Prime periods minimize intersection.
- CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.
- CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.
- CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods (Baker 2005: 233).

[8] See Colyvan 2001: 81-6 and Colyvan 2007: 120-1.

Baker argues that the mathematical explanans, at CP2, supports the "'mixed' biological/ mathematical law" at CP3, which explains the empirical claim CP5. Bangu argues that the explanandum in question at CP5 is, like CP3, a mixed statement, composed of both mathematical and physical facts: a physical phenomenon (the time interval between successive occurrences of cicadas); the concept of a life-cycle period; the number seventeen; and the mathematical property of primeness. The mathematical explanation only explains the mathematical portions of the explanandum. "If the explanandum is the relevance of the primeness of a certain number, since primeness is a mathematical property, it is not surprising that we have to advance a mathematical explanation of its relevance, in terms of specific theorems about prime numbers" (Bangu 2008: 180). Further, it is question-begging to profess ontological commitments to mathematical objects on the basis of their use in mathematics.

Bangu correctly argues that an explanandum with a mathematical element weakens the claim that CP supports EI1. But, his charge of circularity is too strong. CP is, as it stands, a mathematical explanation of a biological fact. Bangu's allegation that CP5 is a mixed fact depends on whether he can analyze, or reformulate, CP5 to separate the mathematical portion. If the mathematical elements of CP5 were inseparable, then we could conclude, with the indispensabilist, that there are essentially mathematical elements of our descriptions of physical phenomena. The mathematical explanations of those elements will thus contribute essentially to our explanations of the phenomenon. If the mathematical elements of the explanandum were truly ineliminable, then we would have reason to believe that the world is essentially as the indispensabilist alleges.

Conversely, if we can, in a dispensabilist spirit, eliminate the mathematical elements of CP, then we can, with Bangu, deny that it provides support to EI. In fact, the elementary uses of numbers in CP are easily excisable. It is only drudgery to remove the adjectival uses of whole numbers in CP4 and CP5. The concept of primeness in CP2 and CP3 requires a bit more machinery, but, as Mary Leng observes, it does not even demand a completed ω-sequence.[9]

Bangu's criticism thus recalls the dialectic between the indispensabilist and the dispensabilist. If we can construct explanations of physical phenomena which eliminate references to mathematical elements, then there are no essentially mathematical explanations of physical phenomena, and EI1 fails. If we can not reformulate our explanations to eliminate references to mathematical objects, then we have support for EI1. But the whole point of introducing EI was to avoid precisely this dispute.

Bangu's argument does not undermine the claim that there are mathematical explanations of physical phenomena, in the sense required by the proponent of EI. By recasting our explanations to remove references to mathematical objects, we trade a satisfying (epistemic) explanation for an austere, parsimonious theory. And the theory we use to specify our ontological commitments may not be most useful when we want to explain facts about the world.

Herein lies the real weakness of EI. EI2 states that we ought to be committed to the theoretical posits postulated by mathematical explanations of physical phenomena. Once we realize that the sense of 'explanation' in question is epistemic, any force that EI2 is supposed to have is lost. Consider whether the following inference should convince someone us there are numbers.

IM I have two mangoes.
Andrés has three different mangoes.
So, together we have five mangoes.

The answer is clearly negative, since such simple uses of arithmetic are easily eliminated.

IN $(\exists x)(\exists y)(Mx \bullet My \bullet Bxm \bullet Bym \bullet x{\neq}y)$
$(\exists x)(\exists y)(\exists z)(Mx \bullet My \bullet Mz \bullet Bxa \bullet Bya \bullet Bza \bullet x{\neq}y \bullet x{\neq}z \bullet y{\neq}z)$
$(x)[(Mx \bullet Bxa) \supset {\sim}Bxm]$
$\therefore (\exists x)(\exists y)(\exists z)(\exists w)(\exists v)(Mx \bullet My \bullet Mz \bullet Mw \bullet Mv \bullet x{\neq}y \bullet x{\neq}z \bullet x{\neq}w \bullet x{\neq}v \bullet y{\neq}z \bullet y{\neq}w \bullet y{\neq}v \bullet z{\neq}w \bullet z{\neq}v \bullet w{\neq}v)$

---

[9] See Leng 2005: 186.

If someone were to present IM in defense of platonism, we would be completely justified in presenting IN as a replacement. Whether we believe in mathematical objects or not, IM is not a good reason for believing in them, since it is just loose talk which does not reflect our serious commitments. If we want to display our actual ontological commitments, we must speak soberly, invoke parsimony, and rewrite our casual sentences. We construct inferences like IN which make it clear that, strictly speaking, the subjects of the given inference are mangoes, not numbers.

Now, consider the question, “Why are there five mangoes here?” A sufficient explanation, in the epistemic sense, is that I brought two and Andrés brought three. That fact is best represented by IM, and only awkwardly demonstrated, if explained at all, by IN. IM is not a complete, best theory of these mangoes, of course. It requires background assumptions about object constancy, and that mangoes do not annihilate each other when, say, there are more than three together. But, it will satisfy any ordinary person, more so than IN. In fact, the only way for IN to have any plausible epistemic explanatory force is for it to be translated back to something like IM.

IM and IN exemplify the two distinct senses of ‘explanation’ I have discussed. IM is a genuinely mathematical explanation, in the epistemic sense, of an empirical phenomenon, but we are not compelled to take its mathematical references seriously. IN is preferable for the purposes of revealing ontological commitments, and is the kind of inference that could be used in a traditional, D-N or related, sense of ‘explanation’, but it contains no mathematical references. There are two morals emphasized by the differences between IM and IN.

Moral 1: We are committed to mathematical objects not by our casual uses of numbers, but only when we are speaking most seriously.

Moral 2: The theory we use to specify our ontological commitments may not be most useful when we want to explain facts about the world, in the epistemic sense of ‘explain’.

Taken together, these two morals present a serious challenge to EI at its second premise. In order to differentiate itself from the traditional indispensability argument, EI needs to rely on an epistemic sense of explanation. But there is little reason to believe that the explanations which facilitate our subjective understanding are ones in which we reveal our ontological commitments by speaking most soberly. Quine's original indispensability argument received essential support from his claim that we find our ontological commitments precisely in our best theories. EI receives no such support, and is thus no improvement on the original argument. The old debate over QI, on which I have taken no position in this paper, remains most salient.

References

Baker, Alan. 2005. "Are there Genuine Mathematical Explanations of Physical Phenomena?" *Mind:* 114: 223-238.

Baker, Alan. 2001. "Mathematics, Indispensability and Scientific Practice." *Erkenntnis* 55: 85-116.

Balaguer, Mark. 1998. *Platonism and Anti-Platonism in Mathematics*. New York: Oxford University Press.

Bangu, Sorin Ioan. 2008. "Inference to the Best Explanation and Mathematical Realism." *Synthese* 160: 13-20.

Burgess, John, and Gideon Rosen. 1997. *A Subject with No Object*. New York: Oxford.

Colyvan, Mark. 2007. "Mathematical Recreation versus Mathematical Knowledge." In Leng, Mary, Alexander Paseau and Michael Potter, eds, *Mathematical Knowledge*, Oxford University Press, 109-122.

Colyvan, Mark. 2002. "Mathematics and Aesthetic Considerations in Science." *Mind* 111: 69-78.

Colyvan, Mark. 2001. *The Indispensability of Mathematics*. Oxford University Press.

Field, Hartry. 1990. "Mathematics Without Truth (A Reply to Maddy)." *Pacific Philosophical Quarterly* 71: 206-222.

Field, Hartry. 1989. *Realism, Mathematics, and Modality*. Oxford: Basil Blackwell.

Field, Hartry. 1980. *Science Without Numbers*. Princeton: Princeton University Press.

Friedman, Michael. 1974. "Explanation and Scientific Understanding." *The Journal of Philosophy* 71.1: 5-19.

Kitcher, Philip. 1981. "Explanatory Unification." *Philosophy of Science* 48.4: 507-531

Leng, Mary. 2005. "Mathematical Explanation." In Cellucci, Carlo and Donald Gillies eds, *Mathematical Reasoning and Heuristics*, King's College Publications, London, 167-189.

Lyon, Aidan and Mark Colyvan. 2008. "The Explanatory Power of Phase Spaces." *Philosophia Mathematica* 16.2: 227-243.

Maddy, Penelope. 1990. "Mathematics and Oliver Twist." *Pacific Philosophical Quarterly* 71: 189-205.

Mancosu, Paolo. 2008. "Explanation in Mathematics." *The Stanford Encyclopedia of Philosophy* (Fall 2008 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2008/entries/mathematics-explanation/>.

Mancosu, Paolo. 2001. "Mathematical Explanation: Problems and Prospects." *Topoi* 20: 97-117.

Melia, Joseph. 2002. "Response to Colyvan." *Mind* 111: 75-79.

Melia, Joseph. 2000. "Weaseling Away the Indispensability Argument." *Mind* 109: 455-479.

Melia, Joseph. 1998. "Field's Programme: Some Interference." *Analysis* 58.2: 63-71.

Quine, W.V. 1986. "Reply to Charles Parsons." In Hahn, Lewis Edwin and Paul Arthur Schilpp, eds. *The Philosophy of W.V. Quine*. La Salle: Open Court, 1986.

Quine, W.V. 1978. "Success and the Limits of Mathematization." In his *Theories and Things*. Cambridge: Harvard University Press, 1981.

Quine, W.V. 1960. *Word & Object*. Cambridge: The MIT Press.

Quine, W.V. 1958. "Speaking of Objects." In his *Ontological Relativity and Other Essays*. New York: Columbia, 1969.

Quine, W.V. 1955. "Posits and Reality." In his *The Ways of Paradox*. Cambridge: Harvard University Press, 1976.

Quine, W.V. 1951. "Two Dogmas of Empiricism." In his *From a Logical Point of View*. Cambridge: Harvard University Press, 1980.

Quine, W.V. 1948. "On What There Is." In his *From a Logical Point of View*. Cambridge: Harvard University Press, 1980.

Quine, W.V. 1939. "A Logistical Approach to the Ontological Problem." In his *The Ways of Paradox*. Cambridge: Harvard University Press, 1976.

Railton, Peter. 1978. "A Deductive-Nomological Model of Probabilistic Explanation." *Philosophy of Science* 45: 206-226.

Shapiro, Stewart. 1983. "Conservativeness and Incompleteness." In Hart, W.D., ed. *The Philosophy of Mathematics*. Oxford: Oxford University Press, 1996.