Quantifiers vs. Quantification Theory

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The syntax and semantics of quantifiers is of crucial significance in current linguistic theorizing for more than one reason. The last statement of his grammatical theories by the late Richard Montague (1973) is modestly entitled "The Proper Treatment of Quantification in Ordinary English". In the authoritative statement of his "Generative Semantics", George Lakoff (1971, especially pp. 238-267) uses as his first and foremost testing-ground the grammar of certain English quantifiers. In particular, they serve to illustrate, and show need of, his use of global constraints governing the derivation of English sentences. Evidence from the behavior of quantifiers (including numerical expressions [1]) has likewise played a major role in recent discussions of such vital problems as the alleged meaning-preservation of transformations, [2] co-reference, [3] the role of surface structure in semantical interpretation, [4] and so on.

In all these problems, the behavior of natural-language quantifiers is one of the main issues. Quantifiers have nevertheless entered the *Methodenstreit* of contemporary linguistics in another major way, too. (These two groups of problems are of course interrelated.) This is the idea that the structures studied in the so-called quantification theory of symbolic logic—otherwise know as first-order logic, (lower) predicate calculus, or elementary logic—can serve and suffice [5] as semantical representations of English sentences. Views of this general type have been proposed by McCawley (1971) [6] and Lakoff (1972) [7] (among others). A related theory of "Deep Structure as Logical Form" has been put forward and defended by G. Harman (1972). Theories of this general type may be compared with the traditional idea that quantification theory can be viewed as an abstraction from the behavior of natural-language quantifiers (as a representation of their "logical form"[8]) and with W.V. Quine's (1960) view of quantification theory as the "canonical notation" of all scientific discourse. It is not clear precisely how much is gained or lost linguistically according to these last two types of views in the transition from ordinary discourse to the language of first-order logic, but obviously some sufficiently loose "congruence of meaning"[9] is being assumed.

It will be shown in this paper that these views are seriously inadequate as they are usually formulated. (I shall not examine whether, and if so how, they can perhaps be repaired.) For this purpose, I will first sketch a more satisfactory semantical theory of certain English quantifiers, viz. those corresponding most closely to logicians' familiar universal quantifier and existential quantifier.[10] I shall indicate the range of problems that can apparently be dealt with by means of this theory, and go on to show how it naturally leads us to consider certain English quantifier expressions whose semantical representations go beyond first-order logic and hence are beyond the purview of the competing theories just mentioned. Indeed, we obtain in this way interesting specific results concerning the logical strength of quantification in English as compared with various kinds of logical systems. Finally, these results will prompt certain conjectures concerning the methodological asymmetry of syntax and semantics.

The semantics of English quantifiers I am about to sketch is formulated in terms borrowed from the mathematical theory of games, and might be referred to as game-theoretical semantics. (For the basic ideas of game theory, see e.g. Luce and Raiffa (1957) or Davis (1970).) The concepts involved are so straightforward, however, that they can be appreciated without any knowledge of game theory. This theory is a direct analogue to a corresponding game-theoretical semantics for formal (but interpreted) first-order languages, sketched in Hintikka (1968a) and (1973, Ch. 3.).[11] Its leading ideas can perhaps be best seen from this somewhat simpler case, which I will therefore first outline briefly.

Let's assume we are dealing with a language with a finite list of predicates (properties and relations). That we are dealing with an interpreted language means that some (non-empty) domain D of entities, logicians' "individuals", has been given and that all our predicates have been interpreted on D. This interpretation means that the extentions of our predicates on D are specified. This specification in turn implies that each atomic sentence, i.e., sentence in which one of our n-place predicates is said to apply to an n-tuple (sequence) of individuals, has a determinate truth-value, true or false. In a sense, what are doing here is to extend this definition of truth to all the other (first-order) sentences. They are obtained from atomic sentences by propositional operations, for instance, by applying "$\sim$" (negation), "$\wedge$" (conjunction), and "v" (disjunction)[12], and/or by applying existential or universal generalization, i.e., by replacing a number of occurences of an individual name by an individual variable (taken from the list "x," "y,"...) bound to an existential quantifier "$(\exists x)$," "$(\exists y)$,"... or to a universal one "(x)," "(y),"..., prefaced to the sentence.[13]

The essential restriction here is that only individuals (members of D) are being quantified over. Intuitively speaking, "every" thus means "every *individual*" and "some" means "some *individual*." Examples are offered by such expressions as (1), (29), and (33) below. (Note that all quantifiers in them range over individuals.) In contrast, as soon as we are binding predicates, predicates of predicates, or any other variables of a higher type[14], for instance quantifying over all the subsets of D, or over all the subsets of some other (infinite) set, we are no longer dealing with first-order logic, and the situation is entirely different. Examples of second-order sentences are offered by (2), (32), and (34) below, with their quantifiers ranging over functions.

Just how different the situation is from first-order logic as soon as we let in any second-order quantifiers is shown by an earlier observation of mine (Hintikka 1955, pp. 100-101) to the effect that even one single universal quantifier with a monadic (one-place) second-order predicate variable in a sense gives us all the power not only of second-order logic but of all of the other (finite) types as well. For any sentence S of the theory of (finite) types, a sentence $r(S)$ involving only one such quantifier (over and above constant predicates and first-order quantifiers) can be found effectively which is valid (logically true) if and only if S is.

The game-theoretical semantics serves to extend the concept of truth from atomic sentences to all others. This is accomplished by correlating with each sentence S of our interpreted first-order language a two-person game $G(S)$. It may be thought of as zero-sum game (i.e., what one player wins, the other one loses). The players will be called Myself and Nature. The former is trying to produce eventually a true atomic sentence, the latter a false one. At each stage of the game, a sentence S' of our language (or of a mild extension of it) is being considered, beginning with S. The next move depends on the form of S' according to the following rules.

(G.E) If S' is of the form $(\exists\ x)\ S''$, the next move is made by Myself: I go to the domain D, choose an individual, give it a name[15] (if it did not have one already), say "b". This name is plugged in for "x" in S'', the result being called "$S''(b/x)$". The game is continued with respect to $S''(b/x)$.

(G.U) If S' is of the form $(x)S''$, the same thing happens except that the individual is chosen by Nature.

(G.v) If S' is of the form $(S'' \vee S''')$, I choose one of the disjuncts S'' and S''' and the game is continued with respect to it.

(G.$\wedge$) Likewise if S' is of the form $(S'' \wedge S''')$, except that the choice is made by Nature.

(G.$\sim$) If S' is of the form $\sim S''$, the roles of the two players (as defined by the game rules and by the rule for winning soon to be defined) are switched and the game continued with respect to S''.

In a finite number of moves, the game yieds an atomic sentence. Winning and losing is defined by reference to it, as already indicated. If the outcome is a true sentence, the winner is Myself and the loser Nature; if false, *vice versa.*

In Hintikka (1973, p. 101), I have shown how the personification of Nature apparently involved in the games just explained can be dispensed with. Hence it does not cause any problems here.

The concept of truth for non-atomic sentences can now be defined. A sentence S is true (in the model constituted by D and by the interpretation of our predicates on D) if and only if I have a winning strategy in the correlated game $G(S)$, that is to say, if I can always choose my moves (depending on what has already happened in the game) so that I will win no matter what Nature does.

Clearly this definition of truth is equivalent to the usual formulations presented in the semantics (model theory) of first-order logic.[16] For if S is true, I can choose my moves so that all the sentences obtained during the game are true (and *mutatis mutandis* when a switch of roles takes place), and *vice versa.* In spite of this equivalence, even in pure first-order logic the game-theoretical approach may offer philosophical, linguistic, and heuristic advantages. For one thing, the games I have defined are the only explicitly defined games so called both in the sense of game theory and in the sense of Wittgenstein's language-games (see Wittgenstein 1953, 1958, Hintikka 1968b) to be found in the literature. (They are language-games because they are rule-governed activities serving to connect our language to the reality it speaks of.) Clearly the activities involved in my moves governed by (G.E) are not random choices of individuals from D, if I want to win the game. Rather, a realistically understood move normally involves careful searching for a suitable individual. Thus the semantical games connected with quantifiers are essentially games of searching and finding. In Hintikka (1973), chapters 5 and 10, some of their philosophical implications are discussed.

Moreover, game-theoretical semantics is much more readily extended beyond first-order logic in certain directions than the usual one, as will be seen later in this paper.

My game-theoretical semantics can be further illustrated by noting that it naturally yields a translation of first-order logic into a fragment of second-order logic. For if the truth of S means the existence of a winning strategy of a certain sort, I can express this by a suitable explicit sentence asserting the existence of such strategy. Now my strategies are (on the classical interpretation of first-order logic) defined by those functions that tell me, as a function of Nature's earlier moves, which individual to choose when I make a move. Such functions are known to logicians as Skolem functions. [17]

The translation we obtain will simply assert the existence of suitable Skolem functions. For instance, I have a winning strategy in the game correlated with the sentence

(1) $$(x)\ (\exists y)\ (z)\ (\exists w)\ F\ (x, y, z, w)$$

(where F may contain propositional connectives but does not contain any quantifiers) if and only if the following second-order sentence is true.

(2) $$(\exists f)\ (\exists g)\ (x)\ (z)\ F\ (x, f\ (x), z, g\ (x, z))$$

Here the function f tells me how to choose the individual serving as the value of "y" in (1), and g how to choose the individual whose name is to be substituted for "w" in (1). The second-order sentence (2) as a whole says that there are functions (strategies) of this sort such that no matter what x and z are chosen by Nature, the outcome is true (i.e., such as to admit a winning strategy in the rest of the game). Hence (2) can serve as a translation (explication) of (1).

In general, the use of Skolem functions can usually be interpreted in terms of my game-theoretical semantics, and conversely this semantics can be viewed as a kind of systematization and generalization of the notion of Skolem function. The main ideas of this kind of game-theoretical treatment of first-order logic have already for some time belonged to the bag of tricks of all sophisticated logicians, although they have not always been formulated systematically. What has not been pointed out before is that this game-theoretical semantics can be extended to natural languages like English. There are of course no variables in English for which names can be substituted, but the basic idea is that a whole quantifier phrase beginning with "some", "every", "any", etc. can be substituted for in the same way as a formal variable.[18] As a special case of the resulting game rules, consider an English sentence of the form[19]

(3) X - some Y who Z - W

(where Y and Z are singular). A game rule (G. some) will now tell me to choose a member of D and give it a proper name (say "b") just as in the case of formal first-order languages. The proper name is then plugged in for the quantifier phrase. Of course extra clauses are now needed to take care of the quantifier phrase itself. Thus the resulting sentence will be

(4) X - b - W, b is a(n) Y, and b Z.

In fact, a general formulation of the rule (G. some) can be gathered from this example. I will not attempt an explicit formulation here, however. The syntactical part of the rule would involve (among other things) directions for taking a relative clause and a noun phrase of a suitable sort and for constructng from them a sentence from which the relative clause could be formed by the usual rules of relative clause formation in English. In brief, it would involve writing down explicit rules for relative clause formation in the reverse direction (i.e., in the form of rules unpacking a relative clause rather than forming one).[20] It is not my purpose to discuss the details of such rules here, and hence I will let a special case of game rules like (G. some) do duty for the whole rule. (Let me nevertheless point out the obvious fact that the explicit rule would of course restrict the above form of (3) to the cases in which "who" in "who Z" occupies the subject position.)

Likewise, the rule (G.every) would take us (via Nature's choice of a member d of the given domain D) from

(5) X - every Y who X - W

to

(6) X - d - W if d is a(n) Y and if d Z.

Likewise, (6) could be the result of applying (G.any) to the sentence

(7) X - any Y who Z - W.

Since (4) and (6) are grammatical English sentences (if (3), (5), and (7) are), the game can be continued with respect to them.

A rule (G.an) similar to (G.some) may be formulated for the indefinite article,[21] and (G.some) extended to plural uses of "some". Numerous further extensions are also possible.

In applying rules for propositional connectives - we shall call them (G.and), (G.or), (G.if), and (G.not) - certain new aspects of the situation (as compared with formal first-order languages) have to be heeded.

(i) The fully explicit form of the rule for negation (G.not) and of the rule for conditionals (G.if) would have to involve directions for forming sentence negation in English, operating in a direction opposite to the usual one (i.e., as rules for going from a negated sentence to the corresponding unnegated one).[22] Again, I will not discuss the details here.

(ii) Sentential game rules for conjunctions, disjunctions, conditional sentences, etc. are normally inapplicable in the presence of pronominal cross-references between conjuncts, disjuncts, etc. They may be applied, however, in the special case in which the pronominalizing NP is a proper name. Then they must be combined with suitable directions for depronominalization.

(iii) It is easily seen that (ii) is not enough to handle all pronominalization problems. In some cases, changes of pronominalization may have to be built into my game rules. This problem will not be examined here, however. One particularly natural course might be to marry my game rules to the treatment of pronominalization which has been recently proposed by R. Grandy and presented in G. Harman (1973). However, I am not ready yet to commit myself to one particular treatment.

The scope of game-theoretical semantics can be expanded by wedding it to a suitable possible-worlds semantics for epistemic and modal notions. (For the basic ideas of the latter, see Hintikka (1969).) The extension is very simple. Instead of just one "world" (D with predicates interpreted on it), we must consider a set of them with suitable alternativeness relations defined on it. At each stage we are considering a world ω and a sentence. The progress of a game is seen from an example. When we have reached "John doesn't know that X" and a world ω_1, I may select an epistemic alternative ω_2 to ω_1 with respect to John, and the game is continued with respect to neg + X and ω_2.[23] When we have reached "John knows that X", Nature may select a similar world ω_3 and the game is continued with respect to X and ω_3. (In each case, pronominal cross-references to "John" in X must also be replaced by this name.)

Examples of what might happen (syntactically speaking) in specific games may help to understand my game-theoretical semantics. The following sentences may come about in the course of a play of the game correlated with first one.

(8) Some gentleman who loves every blonde married a brunette.

My move by (G.some) may yield here

(9) John married a brunette, John is a gentleman, and John loves every blonde.

Nature's move by (G.and) may yield

(10) John loves every blonde.

Nature's move by (G.every) may here yield

(11) John loves Susan if Susan is a blonde.

Here (G.if) gives me (as you can guess from the truth-table of "if") a further choice between

(12) John loves Susan

and

(13) Susan isn't a blonde.

Another example shows that crossing pronominalization does not cause any special problems here. A round of a game might take us successively to the following sentences.

(14) Some boy who was fooling her kissed a girl who loved him.

(15) John kissed a girl who loved him, John is a boy, and John was fooling her.

Here Nature cannot choose by (G.and) the last conjunct because it involves a pronominal reference to "a girl" in the first one. (See point (ii) a few paragraphs back.) However, (G.an) can be applied to "a girl", yielding something with the same form as the following.

(16) John kissed Sally, John is a boy, John was fooling her, Sally is a girl, and Sally loved him.

Here we have utilized a simple convention of omniting "and" between conjuncts other than the last two. In (16), pronominal references are all to proper names, and (G.and) therefore applies. The rest of the game is not of interest here.

Although work is still in progress on the game-theoretical semantics of English quantifiers, it is already clear that it gives us a method of dealing with several different kinds of problems. Some of these applications are immediately suggested by our general game-theoretical point of view.

As a by-product of my semantics, we obtain an approach to the syntax of English quantifier sentences.[24] The game rules involved as their syntactical part a kind of decomposition of sentences with quantifiers into simpler sentences. Turning around this syntactic part of our semantical rules, we can obtain a set of rules for building up English quantified sentences. (For the purposes of explicit syntax the proper names that will be introduced in the course of a round of a game may of course be replaced by variables serving as dummy names.) They differ from the types of rules fashionable in transformational grammar in these days, which are therefore useless for my purposes. One reason for this difference is that much recent syntax has been motivated by a desire to uphold the meaning-preservation of transformations. (Cf. Partee 1971.) Our "transformations" do not preserve meaning because they serve to introduce quantifiers. The example (14) - (16) above suggests that this need not be a drawback syntactically. Certain problems at least become easier to handle.

A major difference as compared with formal first-order languages is that our game rules so far formulated can often be applied in more than one order. This seems to me an advantage rather than a disadvantage. Natural languages clearly employ further principles (ordering principles) to govern this order, which in more familiar logical terminology often amounts to governing the scope of various logical operations.[25]

Such principles can easily be formulated (just because the order of game rules is otherwise left open), and they will (among other things) take over in game-theoretical semantics the role of Lakoff's global constraints for quantifiers. (See Lakoff 1971, and cf. my comments on Jackendoff below.)

It turns out, however, that Lakoff's constraints are not quite adequate in their present form. For one thing, our principles can be applied more flexibly so as to explain certain multiple ambiguities (e.g., "three hunters shot five tigers and four panthers") apparently in a more satisfying manner than Lakoff's account. (Cf. Partee (1970), (1971), and Lakoff (1970).) Moreover, Lakoff's constraints are not universally applicable. On the contrary, we have here an interesting possibility of characterizing the meaning of certain special quantifiers, especially that of "any", by specifying the peculiar ordering principles the game rule (G.any) governing it obeys vis-à-vis other rules. I conjecture that a full characterization of the meaning of "any" can be given in this way[26] and that we can in this way also explain its distribution in the sense that it can apparently ocour only where the ordering principles create a semantical difference between "any" and the "normal" quantifiers "every" and "all".[27] For instance, (G.any) has as a rule the right of way with respect to (G.if) and (G.not), whereas (G.every) does not.

Besides calling our attention to the possible underdeterminacy of meaning due to the vagaries of order of the game rules, the game-theoretical viewpoint suggests that another kind of underdeterminacy may be expected. It is the uncertainty as to which player is to make a move associated with a phrase in English. It turns out that this is in fact the case with the interrogative morpheme "wh-". An essential part of its semantical behavior, besides its operating as a kind of quantifier, is that either player may make a move connected with it. Thus our game rules may take us from a sentence like

(17) John knows who is coming.

either (through a move by Myself) to[28]

(18) John knows that Bill is coming, and Bill is coming.

or (through a move by Nature) to[29]

(19) John knows that Arthur is coming, if Arthur is coming.

These two correspond to the two readings of (17) which may be represented respectively as

(20) $(\exists x)$ [John knows that x is coming $\wedge$ x is coming]

and as

(21) $(\exists x)$ [x is coming $\supset$ John knows that x is coming].

This kind of underdeterminacy of meaning (due to an underdeterminacy of the player in question) is indispensable for understanding the semantics of English questions[30], especially of multiple wh-questions.[31]

Other applications of game-theoretical semantics seem to be in the offing. For instance, the so-called problems of coreference do not cause any special difficulties in our approach.[32]

The most striking observations suggested by our game-theoretical semantics are nevertheless the most straightforward ones Our whole discussion so far has been based on an assumption which is *a priori* completely arbitrary. We have ben assuming that our semantical games are games with perfect information. (For this notion, cf. e.g. Luce and Raiffa (1957), p. 43.) Intuitively speaking, this means that a player always comes to know, and never forgets, what has happened at earlier stages of the game. This is not always the case in real games, for a player may be prevented by the very rules of game from knowing what has happened at certain earlier moves. Then he has to make a move without knowing the outcome of these earlier moves, which of course affects the strategies he has available. These strategies are now defined by functions independent of the unknown earlier moves.

This possibility of the failure of perfect information affects in principle already the games connected with formal first-order languages. However, if the requirement of perfect information is relaxed, we are not dealing with first-order logic any longer, but with an essentially different kind of logic.

What it is can be seen from the idea that in first-order logic a quantifier which lies in the scope of another depends on the latter in the sense that the move connected with the former depends on that associated with the latter. This is reflected by the dependence of Skolem functions on all earlier universally quantified variables. (See, for instance, how the function g in (2) has both x and z as its arguments.) Hence linearly ordered quantifiers cannot express the failure of perfect information. Instead, we must allow branching quantifiers and, more generally, partially ordered (but still finite) quantifiers.[33] We must thus resort to sentences that may look like the following examples.

(22) $$\left.\begin{array}{l}(\exists x)\\(y)\end{array}\right\} F(x, y)$$

(23) $$\left.\begin{array}{l}(x)(\exists y)\\(z)(\exists w)\end{array}\right\} F(x, y, z, w)$$

(24) $$(\exists x)\left\{\begin{array}{l}(y)(\exists u)\\(z)(\exists w)\end{array}\right\} F(x, y, z, u, w)$$

(25) $$\left.\begin{array}{l}(x_1)(x_2)\ldots(x_i)(\exists y_1)(\exists y_2)\ldots(\exists y_j)\\(z_1)(z_2)\ldots(z_i)(\exists w_1)(\exists w_2)\ldots(\exists w_j)\end{array}\right\} \begin{array}{l}F(x_1, x_2,\ldots,x_i, y_1, y_2,\ldots, y_j,\\ \quad z_1, z_2,\ldots, z_i, w_1, w_2,\ldots, w_j)\end{array}$$

(26) $$\left.\begin{array}{l}(x_1)(y_1)(\exists z_1)\\(x_2)(y_2)(\exists z_2)\\ \quad\ldots\\(x_k)(y_k)(\exists z_k)\end{array}\right\} F(x_1, x_2,\ldots, x_k, y_1, y_2,\ldots, y_k, z_1, z_2,\ldots,z_k)$$

Here F is assumed not to contain any quantifiers any longer, although it may of course contain sentential connectives. Notice that only the left-to-right order of quantifiers counts in (22) - (26). The up-and-down order of branches is of course irrelevant.

The totality of sentences obtainable in this way, with the obvious game-theoretical semantics associated with them, is known as the theory of finite partially-ordered (f.p.o.) quantification.[34] The game rules for f.p.o. quantified sentences are the same as of old, except that moves connected with quantifiers in one branch are made without knowledge of those connected with the quantifiers in another.

All f.p.o. quantifier sentences can be translated into the sentences of ordinary (linear) second-order sentences in the same way as first-order sentences. Precisely the same procedure as was used in going from (1) to (2) above is applicable here, too. The only difference is that the Skolem functiin associated with an existential quantifier (say "$(\exists x)$") depends on (i.e., has as its arguments) only variables bound to those universal quantifiers which occur earlier in the same branch (more generally, which bear the partial ordering relation to "$(\exists x)$"). (It is easily shown, as pointed out in Enderton (1970, p. 394), that other sorts of dependencies do not matter anyway.) For instance, in the same way as (1) becomes (2), (23) thus becomes

(27) $(\exists f)(\exists g)(x)(z)F(x, f(x), z, g(z))$

and (25) becomes

(28) $(\exists f_1)...(\exists f_j)\ (\exists g_1)...(\exists g_j)\ (x_1)...(x_i)\ (z_1)...(z_i)$
$F(x_1,..., x_i, f_1(x_1,..., x_i),..., f_j(x_1,..., x_i), z_1,..., z_i, g_1(z_1,..., z_i),..., g_j(z_1,..., z_i))$.

These Skolem function reformulations of f.p.o. sentences also serve to show more fully how semantical games connected with these sentences are played, precisely in the same way as (2) brings out more explicitly the strategies available in the game associated with (1). Skolem functions again serve to define that part of my strategies which deals with the applications of (G.E).

Finite partially ordered quantifiers have not been studied very long yet by logicians. They were introduced in Henkin (1961), and are sometimes referred to as Henkin quantifiers. I have located only two other substantial studies of them in the literature. (See Enderton 1970 and Walkoe 1970.) A few facts are nevertheless known about f.p.o. quantification theory. Of course, not all branching quantifiers result in sentences which have no first-order equivalents. For instance, (22) is easily shown to be equivalent to

(29) $(\exists x)(y)F(x, y)$.

It is known that most of them go beyond first-order logic, however. In fact, (23) represents the simplest kind of sentence which does not have any first-order equivalents,[35] and all the other kinds may be thought of as more complicated versions of (23).[36] For it is shown in Walkoe (1970, p. 542) that any f.p.o. quantifier sentence is equivalent to a sentence of form (25) and likewise to one of form (26).

The greater force of the f.p.o. quantification theory as compared with first-order logic prompts the question whether branching quantifiers (failures of perfect information) can be found among natural-language quantifiers. Quine has argued (1969, pp. 107-112; 1970, pp. 89-91) for a negative answer to this question, but his tentative negative conclusion is a mistaken one. It is in fact easy to find any number of examples of branching quantifiers in perfectly grammatical English, including quantifier sentences which do not have any first-order equivalents. The following example is not the best one intrinsically but nevertheless allows for a more intuitive insight into the semantical situation than many others.

(30) Every writer likes a book of his almost as much as every critic dislikes some book he has reviewed.

I shall argue, if the point is not already obvious, that (30) can be considered an instance of branching quantification representable in the form (23) as follows.

(31) $\left.\begin{matrix}(x)\,(\exists\, y)\\(z)\,(\exists\, u)\end{matrix}\right\}$ [(x is a writer $\wedge$ z is a critic) $\supset$
(y is a book $\wedge$ x has authored y $\wedge$ u is a book $\wedge$ z has reviewed u $\wedge$ x likes y almost as much as z dislikes u)]

This admits of the second-order translation

(32) $(\exists\, f)$ $(\exists\, g)$ (x) (z) ((x is a writer $\wedge$ z is a critic) $\supset$ ($f(x)$ is a book $\wedge$ x has authored $f(x)$ $\wedge$ $g(z)$ is a book $\wedge$ z has reviewed $g(z)$ $\wedge$ x likes $f(x)$ almost as much as z dislikes $g(z)$))

but not of any first-order translation. In a first-order translation the quantifiers will have to be linearly ordered, which creates the kind of dependence which is not being presupposed in (30). In order to see this, notice that since the left-to-right order would presumably be followed here, the prime candidate for a first-order translation of (30) is apparently

(33) (x) [x is a writer $\supset$ $(\exists\, y)$ (y is a book $\wedge$ x has authored y $\wedge$ (z) (z is a critic $\supset$ $(\exists\, u)$ (u is a book $\wedge$ z has reviewed u $\wedge$ x likes y almost as much as z dislikes u)))].

which has a "game-theoretical" second-order equivalent

(34) $(\exists\, f)$ $(\exists\, g)$ (x) (z) [x is a writer $\supset$ ($f(x)$ is a book $\wedge$ x has authored $f(x)$ $\wedge$ (z is a critic $\supset$ ($g(x, z)$ is a book $\wedge$ z has reviewed $g(x, z)$ $\wedge$ x likes $f(x)$ almost as much as z dislikes $g(x, z)$)))].

Here (32) is stronger than (34) in the relevant respect, for a function with fewer arguments can always be thought of as a degenerate case of a function with more arguments. This extra force of (32) is due to the requirement that the values of y and u in (31) are determined on the sole basis of x and z, respectively. Hence the situation in which (32) (and therefore also (34)) is true will be described by the following sentence.

(35) Every writer likes his first book almost as much as every critic dislikes the latest book he reviewed.

Since (35) implies both (32) and (34), it does not yet serve to distinguish them. We can distinguish them intuitively, however, by considering the situation described by the following sentence.

(36) Every writer likes his latest book almost as much as every critic dislikes the first book by that writer he had to review.

It is of course being assumed here that every critic has reviewed at least one book by each writer—a not unimaginable state of affairs.

In circumstances in which (36) is true, (33) - (34) are clearly true. Nevertheless (32) is not. For it may still be true that different writers like their most recent efforts to entirely different degrees, as long as for each separate writer there obtains a nearly uniform degree of disapprobation among critics towards the earliest effort of that particular writer they hat to pronounce themselves on. This is not enough to make (32) true, for it requires that for all pairs of writers and critics books can be found towards which they *all* exhibit about the same intensity of feeling.

The remarkable thing here is that by pretty much the same token it can be seen that (36) does *not* imply (30), either, on the intended, natural reading of (30). This provides strong evidence that (30) can be expressed as (32) but not as (34). The same conclusion follows *a fortiori* if we try other linear orderings of quantifiers in interpreting (30). For instance, if we give the two universal quantifiers the widest scope in (30), we obtain the reading

(33)* (x) (z) [x is a writer $\wedge$ z is a critic) $\supset$ $(\exists y)$ (y is a book $\wedge$ x has authored y $\wedge$ $(\exists u)$ (u is a book $\wedge$ z has reviewed u $\wedge$ x likes y almost as much as z dislikes u))]

which has as its second-order counterpart the following sentence.

(34)* $(\exists f)$ $(\exists g)$ (x) (z) [(x is a writer $\wedge$ z is a critic) $\supset$ ($f(x), z$) is a book $\wedge$ x has authored $f(x, z)$ $\wedge$ $g(x, z)$ is a book $\wedge$ z has reviewed $g(x, z)$ $\wedge$ x likes $f(x, z)$ almost as much as z dislikes $g(x, z)$)].

The same example (36) which showed that (32) and (34) differ in meaning and which showed that (30) cannot naturally be understood as (34) serves to demonstrate the same of (34)* vis-à-vis (32) and (30).

For in the circumstances envisaged in (36) the sentences (33) * - (34) * are clearly true while (32) and (30) may be false by the same token as before. Hence we are again led to conclude that (30) must have the force of (32).

It follows from what has been said earlier that (32) does not have any first-order synonyms. Hence if what we just found is right and (30) and (32) have the same force, we have in (30) an example of an English sentence which is essentially quantificational but which cannot be translated into the first-order notation.

Once this is seen, similar examples can be constructed at will. They cannot be much simpler than (30), however, for (as was already stated) it takes four quantifiers to reach a sentence that cannot be translated into a first-order language.

It is nevertheless clear that we can find more natural examples of "branching quantification" than the single one Quine (1969, p. 109) disparages and dismisses in his discussion of non-standard quantification. The following is about as simple (syntactically simple) an example as I have found.

(37) Some relative of each villager and some relative of each townsman hate each other.

This may even offer a glimpse of the ways in which branched quantification is expressed in English. Quantifiers occuring in conjoint constituents frequently enjoy independence of each other, it seems, because a sentence is naturally thought of as being symmetrical semantically vis-à-vis such constituents.

Notice that in order for (37) to be true the relative of each townsman mentioned in it must not depend on the villager. In other words, (37) need not be true in a situation described as follows.

> The eldest relative of each villager and that relative of each townsman who is closest in age to the villager hate each other.

Yet in such a state of affairs a suitable linear-quantifier sentence analogue to (33) would be true. This example therefore serves to indicate why the linear-quantifier reading of (37) will not quite do.

Other examples can be expressed in the vocabulary which was already studier by Russell (1903). The following is a case in point, assuming that *a* and *b* are sets of sets.

(38) Some element of each *a* contains some element of each *b*.

In (38) and (37) the preposition "of" is used, although no semantical rules are given for it. The reason is simple: in these examples no special rules for it are needer, except for an ordering principle which tells us to unpack iterated of-phrases from right to left. What ordering principles are perhaps needed in other cases and what supplementary rules have to be formulated in an exhaustive treatment of "of" is not discussed here.

In order to avoid unnecessary misunderstanding, it is in order to point out that certain sentences not unlike (30) admit of a first-order translation after all. My discussion has been predicated on treating "x likes y almost as much as z dislikes u" as expressing a relation whose analysis does not involve further quantifiers. Even if this assumption should fail in our particular case, the example still shows how others can be found in which the matrix "F" of (23) does not contain any hidden quantifiers. The example (30) was chosen for expositional convenience only, and I am not putting forward any theses about its ultimate analysis.

In fact, someone might be tempted to analyze (30) as follows.

(30)* $(\exists d)$ [d is a degree of feeling $\wedge$ (x) $(\exists y)$ (x is a writer $\supset$ (y is a book $\wedge$ x has authored y $\wedge$ x likes y almost to the degree d)) $\wedge$ (z) $(\exists u)$ (z is a critic $\supset$ (u is a book $\wedge$ z has reviewed u $\wedge$ z dislikes u to the degree d))].

It is important to realize that the kind of analysis represented by (30)* is not unproblematic but depends on a kind of reification of "degrees of feeling". Such a reification is possible only when the underlying relational structure satisfies suitable, rather strict conditions. Roughly speaking, these conditions are those that would suffice to establish a numerical representation theorem. Normally, these conditions are not satisfied. Whether they are satisfied in the case of (30) is not clear, and depends on the logic of "almost". In case you want (30) replaced by an example which is not subject to this mild uncertainty, the following will do.

(39) Some book by every author is referred to in some essay by every critic.

A situation analogous to that described by (35) is one in which

(40) The bestselling book by every author is referred to in the longest essay by every critic.

By this I mean a situation in which (39) is true (on its natural, f.p.o. reading). A state of affairs in which (39) is false on this reading but in which the most plausible of its putative linear-quantifier readings is true is described by

(41) The bestselling book by every author is referred to in the obituary essay on him by every critic.

This is in other words analogous to (36). The point is that the essay in question is not chosen in (41) on the basis of the critic alone, but depends also on the author in question. Thus we can see that the need of a branching-quantifier interpretation does not depend on the use of comparatives or of the notion of sameness.

The existence of quantificational sentences in English which cannot be translated into the language of first-order logic is not the only consequence of our observations, however, although it may be the most striking one. A no less interesting consequence concerns the overall situation even in the case of those failures of perfect information which do not lead to such breakdowns of translatability. When a breakdown of perfect information occurs, we have to translate an English sentence into the first-order notation in a way different from the one we would have followed if perfect information had prevailed. The possibility of imperfect information therefore affects our ideas of the semantic interpretation of many English sentences whose "logical structure" can be exhibited in first-order terms.

A case in point seems to be the following sentence.

(42) John has not shown any of his paintings to some of his friends.

If translated into first-order notation on the assumption that the requirement of perfect information in satisfied, the preferred reading of (42) will presumably be (because " any" and "some" typically have priority over "not")

(43) (y) [y is a painting of John's $\supset$ $(\exists x)$ (x is a friend of John's $\wedge$ John has not shown y to x)]

Yet the reading of (42) that in fact is the most natural one by far assigns to it the force of

(44) $(\exists x)$ [x is a friend of John's $\wedge$ (y) (y is a painting of John's $\supset$ John has not shown y to x)]

One might suspect that this exception to the general ordering principles of English has something to do with negation. However, the following example exhibits the same peculiarity as (42).

(45) John has shown all his paintings to some of his friends.

One might also try to account for the switch from (43) to (44) by some further complication in our rules and principles. However, no natural revision of our ordering principles that would explain the reading (44) of (42) is in the offing.

The most natural explanation of the actually preferred reading (44) of (42) is to assume that the two quantifiers in (42) are independent. On this assumption, (42) will be (on its most natural reading) of the following "logical form".

(46) $$\left.\begin{matrix}(\exists x)\\ (y)\end{matrix}\right\} [x \text{ is a friend of John's} \wedge (y \text{ is a painting of John's} \supset \text{John has not shown } y \text{ to } x)].$$

This is easily seen to be logically equivalent to a first-order sentence, viz. (44), just in the same way (22) is logically equivalent to (29).

Thus we often have to take into account branching quantifiers even when they do not lead us beyond first-order logic, viz. for the purpose of explaining which first-order semantical representation a given English sentence has.

Although I do not propose to treat nonstandard quantifiers in this paper, it may be worth pointing out in the passing that the phenomenon of branching quantifiers is highly relevant to well-known examples of the type

I told three of the stories to many of the men.

The reading of such sentences which Jackendoff (1972, pp. 307-308) finds impossible to represent in a logical notation results precisely from making the nonstandard quantifiers "three" and "many" independent.[60]

It is indeed easy to ascertain that the "unaccountable" reading is simply an instance of branching quantification. In fact, examples of the kind Jackendoff uses can be employed to show that differences in meaning are often created more readily by branching nonstandard quantifiers than by standard ones.

Thus the realization that formal quantifiers may be unordered thus removes the whole basis of Jackendoff's criticism of attempts to account for the ambiguities which are due to multiple nonstandard quantifiers in terms of different underlying quantifier structures.

What we have seen shows conclusively that all attempts to base the semantical interpretation of natural languages like English on first-order logic alone are bound to be inadequate, and inadequate in a particularly interesting way.

Several logicians and linguists [37] have argued that the semantics of natural languages has to go beyond first-order logic in order to account for the semantics of such expressions as "knows", "believes", "necessarily", "may", etc. However, the extensions of first-order logic (quantification theory) needed to cope with these epistemic and modal notions are entirely different from the ones involved here, and relatively unsurprising. For it has been generally thought of that, whatever else quantification theory perhaps fails to capture, the one thing it does accomplish is to represent the logical form of the basic English quantifiers. This illusion has now been shattered.

It also turns out that our step away from first-order logic is in a sense much longer than those involved, e.g., in ordinary modal or epistemic logic. This point will be argued later in this paper.

Another consequence of our results is that the concept of scope is not alone sufficient to unravel the interplay of quantifiers in English. What we can do by means of the notion of scope is to describe the order of applications of the different rules. However, questions of informational independence of the different applications cannot be formulated by speaking just of the scopes of quantifiers.

We can also push our observations further. Not only do we have some of the irreducible logical forms of f.p.o. quantification theory represented by grammatical English sentences. A plausible (although not completely binding) argument can be put forward to the effect that every formula of f.p.o. quantification theory is reproducible as the semantical structure of some grammatical English sentence.

In order to obtain such an argument let us note that certain grammatical constructions of English either force or at least frequently give a preference to a reading of certain quantifiers which makes them independent. It was already noted that this is the case with quantifiers occuring in conjoint structures. Frequently this also applies to quantifiers occuring in different nested of-phrases. The following example, modified from an example by Julius M. E. Moravcsik (personal communication), is a case in point

(47) Some family member of some customer of each branch office of every bank likes somes product of some subdivision of each subsidiary of every conglomerate.

This sentence has the form

(48)
$$\left.\begin{array}{l}(x_1)\ (x_2)\ (\exists\, y_1)\ (\exists\, y_2) \\ (z_1)\ (z_2)\ (\exists\, w_1)\ (\exists\, w_2)\end{array}\right\} F\,(x_1, x_2, y_1, y_2, z_1, z_2, w_1, w_2)$$

It is also obvious that arbitrarily long branches of quantifiers can be built up in the same way. In fact, any quantifier prefix exemplified by (25) can easily be obtained in this way.

Since any f.p.o. quantifier sentence has an equivalent sentence of the form (25), this seems to imply that the structure of every f.p.o. quantifier sentence is reproducible in English. This conclusion is in fact most likely true, but it does not follow quite strictly from what has been said. For one thing, the special form of nested of-phrases may induce restrictions as to what may occur as the formula F of sentence (25). Further arguments can be given to show that this does not spoil the possibility of finding English sentences of any form (25), but they will not be attempted here.[38]

Moreover, we ought to rule out also the possibility that analytical connections between English substitution-instances may likewise restrict the kinds of F in (25) reproducible in English. Although this objection is clearly less serious than the preceding one, for reasons of space it will not be met in detail here.

Instead, further supporting evidence will be given for the thesis that the logical form of every sentence of f.p.o. quantification theory can be matched by an English sentence. Another grammatical device that induces independence between English quantifiers is the construction known as relative clause with several antecedents. Although the status of this construction is not entirely clear (cf. Perlmutter and Ross 1970), it seems to me impossible to deny that it yields grammatical English sentences. A case in point is the following.

(49) Every villager envies a relative and every townsman admires a friend who hate each other.[39]

An argument similar to the one we conducted in connection with (30) shows that the two pairs of quantifiers in (49), and likewise in the different antecedents of one and the same relative clause in general, are naturally understood as being independent. (Intuitive symmetry considerations have again a valuable heuristic use here in convincing one of this independence.) Moreover, in so far as the grammatical device of split antecedents is acceptable at all, it can obviously be used so as to assign to a relative clause more than two antecedents. Equally obviously, there cannot be any definite limit to the number of antecedents. Moreover, the antecedents can contain three quantifiers instead of two. For instance, the following sentence appears perfectly grammatical, although it is rather complicated.

(50) Each player of every baseball team has a fan, each actress in every musical has an admirer, and each aide of every senator has a friend, who are cousins.

Clearly, (50) is of the form (26) with $k = 3$. It is also easily seen that similar examples can be constructed at will. Since the number of antecedent clauses apparently cannot be restricted, we can in this way see that all prefixes of form (26) are exemplified in English. Since all f.p.o. sentences have equivalents of form (26), we can again conclude, with the same qualifications as before, that the semantics of English quantifiers comprises all of f.p.o. quantification theory.

Nor is this conclusion tied to the particular construction (relative clauses with several antecedents) just used. Although a strict argument would be somewhat messy here, examples can be given which suggest that each quantifier prefix of form (26) can be captured in English even apart from this particular construction. The following example may serve to convey some flavor of the situation.

(51) Some reviewer of every magazine of each newspaper chain admires some book by every author of each publisher although this book is disliked by some proof-reader of each printer in every city.

All these observations lead us toward the conclusion that the logic of idiomatic English quantifiers is at least as strong as f.p.o. quantification theory. If so, striking results follow. For f.p.o. quantification theory is an enormously powerful theory, immensely more powerful than first-order logic ("quantification theory"). Henkin (1961, pp. 182-183) already reported Ehrenfeucht's important result that the set of valid (logically true) sentences of f.p.o. quantification theory is not recursively enumerable. Furthermore, Enderton (1970, pp. 395-396) has shown that every $\bigvee_1^1$ (second order existential) and every $\bigwedge_1^1$ (second order universal) formula[40] admits of an equivalent formula in f.p.o. quantification theory. The argument which Enderton (1970, proof of Theorem 2) and Walkoe (1970, proof of Theorem 4.3) sketch for this result can be carried out in the presence of ordinary free (second-order) predicate variables or similar predicate constants. Hence it may be combined with the result of mine mentioned earlier[41] to the effect that the validity problem for all second order sentences (and for all sentences of any finite type) can be effectively reduced to that for sentences with just one second-order quantifier, over and above first-order quantifiers and free (second-order) predicate variables. Putting these results together yieds the conclusion that the set of valid sentences of pure second-order logic (and of the whole of finite type theory) is recursively isomorphic with the set of valid sentences of f.p.o. quantification theory.[42] In this special but important sense, the whole of second-order logic reduces to f.p.o. quantification theory. In plain English, f.p.o. quantification theory is (semantically speaking) an incredibly powerful and rich logic.

Since our results suggest that the whole of f.p.o. quantification theory is built into the semantics of English quantifiers, it follows that the semantics of a relatively small fragment of English, viz. of English quantifiers plus a few supporting constructions, is a much subtler and much more complicated subject than anyone seems to have suspected. In the eyes of a logician, it seems to be powerful beyond the wildest dreams of linguists and of philosophers of language.

This tentative result is not belied by the fact that only a small part of these riches is actually capitalized in ordinary discourse and that there also seem to exist systematic devices in natural languages like English which often serve to spare us the trouble of going beyond first-order structures. A study of these devices looks extremely interesting, but cannot be attempted here.

Instead, certain methodological speculations are suggested by the great logical power of natural-language semantics. The typical, and in many cases the only kind of evidence actually employed in linguistic discussion in these days is in terms of the classification of sentences or other expressions in certain important categories. In the case of syntax, these categories include prominently grammaticality, and in the case of semantics they include, according to one important type of view, such concepts as analyticity, sameness in meaning, ambiguity, and entailment. (Cf. Katz 1972, pp. 3-7.) The distribution of sentences into these categories is said to be among the most important phenomena to be explained by a semantical theory. Although this kind of program does not necessarily imply that the rock bottom data of semantics are a competent speaker's intuitions about such notions, the cash value of the program indicated often is precisely such a view of the data of semantics.

Now one implication of our results seems to be that in this methodological respect there is an enormous difference between syntax and semantics. In order for the tacit methodology just adumbrated to be theoretically justifiable, one apparently has to assume that there is in principle some sort of procedure backing up a competent speaker's intuitions about the grammatical concept in question. Otherwise, there is no reason to expect that his direct intuitions can serve to characterize the extension of this concept.

In the case of syntax, this does not create any problems, for all the relevant sets of sentences seem to be safely recursive.[43] What we have seen shows, however, that the same presupposition is not satisfied in semantics. There is no effective procedure or anything distantly resembling an effective procedure which a speaker, however competent and well-informed, could in the last analysis rely on in classifying sentences as being analytical or non-analytical, synonyms or non-synonyms, etc. Hence no set of competent speaker's intuitions can constitute a fully sufficient basis for the explication of such concepts as analyticity, synonymy, or other basic concepts of natural-language semantics. Therefore the methodology on which much recent work has been based is bound to be inadequate in principle, however useful it may be in the short run.[44] Syntax, it seems, is a dubious methodological paradigm for semantics.

I am aware of the many pitfalls that lurk in this area, and hence offer these remarks as tentative speculations only. They seem to pose in any case serious problems for the methodology of linguistic semantics.[45]

One particular pitfall is interesting enough to be mentioned here.[46] It is connected with the basic ideas of game-theoretical semantics, and more specifically with the definition of truth for quantified sentences as the existence of a winning strategy in the associated game. If we want to think of our semantical games realistically, it may be suggested, we must restrict the strategy sets available to Myself drastically. For surely it makes no sense for me to try to play a game in accordance with a strategy represented by a function which is not recursive and perhaps not even remotely like a recursive function, it is perhaps thought. In a realistic game-theoretical semantics, the strategies available to Myself—possibly the strategies available to Nature also—must according to this view be restricted to recursive strategies or at least to strategies defined by some "almost recursive" functions and functionals. [47]

It seems to me that this line of thought has to be taken very seriously. It will lead very quickly away from classical logic, however, for already in the classical first-order logic non-recursive strategies must be relied on if the game-theoretical interpretation is to yield the usual semantics. [48] It therefore marks an even more radical departure from traditional ideas than the earlier suggestions explored in this paper. The task of investigating the new perspectives cannot be undertaken here.[49]

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NOTES

[1] Numerical expressions can for many purposes be dealt with in the same way as quantifiers. For instance, they exhibit most of the same ambiguities. This is not surprising in view of the fact that the usual existential quantifier can be read "for at least one" and that it therefore is but a special case of the "numerical quantifier" "for at least a", where a is any cardinal number.

[2] Cf. Partee (1971).

[3] Cf. Partee (1972).

[4] Cf. Chomsky (1971).

[5] Nobody claims that any ordinary formulation of first-order logic is all we need. For instance, it is clear that ordinary language typically uses relativized quantifiers or many-sorted quantification (i.e., logic where different quantifiers may range over different domains) rather than logicians' absolute quantifiers which range over the members of some fixed domain, sometimes called "the universe of discourse." Such discrepancies do not matter for the purposes at hand, however. Where the essential departures from the purview of first-order logic lie will be indicated later. In fact, even ordinary modal logic is not what entails the most striking violations of first-order characteristics.

[6] McCawley writes (p. 219): "I will in fact argue symbolic logic, subject to certain modifications, provides an adequate system for semantic representation within the framework of transformational grammar." I am nevertheless oversimplifying the situation somewhat in that there are all sorts of minor differences ("certain modifications") between McCawley's logic and typical formulations of first-order logic.

[7] Qualifications resembling those mentioned in the preceding two notes are needed here, too. In particular, we should not forget departures from first-order logic in the direction of modal logic.

[8] Views of this type are frequent in expositions and discussions of elementary logic. See e.g. Quine (1951, pp. 2-3) or von Wright (1957, pp. 1-6).

[9] I borrow this apt term from Patrick Suppes' Presidential Address, APA Pacific Division Meeting, Seattle, March 30, 1973. (Forthcoming in the *Proceedings and Addresses of the APA*.)

[10] Much of what will be said in this paper applies to other types of English quantifiers, too. Most of their peculiarities are disregarded here, however.

[11] There is no good exhaustive survey of the increasingly important uses of game-theoretical ideas in logic. Interesting glimpses are nevertheless offered e.g. by Ehrenfeucht (1960) and Mycielski (1964). In first-order logic, game-theoretical concepts have been employed earlier by Paul Lorenzen. The philosophical perspective from which he looks at the situation is different from mine, however. His "dialogical games" are interpretationally quite unlike my language-games of seeking and finding. The former are indoor games, played by successive verbal "challenges" and "responses". They cannot therefore be put to use in semantics, i.e., to discuss the relation of language to the reality it can convey information about. In contrast, my semantical games are "outdoor" games, played among the entities our first-order language speaks of.

[12] The precise formulation of the formation rules involved here might be the following:

(F. A) An n-place predicate followed by a sequence of n names of individuals (not necessarily different) is a sentence.

(F. &) - (F.v) If S and S' are sentences, then so are ($S \wedge S'$ and $S \vee S'$).

(F. $\sim$) If S is a sentence, then so is $\sim S$.

Further rules may be introduced for other sentential connectives, although they can all be defined in terms of say) "$\sim$" and "v".

[13] The rule involved hers may be formulated as follows:

(F. E) - (F. U) If S is a sentence containing a name "b" but not containing an individual variable "x", then $(\exists x) S(x/b)$ and $(x) S(x/b)$ are sentences.

Here $S(x/b)$ is the result of replacing "b" everywhere by "x" in S.

[14] For the notion of type, see e.g. Henkin (1950).

[15] Of course, this proper name must not have been used earlier.—It is clear that the mild extension of our original language which is involved here is not problematic. For instance, in each round of a game only a finite number of new individuals are needed.

[16] Cf. Mostowski (1965), Chapters 3, 6, and 13.

[17] Of course together with the function that tells me how to choose a disjunct when it comes to that. However, in this paper I shall look away from the strategic problems connected with sentential connectives.

[18] In this respect, my approach is not unlike Montague's, who treats quantifier phrases like "every man" on a par with other name phrases. See e.g. Montague (1973).

[19] Here I am using an informal linguistic notation, not a logical one. For instance, in (3) "X - W" marks any context in which a quantifier phrase of the form "some Y who Z" can occur (subject to the restrictions to be indicated later).

[20] The problem of formulating such rules seems to be a syntactical, not a semantical one. I shall not try to treat it here.

[21] This rule does not handle generic uses of the indefinite article, which must be treated separately. Furthermore, allowance must be made for the fact that in unanalyzable sentences of the form

(*) b is a(n) X

"is a(n)" may be thought of as expressing predication rather than identity *cum* existential quantification. Nothing untoward will happen, however, even if the latter interpretation is attempted. What we obtain by applying the rule (G.an) to (*) is a redundancy but not any mistake. In other words, in (*) "is" may be thought to express either predication or identity, which goes some way towards explained why English can use the same word to express both. Hence the only caution needed here is not to apply (G.an) needlessly to (*) when it is grammatically unanalyzable further.

[22] Again, the details do not matter for my purposes at hand. Cf. Klima (1964).

[23] Thus in a negated "knows that" statement there may in effect be a tacit negation *inside* the epistemic operator. This observation is important for the purpose of understanding the interplay of epistemic operators and quantifiers.

[24] I realize that this approach touches only some of the gross overall features of English quantifiers and leaves much of their finer features unaccounted for. Some idea of their actual complexity can be gathered from Jackendoff (1968).

[25] The limits of the applicability of the usual concept of scope will be examined later.

[26] A related suggestion is made in Lasnick (1972). It is not clear yet, however, what all the rules are with respect to which (G.any) behaves differently from (G. every). Further work is thus needed here.

[27] Reference to (G.all) is needed because "any" can be plural, unlike "every". In its plural uses, (G.any) therefore has to compared with (G.all) rather than (G. every).

[28] In (18) the second conjunct is of course semantically redundant, and is therefore naturally omitted in natural discourse I am inserting it to bring out the parallelism between (18) and (19).—Certain additional restrictions are also needed when a proper name (like "Bill" in (18)) is substituted for a quantifier phrase which involves "quantifying into" an epistemic or modal context. These "uniqueness conditions" will not be discussed in this paper, however. They would be analogous to their namesakes in formal languages, discussed in Hintikka (1969).

[29] More idiomatically, we would of course say instead of (19), "If Arthur is coming, John knows that he is."

[30] It also supplements in an important way my own earlier accounts of the construction *an epistemic verb + an interrogative clause.* Cf. Hintikka (1969).

[31] See e.g. Kuno and Robinson (1972), which gives references to earlier literature. I am preparing an analysis of the semantics of English questions.

[32] The reader may check that the semantical interpretation any games assign to such pairs of sentences as the following is precisely the right one.

John saw a warbler yesterday, and Bill saw a warbler, too.
John saw a warbler yesterday, and Bill saw it, too.

A great deal of further discussion is of course needed to show how all the different kinds of problems can be handled.

[33] A partially ordering relation is one which is (i) transitive, (ii) reflexive, and (iii) antisymmetric.

[34] Strictly speaking, we obtain here two slightly different generalizations of first-order logic. One is obtained by allowing the replacement of the quantifier prefix of a first-order sentence in the prenex normal form to be replaced by a partially ordered quantifier prefix in one fell swoop. The other allows for the introduction of a partially ordered quantifier prefix as a rule of formation which can be used repeatedly, like the other formation rules. Cf. Walkoe (1970), p. 538, definitions of H and H'.

In this paper it suffices to restrict our attention to the former, narrower concept of a f.p.o. sentence.

[35] Henkin (1961), p. 183 (Ehrenfeucht's result). Cf. Walkoe (1970), Theorem 4.1 (p. 540).

[36] In the sense that their quantifier orderings are extensions of that of (23).

[37] Anyone who for instance takes possible-worlds semantics seriously as an essential tool for explicating the semantics of natural languages is committed to this type of a position.

[38] The conceptual situation here is illustrated by Enderton's reduction (pp. 396-397) of all f.p.o. prefixes of elementary number theory to the form (23). This shows that the essential question here is whether natural language is subtle enough to reproduce those combinatorial operations on which this further reduction depends—or to accomplish something equivalent. Although the details are messy, it seems to me quite unreasonable to doubt that this can be done.

[39] In order to spare the reader the trouble of restoring an innocuous ellipsis, I perhaps ought to write here "...a relative of his..." and "...a friend of his...."

[40] For this notation, see Addison (1962).

[41] See Hintikka (1955). The result is generalized in Montague (1965).

[42] The term "a language of order 1 1/2", which has sometimes been used for f.p.o. quantification theory, is thus profoundly misleading, even though my results do not preclude other sorts of differences in power between f.p.o. quantification theory and second-order logic.

[43] This is illustrated by the well known fact that even type 1 or context-sensitive grammars (constituent structure grammars) admit a decision procedure for membership. See Wall (1972), pp. 212, 234-235.

[44] This still leaves largely open the question of its empirical adequacy. I hope to be able to return to this question on another occasion.

[45] The whole question as to how a finite language user can as much as form the concept of nonrecursive entities is in need of further analysis.

[46] This line of thought is the analogue in formal semantics to constructivistic views on the foundations of logic and mathematics. For them, cf. Mostowski (1965), chapters 1 and 10-11.

[47] What possibilities there are in this direction is indicated by Gödel's (1959) "extention of the finitistic viewpoint" which can be construed in game-theoretical spirit.

[48] Cf. e.g. Mostowski (1965), pp. 58-59.

[49] Much of the material in this paper was first presented in seminars and lectures at the University of Helsinki, Stanford University, NYU, and the Third Scandinavian Logic Symposium in Uppsala, Sweden. It is also appearing, with the appropriate permission, in *Linguistic Inquiry* 5 (1974). In working on it, I have profited from contacts with more people than I can name here. Two of them must nevertheless be mentioned. They are Erik Stenius, whose criticism first prompted me to explore the sufficiency of first-order logic for the representation of English quantifiers, and Joan Bresnan, whose expertise has proved invaluable in several ways. Neither is responsible for my shortcomings, however, nor are the anonymous referees of *Linguistic Inquiry* whose comments have proved extremely useful.

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