Brit. J. Phil. Sci. **50** (1999), 431–455

SURVEY ARTICLE

Listening to Fictions: a Study of Fieldian Nominalism[1]

Fraser MacBride

One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers. (Heinrich Hertz)

1 Introduction

Recent philosophy of mathematics has concerned itself a good deal with metaphysics. Are there any mathematical entities? If such entities exist what kind of thing are they? What relation do these entities bear to the physical world we daily encounter? These are amongst the basic metaphysical questions that recent philosophy of mathematics has sought to address. It is the aim of this paper and its sequel (MacBride [forthcoming]) to capture some of what is distinctive of recent philosophy of mathematics by outlining two of the ingenious strategies developed to answer these metaphysical questions: Fieldian nominalism and Neo-Fregeanism. It is important that these strategies are properly assessed. If they are effective they extend our understanding not only of the metaphysical foundations of mathematics but also of scientific schemes that employ mathematics.

The felt need for an articulated metaphysic of mathematics has arisen from reflection on what simple mathematical sentences say and what it takes for such sentences to be true. The sentence ‘There are prime numbers greater than 17’ expresses a claim of arithmetic. *Prima facie* this sentence shares its form

[1] This paper was commissioned under the editorship of David Papineau. Received July 1998, revised January 1999.

with another: 'There are horses faster than Zev'. This latter sentence expresses an existential claim. It is true only if there is at least one entity that possesses the properties of being a horse and being faster than Zev. Since the former sentence appears to be built up in just the same way from just the same sorts of expressions as the latter, it appears that the former sentence also expresses an existential claim. It is true only if there is at least one entity that possesses the properties of being a prime number and being greater than 17. But what sorts of entities might these things be that possess the property of being a number? They do not seem to be the sorts of entities we could encounter in experience. They do not appear to be the sorts of entities that could figure in the manifold of space-time. So if there are numbers, they are, apparently, nowhere. If the claims of arithmetic are true the numbers these sentences (apparently) assert to exist must, it seems, be abstract.

But the supposition that arithmetic—and mathematics generally—describes a realm of abstract entities is deeply problematic. On the one hand, science appears to reveal that we are wholly natural creatures, creatures who inhabit a world of space-time, whose powers are physically constrained. Our capacity to refer to and acquire information about the elements of reality is consequently constrained by our capacity to physically interact with its constituents. So if mathematical entities are abstract we cannot physically interact with them. It follows that we can neither refer to mathematical entities nor know what they are like. On the other hand, science also appears to inform us that there are mathematical entities. For example, elementary physics makes claims that concern numbers ('the distance-in-metres between two particles = 3.56'). And, at more sophisticated levels, physics makes claims that concern such *recherché* mathematical entities as vectors, phase spaces and tensor fields. These claims appear to form indispensable components of our best scientific theories. We have—it seems—no idea of how to construct scientific theories that do not refer to mathematical entities. And since our best scientific theories are surely warranted by the evidence we have discovered for them it follows that we are warranted in supposing that the mathematical entities these theories mention really do exist. Arguments that appeal to the indispensability of mathematics in science to demonstrate that there are mathematical entities appear to show that we know about and refer to mathematical entities after all.[2]

A certain metaphysical picture is sometimes used to articulate the difficulties the (apparently) abstract nature of numbers presents. According to this picture, there are two realms—the concrete and the abstract. We exist in the concrete realm, a realm causally and explanatorily self-contained. Numbers

[2] This basic problem is developed in Benacerraf ([1973]) and Putnam ([1979]). For discussion, see Field ([1989], pp. 25–30), Chihara ([1990], pp. 3–23), Maddy ([1990c] pp. 36–48), Resnik ([1997], pp. 44–8, pp. 82–92) and Shapiro ([1997], pp. 45–6). Burgess and Rosen ([1997], pp. 3–66) provides a splendid introduction to the debate.

exist in an entirely distinct abstract realm. When our thinking is conditioned by this picture it is mysterious how we concrete creatures could ever interact with entities from the other realm—the abstract entities that mathematics appears to describe. It is unfathomable how we could ever be reliable guides to that realm we cannot visit. And it is just as mysterious how numbers could ever come to perform an indispensable role in describing a concrete realm they do not inhabit. The doctrines this paper will discuss aim to provide an account of the nature of mathematical entities that avoid the mysteries attendant on the metaphysic this picture portrays.

2 Mathematical fictionalism

Taken at face value the sentences of mathematics refer to and assert the existence of abstract entities (such as the prime numbers greater than 17). So if any mathematical sentences (understood in this face value way) are true we are faced with questions that apparently we have no idea of how to answer. For example, we are faced with the perplexing question: what mechanism could possibly ensure that concrete creatures are reliable indicators of the states of an abstract realm? Mathematical fictionalism ('fictionalism' for short) seeks to avoid the need to answer such questions by denying that any mathematical sentences are non-trivially true.[3] For if all sentences that purport to refer to abstract mathematical entities or assert their existence are false, then there are no such entities. And if such entities do not exist there are no puzzles to fathom concerning the capacity of concrete creatures to interact with them. According to fictionalism, there is only one realm, the concrete. The abstract realm of mathematical entities is a fiction that has somehow been confused for reality.

The difficulties that beset our understanding of mathematics cannot, however, be resolved by simply denying any mathematical sentences are true. Fictionalism faces the following powerful objection: Mathematics is indispensable to science. Mathematical vocabulary is used to frame scientific hypotheses and mathematical sentences serve as premises in, apparently, sound scientific deductions. How could these deductions be sound if their mathematical premises are, as fictionalism claims, false? How could mathematics have gained wide and successful application in science unless, contrary to fictionalism, what mathematics says is true?

[3] There may be other reasons for adopting fictionalism. Wagner ([1982]) argues that fictionalism can accommodate the multiple reducibility of arithmetic to alternative set theories by treating each putative reduction as an alternative elaboration of a story. Papineau ([1988], [1990] and [1993], pp. 171–97) argues that mathematicians only ever establish conditional claims (demonstrating that certain theorems follow from certain axioms). They do not establish any categorical claims about mathematical objects (e.g. that such objects exist). Papineau therefore recommends that 'we adopt an attitude of sceptical disbelief to mathematics, and content ourselves with accepting it as a fiction'.

Hartry Field has sought to defend fictionalism against such objections.[4] First, Field argues, mathematics is—despite appearances—*dispensable* to science. It is possible, Field claims, to formulate science 'nominalistically'. In a nominalized science no reference is made to mathematical entities. Second, in order for mathematics to gain successful application in science, mathematics is not required to be true. Mathematics requires, Field claims, only to be *conservative*. Mathematics is conservative if, roughly, it allows us to draw only those nominalistic conclusions about the concrete world that we could have drawn anyway (by applying logic). More precisely, Field claims:

> (C) A mathematical theory M is conservative if and only if for any assertion A about the concrete world and any body N of such assertions, A doesn't follow from N + M unless it follows from N alone.

Field goes on to prove a 'conservation theorem', a theorem that says mathematics really is conservative (Field [1980], pp. 16–9).

According to Field, therefore, the function of mathematical theories in science is not to allow us to draw nominalistic conclusions that cannot otherwise be obtained. Mathematical theories cannot perform such a function because they are both dispensable and conservative. Rather, Field claims, their function is to 'facilitate inferences': it is often easier to see that a nominalistic conclusion follows from the conjunction of nominalistic premises and a mathematical theory than it is to see that the conclusion follows from the nominalistic premises alone. It is because mathematical theories facilitate inferences that the fiction of abstract mathematical entities is employed so widely in science.

Field offers the following general picture. Suppose we wish to determine whether the nominalistic statement A follows from the nominalistic premises $N_1 \ldots N_n$. One way to achieve this would be to apply logic directly to $N_1 \ldots N_n$. But another way would be to proceed indirectly *via* mathematized science. For corresponding to A, $N_1 \ldots N_n$ are a body of 'abstract counterparts', scientific statements such as we are familiar with that purport to refer to and quantify over mathematical entities. So to determine whether A follows from $N_1 \ldots N_n$ we may proceed as follows. First, we 'ascend' from $N_1 \ldots N_n$ to their abstract counterparts $N_1^* \ldots N_n^*$ (where the leap from concrete to abstract is guided by 'bridge laws'). Second, applying the mathematics embodied by the scientific theory in question to $N_1^* \ldots N_n^*$ we determine what (mathematized) conclusions

[4] See Field ([1980], [1989]). Field ([1982]) provides a non-technical introduction to his views. Field ([1989], pp. 1–52) contains a useful overview of Field's fictionalism and criticisms that have been made of it. For Field's general perspective on the philosophy of mathematics, see his ([1998]). Lear ([1982]) contains an informative comparison of Field and Aristotle's treatments of mathematics. Hallett ([1990]) usefully compares Field's programme to that of Hilbert's.

(say A*) may validly be drawn from these premises. Third, we 'descend' from A* to its concrete counterpart (using the bridge laws once more). If A is the concrete counterpart of A* then A follows from $N_1 \ldots N_n$. The conservation theorem guarantees that any nominalistic conclusions derived in this way from mathematized science could have been derived directly from nominalistic premises. But, Field claims, sometimes the direct derivations are incredibly long and tedious. By contrast, the indirect derivations may be both shorter and easier to grasp (Field [1980], pp. 20–3).

If Field's defence of fictionalism is to succeed then he must demonstrate not only that mathematics is conservative but also that mathematics is—at least in principle—dispensable in science. These are separate tasks. For even if mathematics is conservative it does not follow that it is dispensable. Adding mathematics to nominalistic theories may not allow us to derive anything we couldn't have derived already from the nominalistic theories. But nominalistic theories may still fail to allow us to derive any interesting scientific results. It may be that only mathematized theories (that *are not* formed from the conjunction of mathematics and some nominalistic sub-theory) are sufficiently rich to allow such derivations.

To demonstrate that mathematics is dispensable Field undertakes a programme of producing nominalistic versions of interesting scientific theories. Field appeals to the following general methodological principle to suggest that nominalistic versions of these theories are available: 'underlying every good extrinsic explanation there is an intrinsic explanation' (Field [1980], pp. 43–6, [1989], pp. 18–9, pp. 192–3]. The explanations of mathematized science are extrinsic in the following sense. They invoke entities that bear no causal relation to what is being explained. For example, the mathematized explanation of a body's acceleration in terms of a relation (a mass-in-some-units relation) to a real number outside space-time is extrinsic. By contrast, nominalized explanations are intrinsic in the following sense. They invoke only entities that are causally relevant to what is being explained. For example, the nominalistic explanation of a body's acceleration will invoke the body's mass. Since, Field claims, intrinsic explanations are more 'satisfying', more 'informative' than extrinsic explanation it is 'plausible' to suppose that every good extrinsic explanation has an intrinsic explanation underlying it (a principle of sufficient satisfaction?). *Eo ipso* underlying the effective explanations of mathematized science must be good nominalistic explanations.

It is important to realize that such reflections do not establish that there *must* be underlying nominalistic explanations. After all, the fact that mathematics is apparently indispensable to science provides us with strong *prima facie* reasons for supposing both that there are not underlying nominalistic explanations and that our intuitions concerning what it takes for an explanation to be 'satisfying' are not a reliable guide to what explanations there are. To show that mathematics is dispensable to science there is nothing for it but to roll up the sleeves and provide nominalistic versions of scientific theories.

Field concentrates on Newtonian gravitational theory. He suggests that the methods used to nominalize this theory may also serve more widely to nominalize other physical theories. Newtonian gravitational theory has a basic ontology of ordered quadruples of real numbers and sets thereof. The vocabulary of the theory includes functional expressions that purport to denote functions whose values are numbers (e.g. 'the gravitational potential of x'). Field sketches a nominalized version of this theory in which quadruples of real numbers are replaced by space-time points. In effect, the quadruples are used to describe the three spatial co-ordinates and the one temporal co-ordinate of the space-time points to which they correspond. Sets of quadruples are replaced by regions formed from the space-time points that correspond to the member quadruples. The functional expressions of the Newtonian theory (that when completed designate numbers) are replaced with comparative predicates true or false of space-time points (e.g. 'the difference in gravitational potential between x and y is less than that between z and w'). Field's idea is that comparative claims about space-time points are concrete counterparts of the abstract claims made by Newtonian gravitational theory.

In order to explain why Newtonian gravitational theory has been so useful in deriving nominalistic results, Field introduces bridge laws to connect the abstract claims of the Newtonian theory with their concrete counterparts. These laws are provided by 'representation theorems', theorems that show how the abstract claims of mathematized science may be used to 'represent' concrete facts. A simple case of such a theorem connects numerical claims about distance with comparative claims about points (Field [1980], pp. 24–9). Suppose the nominalized theory contains the comparative predicates 'x Bet yz' ('x is a point on the line-segment whose end points are x and z') and 'xy Cong zw' ('the line-segment with end-points x and y is congruent to the line-segment with end-points z and w'). It can be proved—using mathematics—that there is a 'distance' function d that maps pairs of space-time points into the real numbers such that:

(1) for any points x, y, z and w, xy Cong zw iff $d(x,y) = d(z,w)$

(2) for any points x, y and z, y Bet xz iff $d(x,y) + d(y,z) = d(x,z)$.

If d is to taken to represent distance, then the representation theorem shows that claims about segment congruence and between-ness are 'equivalent' to claims about distance. So the theorem that proves function d exists allows us to pass from comparative claims about space-time points to abstract numerical claims about distances and back again. Field goes on to provide an 'extended-representation theorem' (Field [1980], pp. 61–91). This theorem proves there is a 'spatio-temporal co-ordinate' function that maps 1–1 space-time points into quadruples of reals. It also proves there are 'gravitational potential' and 'mass density' functions that (respectively) map space-time points into intervals of reals and intervals of non-negative reals. The existence of these functions establishes that abstract claims concerning gravitational potential and mass density are equivalent to comparative claims concerning space-time points. With the aid of the extended-representation theorem, Field claims, we may ascend from the nominalistic theory of space-time points into the mathematized theory of Newtonian gravitational theory and then descend again to draw nominalistic conclusions.

If Field is to make good his defence of fictionalism then he is required to establish three claims. First, that mathematics really is dispensable. Second, that mathematics really is conservative. And, third, there really are representation theorems that connect mathematized science and nominalistic theories. All three claims have been contested. As we shall see, it is doubtful whether fictionalism—in anything like the form that Field originally presented it—can be sustained (Chihara [1990], pp. 146–73, provides a useful overview of several objections to Field's fictionalism).

3 Dispensability

Does Field really show that mathematics is dispensable? Field does show how to relate quadruples-of-reals-talk to space-time-points-talk. But this will only establish that mathematical entities are dispensable if it also shown that there are space-time points and that space-time points are not themselves a species of mathematical entity.

Field undertakes to defend the claim that there are space-time points by arguing for a substantivalist theory of space-time. According to substantivalism the physical world contains not only physical objects and the matter from which they are composed but also a further entity, space-time itself. Substantivalism allows that there are space-time points: they are infinitesimal shards of the entity space-time. Opposed to the substantivalism is the relational view. According to relationalism there is no such entity as space-time: the physical world contains only physical objects and their matter related in a variety of ways. Clearly, if space-time does not exist then space-time points cannot be its shards. So relationalism must treat points in a different way. Reductive relationalism says that points are abstract objects, in fact, set-theoretic constructions from physical objects. Eliminative relationalism says there simply are no points. Neither form of relationalism coheres with Field's fictionalism. If reductive relationalism is true then Field's nominalizations succeed only in replacing one sort of abstract entity (quadruples) with another (sets). This brings us no closer to understanding how concrete creatures can apply abstract forms to a concrete world. If eliminative relationalism is true then Field fails to replace the mathematical entities with anything. Mathematical entities will not have been dispensed with.

Field defends substantivalism by arguing that relationalism cannot accommodate the role that field theory performs in modern science.[5] According to Field, field theories employ causal predicates that are true or false of space-time points. Field theories use these predicates to attribute causal powers to both points that are occupied by matter and points that are not. For example, electromagnetic theories ascribe an electromagnetic intensity to all space-time points, regardless of whether they are occupied. It follows that relationalism cannot easily account for the attributions of intensity made by field theories. For relationalism denies that there are any unoccupied points to which such attributions may be made. Substantivalism, by contrast, can readily make sense of these attributions. If space-time is an independently existing entity then its components—space-time points—can exist and possess causal powers regardless of whether they are occupied (Field [1989], pp. 181–4).

This way of understanding field theories (classical electromagnetism, general relativity and quantum field theory are mentioned) is central to Field's argument for the dispensability of mathematics. Field also bases his claim that space-time points are concrete—unlike the abstract quadruples they replace—upon this understanding (Field [1989], pp. 46–7, pp. 67–73). Field offers two reasons for supposing that space-time points are concrete. First, he claims, points figure in spatio-temporal relations to us. Second, he claims, points possess the causal powers that field theories attribute to them. The latter claim is the more important. The mere fact that space-time points figure in spatio-temporal relations does not provide an explanation of how we can reliably acquire information about these entities. Such an explanation must account for how these relations can serve as a conduit for information about space-time points. If such an explanation is not provided then the supposed fact that we know and refer to space-time points is just as mysterious as the putative fact that we know and refer to mathematical entities. By contrast, the fact—if it is a fact—that points figure in causal relations appears to remove the mystery of how we interact with them.

Field's understanding of field theories has been contested. Sometimes it is claimed field theories attribute powers to 'fields' (entities that occupy space-time) rather than space-time itself (Malamet [1982], p. 532). Field

[5] See Field ([1980], pp. 34–6, [1985a]). Field also develops a version of Newton's famous 'bucket argument'. This argument is designed to show that, by contrast to substantivalism, relationalism cannot provide an adequate definition of absolute acceleration. Field does not think the argument insurmountable. Nevertheless, he claims, the only adequate relationalist definitions invoke modal notions or abstract entities that render those definitions otherwise unappealing. Field ([1989], pp. 184–6) provides an overview of the argument.

argues this position is merely a 'verbal' variation of his own. If fields are objects then they are located throughout space-time and their spatio-temporal parts share all the geometric properties of the parts of space-time they occupy. So if fields are objects then there is no point in positing a further causally inert geometric entity—space-time—for those objects to occupy. We may as well think of fields as space-time endowed with causal powers.

Resnik has also contested Field's view (Resnik [1985b], pp. 165–9). He claims: 'The explanatory, historical and evidential place of space-time points in physics is much closer to that of standard mathematical objects than it is to that of standard physical objects.' By contrast to physical objects and events, space-time points are not posited to explain occurrences. Like mathematical entities, points are posited to 'structure and organise' occurrences. Resnik also claims that whilst the attribution of causal powers to the space-time regions occupied by fields makes our epistemic access to regions unproblematic it does not follow that our access to the points that compose those regions is also unproblematic (Resnik [1997], pp. 108–10). If Resnik is correct then it is unclear what epistemological advantage is gained by substituting space-time points for numbers. Field responds to Resnik's first claim by suggesting that space-time points do not merely serve to structure happenings. Rather field theories attribute causal powers to points that serve to explain those happenings (Field [1989], p. 47). Presumably, Field will respond to Resnik's second claim by suggesting that our epistemic access to points is mediated by the causal powers field theories attribute to them. The debate between Field and Resnik reduces to assertion and counter-assertion. These matters cannot be resolved until a more detailed investigation of the ontology of field theories is undertaken.

Two other doubts have been raised concerning whether Field's nominalistic theory dispenses with mathematical entities. The first doubt suggests that Field's space-time has too much *structure* in common with the real numbers to be considered a genuine nominalistic replacement for them (see Shapiro [1983a], pp. 544–7; Resnik [1985a], pp. 195–6; Shapiro [1993], pp. 472–479, [1997], pp. 235–42). In fact, Field builds into space-time almost all the structure and complexity of the real number system of which it is a concrete counterpart. And it turns out—not surprisingly—that by adding some extra vocabulary to Field's nominalistic theory a concrete counterpart of classical analysis can be developed in which reals are replaced by points and sets of reals by regions (Resnik [1985a], pp. 192–5). The fact that Field's theory of space-time can be extended in this way does not by itself show Field's theory fails to dispense with mathematical entities. For as Field remarks, points still differ from numbers in a crucial *non-structural* respect: points, unlike numbers, are physical entities with causal powers (Field [1980], pp. 31–4).

What is more worrisome is that undecidable set-theoretic hypotheses are transformed in Field's theory into hypotheses concerning regions of space-time (the continuum hypothesis, for instance). Field writes:

> It may seem hard to believe, though, that the question of whether there are sub-regions of a line with intermediate cardinality can be 'physically real'. If this does seem hard to believe, then the development of a point-free physics should be welcome, since presumably no such physical analogue of the continuum hypothesis would arise in it (Field [1989], p. 48; Resnik [1985a], p. 198).

The worry here might be put like this. In order for space-time to serve as the concrete counterpart of the reals it will have to incorporate structure of a kind (e.g. the structure of the continuum) that is more evidently mathematical than physical. This worry raises a challenge for Field. Either space-time points serve as the 'concrete' counterpart of the reals or they do not. In the former case, space-time turns out to have properties that are mathematically but not 'physically real'. It then appears that Field has failed to dispense with mathematical entities after all. In the latter case, the reals that are posited in mathematized science possess no concrete counterparts. But if the reals have no concrete counterparts then it is inexplicable how talk of reals can have served so well to facilitate the drawing of nominalistic inferences. Clearly the force of this challenge (and consequently the extent to which Field succeeds in dispensing with mathematical entities) cannot be properly assessed until we possess what we currently lack: an account of what the notion of physical space amounts to and what it is for structure to be 'physically real'.

The second doubt suggests that the methods Field exploits to nominalize Newtonian gravitational theory cannot be used to nominalize other scientific theories. Discussion has focused on the case of quantum mechanics (Malamet [1982], pp. 532–4). Quantum mechanics has a far richer mathematical ontology than Newtonian mechanics. It includes not only the reals but also Hilbert spaces and vectors. To nominalize this theory—using Field's methods—some concrete counterparts for Hilbert spaces and vectors must be uncovered. But it appears that there are no concrete counterparts to be uncovered. The Hilbert spaces and vectors used in quantum mechanics are usually taken to represent quantum *propositions* and *possible* pure states of quantum systems. And propositions and possibilities can hardly be deemed concrete. (Resnik [1997], pp. 56–8, suggests that similar difficulties attend any attempt to nominalize theories that employ statistical explanations. Resnik claims we cannot understand statistical inference without invoking abstract entities like probabilities, sample spaces or sets of events.) However, Balaguer ([1996a]) has suggested that 'propensities' provide an alternative source of concrete counterparts for the abstract claims of quantum mechanics. Balaguer offers two options. Either we can nominalize propensities by appealing to propensity *predicates* that are true or false of quantum systems. Or we can think of propensities as *concrete* properties of those systems, properties that inhabit space-time.

Despite such proposals the claim that *all* scientific theories may be nominalized must remain highly speculative. The success of Balaguer's strategy depends on details that (as Balaguer admits) have not been worked out. It needs to be shown that terms which apparently refer to propensities can be semantically parsed as predicates true or false of quantum systems. It needs to be shown that propensity properties, if there are such, are best understood as concrete. These are not trivial matters. For example, it has been argued that the best way to understand the role *possible* states of quantum systems perform in physical explanations is to treat states of quantum systems as abstract. There are also arguments that suggest the role properties perform in laws of nature also determines they are abstract (see Forrest [1993], pp. 50–4; Tooley [1987], pp. 113–20). More generally, it is apparent that there can be no assurance that mathematics is dispensable in advance of actually constructing nominalized versions of a wide range of different scientific theories.[6]

4 Conservativeness

Does Field really show that mathematics is conservative? If mathematics is conservative then whenever a nominalistic assertion A is a consequence of a mathematical theory M and a body of nominalistic assertions N, then A is a consequence of N alone. So if mathematics is conservative there is no need to think that using a mathematized science (that results from the combination M+N) commits us to the mathematical entities the science invokes. But in order to say that mathematics is conservative, and prove that it is so, we appear to incur a commitment to just the sort of entities that the conservativeness of mathematics was supposed to obviate. The problem is that the usual notions of consequence in terms of which conservativeness may be defined incorporate a commitment to abstract entities.

Consequence is usually defined either deductively (proof-theoretically) or semantically. According to the deductive definition of consequence, mathematics is *deductively conservative* if whenever A can be deduced from M + N, A can be deduced from N alone. According to the semantic definition of consequence, mathematics is *semantically conservative* if A is true in all models of M + N only if A is true in all models of N. Clearly the semantic definition of conservativeness invokes abstract entities (*viz*. models). But it is

[6] Balaguer ([1996b]) argues that fictionalism may be tenable even if mathematics is indispensable to science. Balaguer claims—contentiously—tha whilst mathematics is indispensable to science as a 'heuristic device', mathematics makes no contribution to the 'content' of scientific theories. Consequently, he argues, it is open to the fictionalist to accept that mathematics is indispensable to science whilst denying that what mathematics says is true.

plausible to claim that the deductive definition does too. For if mathematics is deductively conservative then there are a range of nominalistic conclusions (that flow from the results of mathematized science) that may be deduced from nominalistic premises. Nevertheless, there are no concretely inscribed deductions of these conclusions from such premises. After all, no one has ever written down (and perhaps never will write down) the nominalized theories that underlie the science and which would form the premises of such proofs. So if mathematics is deductively conservative, deductions cannot be concrete inscriptions. The obvious alternative is to understand deductions as abstract types that exist independently of whether they possess any concrete tokens. Supposing then that consequence is defined either deductively or semantically, it follows that we can only know that mathematics is conservative if we know what abstract deductions (or models) there are. In fact Field explicitly invokes models and proofs to prove his conservativeness theorem (Field [1980], pp. 17–9). But it is just as problematic to suppose that we possess knowledge of these mathematical entities as it is to suppose that we have knowledge of the mathematical entities invoked in science.

This is an instance of a general difficulty for fictionalism.[7] Mathematics not only gains useful application in science. It also gains useful application in metalogic. In metalogic mathematical entities (proofs and models) are invoked to define the notions of logical consequence and consistency and to determine results concerning those notions (e.g. the completeness of first-order logic). In order to show that we are not committed to the existence of mathematical entities the fictionalist must therefore demonstrate that mathematics is dispensable to metalogic as well as to science (see Field [1984], [1989], [1991] and [1992]).

If mathematics is—despite appearances—dispensable to metalogic then it must be possible to formulate metalogic nominalistically. The basic subject matter of metalogic is the relation of logical consequence. So nominalized metalogic must somehow define that relation without invoking abstract entities (i.e. models and proofs) in the usual way. Field suggests that the notion of *logical* consistency (hereafter 'consistency') be taken as primitive. Logical consequence can then be defined (assuming compactness) in terms of this primitive: p is a consequence of $q_1 \ldots q_n$ iff the conjunction $q_1 \& q_2 \ldots q_n$ with p's negation is not consistent. Since no appeal is made to abstract entities the resulting definition is nominalistic. The notion of logical consequence may therefore be used to provide a nominalistic definition of conservativeness.

[7] Hawthorne ([1996]) elaborates a further objection to the conservativeness of mathematics. It is common scientific practice to conjoin two theories to form a new unified theory. However, where the mathematics embedded within the two sub-theories are distinct, the conjoined theory may have nominalistic consequences that do not follow from either sub-theory taken in isolation. Hawthorne argues—plausibly—that there are formal precautions the fictionalist can adopt to avoid the formation of unified theories giving rise to such non-conservative consequences.

More generally, the fictionalist must claim, the primitive notion of consistency may be used to articulate a nominalistic version of metalogic.

An explanation is now required of the usefulness of mathematics in metalogic. To do this, bridge principles must be given to connect the abstract claims of mathematized metalogic with their nominalistic counterparts that invoke the primitive notion of consistency. Field suggests there are two relevant bridge laws (Field [1989], p. 108, [1991], pp. 12–3). First, there is a *modal soundness principle*:

(MS#) If it follows from S that there is a refutation in F of A then A is not consistent

(where S is a finite axiomatization of some set theory and the refutation of A is a deduction couched in a standard formalization F of first order logic). Second, there is a *model-theoretic possibility principle*:

(MTP#) If it follows from S that there is a model of A then A is consistent.

Using these principles the fictionalist can deploy proof theory and model theory in the usual way to determine which theories are consistent. It can be shown (using (MTP#)) that a theory is consistent by showing it has a model. It can be shown (using (MS#)) that a theory is inconsistent by showing it has a refutation.

In order to apply these bridge principles the fictionalist must be able to know what they say is true. But can the fictionalist legitimately claim to know all the conditional facts concerning models and proofs that (MS#) and (MTP#) schematise? Field argues that these principles may be derived from modal knowledge we already possess—our knowledge of consistency (Field [1991], pp. 14–7). Consider (MTP#). It follows from the consistency of S and the logically valid schema:

(MTP*) If S and the claim there is a model of A are consistent then A is consistent.

The antecedent of (MTP*) follows from the consistency of S and the antecedent of (MTP#). The claim that A is consistent then follows by (MTP*). So the consequent of (MTP#) follows from its antecedent, given (MTP*) and the consistency of S. A fictionalist can therefore claim knowledge of (MTP*) by claiming knowledge of the consistency of the axioms of set theory.

But how can a fictionalist claim to know, for example, that the axioms of Gödel–Bernays set theory are consistent? There are metaphysical and epistemological difficulties here. The former are raised by Hale and Wright (Hale [1987], pp. 106–15, [1990], pp. 121–9; Wright [1988], pp. 462–6; Hale and Wright [1992]). Suppose that some axiomatization S of set theory is consistent. Since S is consistent it must be possible for the axioms of S to be true together. So even though Field claims that all the axioms of S are false, Field can only claim that they are *contingently* false. They are only contingently false because the consistency of S implies that they could have been true. Hale and Wright deny, however, that Field can grant that the axioms of S (or the claims of mathematics generally) are contingent.

According, they claim, to our usual way of thinking, contingencies are either contingent *on* something or something else contingently *depends* on them. But—if Field is correct—mathematical claims cannot be contingent in this way. If mathematics is conservative then mathematical claims cannot be contingent on any nominalist claims. And if mathematics is dispensable then nominalist claims cannot depend on any mathematical claims. Call a claim 'brute' if its obtaining doesn't depend on anything else. Call a claim 'barren' if the obtaining of any other claim does not depend on it. An 'absolutely insular conceptual contingency' is a contingency that is both brute and barren. Hale and Wright ([1992], p. 134) suggest that our usual thinking about contingency is governed by the following principle:

(CON) There are no absolutely insular conceptual contingencies.

But according to Field's nominalism the contingent claims of mathematics are absolutely insular. Hale and Wright conclude that unless Field provides some independent motivation for allowing mathematics to constitute an exception to (CON) Field's nominalism should be rejected.

Field responds by arguing that whilst (CON) may be true of many sorts of contingencies, there is no reason to suppose it applies to all contingencies (see Field [1989], p. 43, [1993], pp. 291–3, and Papineau [1993], p. 197). In particular, he argues, there is no reason to suppose that it applies to logical contingencies. After all, logical contingencies are just those claims that are neither logically true nor logically contradictory; their contingency amounts to just the fact—and no more than the fact—that they are neither of those things. Fictionalism supposes that mathematical claims are contingent only in the logical and not in any more robust sense. So it is no objection to fictionalism that it characterizes those claims as absolutely insular. (Hale and Wright [1994] do not concur.)

Field also provides a *reductio* argument against (CON) (Field [1993], pp. 296–7). The argument introduces the concept of a *surdon*: the concept of an entity whose existence and nature are entirely independent of the existence and nature of any other entity. It follows from this definition that the claim that says surdons exist will be absolutely insular. (For if it were not then the existence and nature of surdons would be dependent upon the existence and nature of the entities described by the other claims upon which the claim that they exist depends. Since, *ex hypothesi*, surdons do not depend for their nature or existence on anything else this dependency between claims cannot obtain.) However, (CON) tells us that the claim surdons exist cannot be a contingent one. Since claims that are not contingent are either necessary or impossible it follows that the sentence that says surdons exist must either be necessary or impossible. But it cannot be impossible; the concept of a surdon is a consistent concept. So the claim that surdons exist must be a necessary one!

Hale and Wright attempt to avoid this *reductio* by denying that the apparent consistency of the concept of a surdon justifies the claim that surdons are possible (Hale and Wright [1994], pp. 182–3). Field may grant Hale and Wright this response. But by making this response, Field may argue, Hale and Wright make just the distinction between kinds of possibility that Field has been insisting on all along. For all that Field means by logical possibility is what Hale and Wright mean by apparent conceptual consistency (i.e. a contradiction cannot be derived). And, just as Hale and Wright argue, it does not follow from the consistency of a claim in this sense that it is possible in any more robust sense governed by (CON).

It may be that Hale and Wright's objection is better put in a different way. They are merely pointing out that Field's fictionalism has an unmotivated metaphysical consequence. It has as a consequence a commitment to a species of possibility (absolutely insular possibility), a species of which we had no inkling prior to the articulation of Field's theory. So unless Field provides some independent motivation for accepting this commitment—independent that is of its simply being a consequence of adopting his fictionalism—Field will beg the question against an opponent who refuses to accept fictionalism simply because of its untoward commitment to absolutely insular possibilities. But if this is Hale and Wright's objection it cannot be effective unless other independent objections to fictionalism succeed. For Field may reply that because the theoretical benefits of adopting fictionalism outweigh the theoretical cost of recognising a novel species of possibility no question is begged. So unless Hale and Wright provide some independent motivation for supposing that fictionalism does not possess the theoretical benefits that Field intends they will have made no effective case against him.

The epistemological difficulties Field faces are more evidently critical. They arise from the fact that Field offers no account of how knowledge of the consistency of the axioms of set theory may be acquired. Indeed Field concedes that 'neither I nor anyone else I know of has a great deal to say about the epistemology of modal claims' (Field [1989], p. 140). Instead Field argues that whatever assumptions the nominalist may require to prove the consistency of a given claim they will always be strictly weaker than those required by a non-fictionalist (Field [1991], pp. 16–7). For whereas a non-fictionalist will have to assume that set-theory is *true* in order to derive the result that a claim is consistent, the fictionalist will only have to assume that the set-theory in question is *consistent*. Field concludes that it is hard to see how the epistemological difficulties that face the fictionalist can be any worse than those that face the non-fictionalist.

This conclusion may be contested in a variety of ways.[8] Here are just three relevant considerations. First, it may be claimed quite generally that modal notions (including the notion of consistency) are best understood as implicitly involving quantification over certain abstract entities, namely possible worlds. But if that is the case no epistemological gain will have been achieved by founding our knowledge of mathematics on our knowledge of modality. For the acquisition of the latter form of knowledge will—apparently—involve just the same sort of ineffable interaction between concrete subjects and abstract objects as that to which a non-fictionalist account of the former appears to commit us.

Second, even if it is granted that *some* modal notions are best understood in terms that do not involve quantification over abstract entities, it may be doubted that the notion of consistency upon which Field's account of conservativeness relies is amongst them. Field appeals to the fact that the untutored mind possesses a notion of possibility that does not apparently involve commitment to abstract entities (Field [1989], pp. 33–4). For this reason such a mind may grasp that it is possible for two sentences ('grass is green', 'snow is white') to be true together without having any thought of sets or possible worlds. However, it is unclear whether the untutored mind possesses a notion of possibility that is sufficiently articulate to comprehend the question whether the axioms of Gödel-Bernays set theory are consistent.

[8] Maddy ([1990a], [1990b] and [1990c], pp. 159–70), offers a sustained critique of this conclusion. One of Maddy's central arguments rests on the following consideration. Where second-order logic is deployed, a physical equivalent of either the axiom of choice (for example) or its negation may be stated within a nominalistic theory. Consequently, a nominalistic theory which makes such a statement will be committed to either the existence of a physical counterpart of the axiom of choice or of its negation. But in that case, only the set theory which contains the axiom that has an equivalent within the nominalistic theory will be conservative; the set theory which contains that axiom's negation will—when added to the nominalistic theory—generate nominalistic consequences (e.g. a description of the physical counterpart of the axiom's negation) that conflict with the claim of the original theory that describes the physical counterpart of the axiom in question. Hence, Maddy argues, the fictionalist has as much of an uphill epistemological task to establish which of the axiom of choice or its negation is conservative as the non-fictionalist has to establish which is true. Field ([1990], pp. 210–6) concurs but argues in response that nominalistic theories should deploy only first-order logic. Field goes on to claim that even if nominalistic theories are formulated in second-order logic Maddy's conclusion may be resisted. He suggests that the fictionalist need only show that alternative set theories are 'quasi-conservative', rather than conservative. Roughly, a mathematical theory is quasi-conservative if when added to a nominalistic theory in *a restricted way* it generates no new nominalistic consequences. The restriction in question is designed to prevent set-theoretic vocabulary appearing in second-order quantified sentences of the resulting mathematized theory. (It is such sentences that generate novel nominalistic consequences.) Since conflicting set theories may all be quasi-conservative even when—in a second-order context—they are not all conservative, the epistemological burden shouldered by the fictionalist in determining which set theories are quasi-conservative will always be strictly less than the non-fictionalist who must establish which theory is true (Field [1990], pp. 218–9).

What, after all, would the untutored mind say in response to such a question? It may be that only after learning some set theory that a mind becomes sufficiently tutored to make sense of the species of possibility such questions concern. But in that case the relevant grasp of possibility flows from a prior grasp of mathematics. So, *contra* Field, knowledge of the latter cannot be founded upon knowledge of the former.

Hale and Wright's concern with absolutely insular possibilities deserves re-attention here. The notion of possibility the untutored mind employs may—as they conjecture—not allow for possibilities that are both brute and barren. But then it follows that the primitive notion of modality the untutored mind employs cannot be that upon which Field's account of conservativeness relies. Consequently, it remains for Field to establish that there is any notion of possibility that both serves his theoretical purpose and may be understood without appeal to abstract entities.

Third, until some account is given of how the modal knowledge to which Field appeals is acquired we can have no assurance that the fictionalist has not simply replaced one intractable problem (explaining how we can have knowledge of mathematical entities) with another (explaining how we can have knowledge of consistency). On the face of it we have no more idea of how such creatures as ourselves—creatures that apparently encounter the world simply as it *is*—can comprehend how the world *might* have been and *must* be, than we have inkling of how concrete thinkers can comprehend abstract truths. And without such assurance the fictionalist can hardly appeal to the fact that mathematics is conservative in order to show that our knowledge of mathematics is tractable.[9]

5 Representation theorems

Does Field's defence of the claim that there are representation theorems connecting nominalistic theories and mathematized science fare any better? The availability of such theorems is determined (in part) by the logic of the underlying nominalistic theories. If these theories are second-order then representation theorems may be available. But there are considerable costs attached to making theorems available in this way: it requires the qualification of the claim that mathematics is conservative; nominalistic theories are made to take on the existential commitments of second-order logic. Alternatively, if

[9] For a battery of related criticisms, see Resnik ([1985a], pp. 200–4, [1985b], pp. 169–75); Chihara ([1990], pp. 261–72); Shapiro ([1993], pp. 459–65, [1997], pp. 219–28). Field has also been criticized for his use of substitutional quantification and infinite conjunction. Such logical devices are not, it has been argued, anymore available to the nominalist than the notion of consistency. See Field ([1980], pp. 93–8, [1984b]); Resnik ([1985a], pp. 204–6); Field ([1989], pp. 48–52), and Burgess ([1993], pp. 181–3).

these theories are first-order then the sorts of representation theorems Field requires are not available at all.[10]

In order to evaluate this complex situation its logical components must be assembled. First, note that the natural number series can be modelled within the universe of points that Field's nominalistic gravitational theory (hereafter 'NT') describes (regardless of the logic the theory assumes). This is hardly surprising since NT was originally designed so that its points corresponded to the far more complex structure of the real numbers. The natural numbers form a progression that begins with 0, where every number has a unique successor and which carries on to infinity. NT possesses the capacity to describe the concrete counterpart of this progression. Field introduces the notion of an 'infinite spatio-temporally equally spaced region' R.[11] R is a region composed of all those infinitely many points that lie on a single line l_1. The points in R are separated by segments of l_1 that are all congruent to one another. Suppose that l_1 begins at a point p. We can think of p and the equally spaced points of R that follow p along l_1 as corresponding to 0 and the successive numbers that follow 0 along the natural number progression. Concrete analogues of the successor relation and the arithmetical operations of addition and multiplication may now be defined. (For example, we can say that point x is the successor of point y iff x and y are both in R, there is no distinct point of R between x and y, and if y is distinct from p then y is between x and p. For more details, see Shapiro ([1983b], pp. 526–7). As a result, the concrete counterparts of the axioms of Peano arithmetic (PA) may be proved in NT.

Second, Gödel's incompleteness theorems tell us that any theory that embodies PA is incomplete. Such a theory will contain sentences, in particular the sentence that says the theory in question is consistent, that are not provable within that theory. Since NT embodies the concrete counterpart of PA we may ascend (using appropriate representation theorems) to the abstract claims of PA. Gödel's incompleteness theorems can then be applied to yield the result that there is a sentence (CPA) that says PA is consistent and cannot be proven in PA. Using representation theorems to descend we can conclude that there is a concrete counterpart of CPA that is unprovable. The counterpart is a sentence (CNT) that says NT is consistent.

However, the fact that CNT cannot be proved in NT poses a threat to Field's claim that mathematics is conservative over NT. For the consistency of NT may be proved by ascending to the mathematized version of NT and using set theory. So it appear that there is at least one sentence (*viz.* CNT) that is provable in NT + set-theory but is not provable in NT alone. It turns out that

[10] See Shapiro ([1983b]). Field ([1980], pp. 104–6) anticipates this difficulty. See also Resnik ([1985a], p. 199–200); Chiharha ([1990], pp. 154–7); Urquhart ([1990], pp. 152–4); and Papineau ([1993], pp. 208–9).

[11] See Field ([1980], pp. 65–6).

this threat to the conservativeness of mathematics can be contained. But it can only be contained by either adopting second-order logic or abandoning the claim that there are representation theorems connecting NT with its abstract counterparts.

Let us assemble some more logical pieces. As we saw earlier, there are semantic and deductive definitions of conservativeness that respectively employ semantic and deductive definitions of consequence. And, as was also remarked, Field has used set theory to prove the semantic conservativeness of mathematics. What this proof shows depends on whether the nominalistic theory to which it is applied is first or second-order.

Now suppose NT is a second-order theory. We have established—on the basis of the incompleteness theorems—that there is a sentence of NT (*viz.* CNT) that is a deductive consequence of a mathematized version of NT but not of NT alone. But if NT is second-order this fact does not conflict with Field's proof that mathematics is semantically conservative. For what the incompleteness theorems also show is that the notions of semantic and deductive consequence do not extensionally coincide in second-order languages. Consequently the fact that CNT is not a deductive consequence of NT alone does not imply that CNT is not a semantic consequence of NT. Field's claim that mathematics is conservative needs to be qualified. Mathematics may be semantically conservative, but it certainly is not deductively conservative.

Now suppose that NT is first-order. By contrast to higher-order logics, first order logic is sound and complete. So the notions of deductive and semantic consequence extensionally coincide in first order theories. It follows that if NT is first-order Field's proof that mathematics is semantically conservative over NT can be converted into a proof that mathematics is deductively conservative over NT (by the substitution of co-extensive terms). But this conflicts with a result proved earlier on the basis of ascent from NT to PA, the application of the incompleteness theorems to PA, and then descent from the results achieved to NT. This result showed, *contra* the conservativeness of mathematics NT, that there is one sentence (*viz.* CNT) provable in the mathematized version of NT that is *not* provable in NT alone. In order to remove this conflict there appears to be only one option: to deny there are representation theorems available to allow us to ascend and descend from NT to its mathematized counterpart.

Field is thus presented with a dilemma. Either adopt second-order logic or abandon the project of providing representation theorems for mathematizsed science. Field's response is striking (Field [1985b]). It does not concern Field that if NT is second-order the addition of mathematics will serve to extend the range of deductive inferences that may be drawn from NT. His nominalist account of metalogic explains—Field supposes—how the nominalist may legitimately exploit the notion of semantic consequence. So it suffices for nominalist purposes that mathematics is semantically conservative. Nevertheless, Field is wary of the second-order option. Standard second-order logic is not acceptable to the nominalist. It quantifies over sets or properties. However, the only sets that are required in proving representation theorems are non-empty, having points as members. Field consequently argued, the nominalist may use a non-standard second-order logic that quantifies only over mereological sums of these points (Field [1980], p. 38). But Field has gone on to reject the 'complete logic of Goodmanian sums' (see Field [1989], pp. 135–42, [1990], p. 212 and—for further critical discussion—Parsons [1990], pp. 323–5). Logic, Field later claimed, should not make existential assertions. But the logic of Goodmanian sums does make such assertions (for example, the instances of the Goodmanian schema:

(C_s) If $\exists x F(x)$ then there is a u such that u is the mereological sum of those entities y such that $F(y)$.

Field therefore denied that NT can be second order and claimed it was first order instead. He then abandoned the project of providing representation theorems for mathematized science (e.g. the extended-representation theorem for Newtonian gravitational theory).[12]

Representation theorems were originally introduced to provide an explanation of how a mathematical theory could be false and yet gain useful application in science. Moreover, they appeared to allow the derivation of satisfying conservative sub-theory results. For example, the extended representation theorem appeared to show that Newtonian gravitational theory is entailed by NT and standard mathematics. Since NT is nominalistic and mathematics is conservative it follows that NT has all the nominalistic consequences of the Newtonian theory. Consequently, the Newtonian theory can be taken to be a conservative extension of NT and NT can be taken to be a conservative sub-theory of that theory.

In the light of such results the fact that mathematics gains application in science provides us with no reason to suppose that the mathematical entities invoked really exist. We can think of the mathematized theory as serving merely to cloak a nominalistic sub-theory that captures the real content of what science expresses by its mathematized claims. But without representation theorems such results cannot be proved and we cannot think of mathematized

[12] See Field ([1989], pp. 141–6). Maddy ([1990b], pp. 194–6) argues, *contra* Field, that scientific practice can only be rendered intelligible if certain very general principles (e.g. Dedekind's Continuity Postulate) are understood as second-order axioms rather than the corresponding schemas that are all first-order languages make available. See also Shapiro ([1985]). Field ([1990], pp. 213–6) offers counter considerations. Field ([1990], p. 211, pp. 216–220) also contains an interesting discussion of the possibility that fictionalism adopt a nominalist friendly, plural quantification formulation of second-order logic.

science as resulting from a conservative extension of a nominalistic theory. We may have to believe that what mathematized science literally says—that mathematical entities exist—is true after all.

Field responds to this objection by distinguishing between full and weak representation theorems. Full representation theorems map all the claims of mathematized science into conservative sub-theories. These are theorems we cannot have. Nevertheless, Field argues, we can have weak representation theorems that map a large body of the claims of mathematized science into a nominalistic theory. Such theorems will fail to map mathematized claims about consistency (CPA) into their nominalized counterparts (CNT). Therefore they will not conflict with Gödel's incompleteness theorems. But they will map all those mathematized claims that have empirical support into the nominalistic theory. The resulting nominalistic theory will therefore 'explain all our observations in a satisfactory fashion'. Mathematized science will contain further 'arcane' claims that are not captured in any way by the nominalistic theory. But since these claims are arcane and possess no empirical support it is not a failing of nominalistic theories that they fail to incorporate their concrete analogues. What mathematized theories have that nominalist theories lack the scientist has no reason to believe in anyway.

If weak representation theorems are available then the fictionalist can continue to hold that what mathematized science literally says is false. The truth will be encapsulated in what the corresponding nominalistic theories say. Moreover, if the theorems are available the nominalist can provide *some* explanation of why mathematics has been so useful in science. For the theorems will establish that the contentful claims of nominalistic theories have abstract counterparts in mathematized science.

However, substantial difficulties remain. First, the fictionalist must provide some principled account of what it is for a claim to have 'empirical content'. Such an account must not presuppose that only those claims a nominalist can consistently suppose to have concrete counterparts are the claims that possess empirical content. Moreover, such an account of content must deal with the following sceptical doubt: there may be no sentences that are by their nature arcane and without empirical content; for it may turn out by accident, as it were, that upon an interpretation any sentence may have a concrete model and thereby make a contentful empirical claim about the world.

Second, it remains a substantial task to actually discover and detail theorems that are weak enough to avoid conflicting with the incompleteness theorems but also strong enough to capture all the empirical content of mathematized science. And at this particular point we can only be agnostic concerning whether there are weak representation theorems available for a sufficiently wide range of scientific theories to vindicate the fictionalist stance (although the work of John Burgess may suggest that a more optimistic judgement is in order).[13]

Fieldian nominalism is also open to a more general methodological criticism. According to the doctrine of naturalised epistemology, the epistemic norms and methods to which we should adhere are those employed by the scientific community. However, the norms to which Field seeks to cleave are different. It is epistemic scruples concerning abstract entities that drive Field to construct scientific theories that abjure them. But avoiding commitment to abstract entities is no concern of scientists. And scientists do not recognize nominalistic theories as progressive; nominalistic theories are less pellucid and more prolix than their mathematized brethren. Since the epistemic standards Field uses to cast doubt on mathematized theories are different from those used by practising scientists, Field fails to offer the naturalized epistemologist any reason to doubt that the usual mathematized sayings of scientists are true. On this basis, Burgess and Rosen reject Fieldian nominalism, branding it 'anti-scientific' (see Burgess and Rosen [1997], pp. 32–4, pp. 205–25; Burgess [1983], [1990b]).

Were the doctrine of naturalized epistemology to assume a form that served to substantiate such criticisms it would be difficult not to wonder whether the doctrine was tenable. One might wonder whether naturalized epistemology understood to incorporate such reverence for what scientists say simply passes by (rather than replace or undermine) the intellectual concerns of traditional epistemology that motivate Field. Moreover, it may be doubted whether scientific method forms so seamless a web that the naturalized epistemologist can make such sweeping claims concerning the nature of science's epistemic standards. Nevertheless there is a related worry that is worth pursuing which—by contrast to Burgess and Rosen's—does not presuppose the doctrine of naturalised epistemology.

Even if there are weak representation theorems available there remains a further explanatory challenge for the fictionalist to overcome. Science does not advance nominalistically. Novel results are achieved in science by mathematized theorizing. It may be true in retrospect that nominalistic versions of these results may be constructed. Nevertheless some explanation is needed of why it is invariably mathematized rather than nominalized scientific theories that lead us forward to new discoveries. Perhaps the best explanation is that what mathematics says is true. And perhaps it would be ungrateful to think otherwise.

Of course, such an explanation would not be convincing were it offered on the assumption that the entities mathematics describes belong to a different world from the one inhabited by the concrete objects whose behaviour science seeks to predict and explain. For—as we reflected at the start—it is mysterious how abstract entities could ever serve an indispensable role in the successful

[13] See Burgess ([1984], [1990a], [1991], pp. 97–123); Burgess and Rosen ([1997], pp. 97–123); and—for related work—see Mundy ([1987a], [1987b]).

description of a world to which they do not belong. But such an explanation of the productivity of mathematized science might well induce credence if abstract numbers and concrete objects could be understood to inhabit the same world. It is just such a metaphysic that Neo-Fregeanism seeks to portray and will be the focus of the sequel of this paper.

Acknowledgements

I am grateful to Keith Hossack, Iain Law, Mike Martin, Alex Oliver, David Papineau, Stephen Read, Stewart Shapiro, and Crispin Wright for their comments on an earlier version of this paper.

Department of Logic and Metaphysics
University of St. Andrews
St. Andrews KY16 9AL
UK
fipm@st-andrews.ac.uk

References

Balaguer, M. [1996a]: 'Towards a Nominalisation of Quantum Mechanics', *Mind*, **105**, pp. 209–26.

Balaguer, M. [1996b]: 'A Fictionalist Account of the Indispensable Applications of Mathematics', *Philosophical Studies*, **83**, pp. 291–314.

Benacerraf, P. [1973]: 'Mathematical Truth', *Journal of Philosophy*, **19**, pp. 661–79.

Burgess, J. [1983]: 'Why I Am Not a Nominalist', *Notre Dame Journal of Formal Logic*, **24**, pp. 93–105.

Burgess, J. [1984]: 'Synthetic Mechanics', *Journal of Philosophical Logic*, **13**, pp. 379–95.

Burgess, J. [1990a]: 'Synthetic Mechanics Revisited', *Journal of Philosophical Logic*, **20**, pp. 121–30.

Burgess, J. [1990b]: 'Epistemology and Nominalism', in Irvine (*ed.*) [1990], pp. 1–15.

Burgess, J. [1991]: 'Synthetic Physics and Nominalist Realism', in C. W. Savage and P. Ehrlich (*eds*), *Philosophical and Foundational Issues in Measurement Theory*, Hillsdale, MI, pp. 119–38.

Burgess, J. [1993]: 'Review of Field', *Philosophia Mathematica*, **3**, pp. 180–88.

Burgess, J. and Rosen, G. [1997]: *A Subject with No Object: Strategies for Nominalistic Interpretation of Mathematics*, Oxford: Clarendon Press.

Chihara, C. [1990]: *Constructability and Mathematical Existence*, Oxford: Clarendon Press.

Field, H. [1980]: *Science without Numbers: A Defence of Nominalism*, Oxford: Blackwell Publishers.

Field, H. [1982]: 'Realism and Anti-Realism about Mathematics', *Philosophical Topics*, **18**, pp. 45–69; reprinted with postscript in Field [1989].

Field, H. [1984a]: 'Is Mathematical Knowledge Just Logical Knowledge?', *Philosophical Review* **93**, pp. 509–52; reprinted with postscript in Field [1989].

Field, H. [1984b]: 'Review of Gottlieb', *Nous*, **28**, pp. 160–6.

Field, H. [1985a]: 'Can We Dispense with Space-Time?', in P. Asquith and P. Kitcher (*eds*), *PSA 1984: Proceedings of the 1984 Biennial Meeting of the Philosophy of Science Association*, ii, pp. 30–90; reprinted with postscript in Field [1989].

Field, H. [1985b]: 'On Conservativeness and Incompleteness', *Journal of Philosophy*, **81**, pp. 239–60; reprinted in Field [1989].

Field, H. [1988]: 'Realism, Mathematics and Modality', *Philosophical Topics*, **9**, pp. 57–107; reprinted in Field [1989].

Field, H. [1989]: *Realism, Mathematics & Modality*, Oxford: Blackwell Publishers.

Field, H. [1990]: 'Mathematics Without Truth [A Reply To Maddy]', *Pacific Philosophical Quarterly*, **71**, pp. 206–22.

Field, H. [1991]: 'Metalogic and Modality', *Philosophical Studies*, **62**, pp. 1–22.

Field, H. [1992]: 'A Nominalistic Proof of the Conservativeness of Set Theory', *Journal of Philosophical Logic*, **21**, pp. 11–23.

Field, H. [1994]: 'The Conceptual Contingency of Mathematical Objects', *Mind*, **102**, pp. 285–99.

Field, H. [1998]: 'Mathematical Objectivity and Mathematical Objects' in S. Laurence and C. MacDonald (*eds*), *Contemporary Readings in the Foundations of Metaphysics*, Oxford: Blackwell Publishers, pp. 387–403.

Forrest, P. [1993]: 'Just Like Quarks? The Status of Repeatables', in J. Bacon, K. Campbell and L. Reinhardt (*eds*), *Ontology, Causality and Mind: Essays in Honour of D. M. Armstrong*, Cambridge: Cambridge University Press, pp. 45–65.

Hale, B. [1987]: *Abstract Objects*, Oxford: Blackwell Publishers.

Hale, B. [1990]: 'Nominalism', in Irvine (*ed.*) [1990], pp. 121–44.

Hale, B. and Wright, C. [1992]: 'Nominalism and the Contingency of Abstract Objects', *Journal of Philosophy*, **89**, pp. 111–35.

Hale, B. and Wright, C. [1994]: 'A Reductio Ad Surdum? Field on the Contingency of Mathematical Objects', *Mind*, **103**, pp. 169–84.

Hallett, M. [1990]: 'Physicalism, Reductionism and Hilbert' in Irvine (*ed.*) [1990], pp. 183–257.

Hart, W. D. (*ed.*) [1996]: *The Philosophy of Mathematics*, Oxford: Oxford University Press.

Hawthorne, J. [1996]: 'Mathematical Fictionalism Meets the Conjunction Objection', *Journal of Philosophical Logic*, **25**, pp. 363–97.

Irvine, A.D. (*ed.*) [1990]: *Physicalism in Mathematics*, Dordrecht: Kluwer Academic Publishers.

Lear, J. [1982]: 'Aristotle's Philosophy of Mathematics', *Philosophical Review*, **XCI**, pp. 161–92.

MacBride, F. [forthcoming]: 'Speaking with Shadows: A Study of Neo-Fregeanism', *British Journal for the Philosophy of Science*.

Maddy, P. [1990a]: 'Physicalistic Platonism', in Irvine (*ed.*) [1990], pp. 259–89.

Maddy, P. [1990b]: 'Mathematics and Oliver Twist', *Pacific Philosophical Quarterly*, **71**, pp. 189–205.

Maddy, P. [1990c]: *Realism in Mathematics*, Oxford: Clarendon Press.

Malamet, D. [1982]: 'Review of Field', *Journal of Philosophy*, **79**, pp. 523–34.

Mundy, B. [1987a]: 'Faithful Representation, Physical Extensive Measurement Theory and Archimedean Theory', *Synthese*, **70**, pp. 373–99.

Mundy, B. [1987b]: 'The Metaphysics of Measurement', *Philosophical Studies*, **51**, pp. 29–54.

Papineau, D. [1988]: 'Mathematical Fictionalism', *International Studies in the Philosophy of Science*, **2**, pp. 151–73.

Papineau, D. [1990]: 'Knowledge of Mathematical Objects', in Irvine (*ed.*) [1990], pp. 155–82.

Papineau, D. [1993]: *Philosophical Naturalism*, Oxford: Blackwell Publishers.

Parsons, C. [1990]: 'The Structuralist View of Mathematical Objects' *Synthese*, **84**, pp. 303–46.

Putnam, H. [1979]: *Mathematics, Matter and Method: Philosophical Papers, Vol. 1*, Cambridge: Cambridge University Press.

Resnik, M. [1980]: 'Review of Field', *Nous*, **27**, pp. 514–9.

Resnik, M. [1985a]: 'Ontology and Logic: remarks on Hartry Field's Anti-platonist Philosophy of Mathematics', *History and Philosophy of Logic*, **6**, pp. 191–209.

Resnik, M. [1985b]: 'How Nominalist is Hartry Field's Nominalism?', *Philosophical Studies*, **47**, pp. 163–81.

Resnik M, (*ed.*) [1995]: *Mathematical Objects and Mathematical Knowledge*, Dartmouth: Dartmouth Publishing Co.

Resnik, M. [1997]: *Mathematics as a Science of Patterns*, Oxford: Oxford University Press.

Shapiro, S. [1983a]: 'Mathematics and Reality', *Philosophy of Science*, **50**, pp. 523–48.

Shapiro, S. [1983b]: 'Conservativeness and Incompleteness', *Journal of Philosophy*, **80**, pp. 521–31.

Shapiro, S. [1984]: 'Review of Field', *Philosophia*, **14**, pp. 437–44.

Shapiro, S. [1985]: 'Second-order Languages and Mathematical Practice', *Journal of Symbolic Logic*, **50**, pp. 714–42.

Shapiro, S. [1993]: 'Modality and Ontology', *Mind*, **102**, pp. 455–81.

Shapiro, S. [1997]: *Philosophy of Mathematics: Structure and Ontology*, Oxford: Oxford University Press.

Tooley, M. [1987]: *Causation: A Realist Approach*, Oxford: Clarendon Press.

Urquhart, A. [1990]: 'The Logic of Physical Theory', in Irvine (*ed.*) [1990], pp. 145–54.

Wagner S. [1982]: 'Arithmetical Fictionalism', *Pacific Philosophical Quarterly*, **63**, pp. 255–69.

Wright C. [1988]: 'Why Numbers Can Believably Be', *Revue Internationale de Philosophie*, **42**, pp. 425–73; reprinted in Resnik [1995].