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Rules of Inference I, §7.1

I. **A System of Natural Deduction**
This is a method of proof of validity.
We use the Language of Propositional Logic, plus 8 Rules of Inference, and 10 Rules of Replacement.
Once we've proven a rule valid, using the indirect truth table method, then we can use it in a proof.

Deductions generally begin with any number of premises, and end with a conclusion.
A deduction is valid if every step is either a premise or derived from premises or previous steps using valid rules of inference.

II. **Some Rules of Inference**
Consider the validity of each of the following:

1)
$A \supset B$
A / B

2)
$(E \cdot I) \supset D$
$(E \cdot I)$ / D

3)
$\sim G \supset (F \cdot H)$
$\sim G$ / $F \cdot H$

What does each have in common?
The (valid) form:
$P \supset Q$
P / Q
This is called *Modus Ponens*, and abbreviated (MP).
For example, "If I own a Toyota, then I own a car. I own a Toyota. So, I own a car."
Note that P and Q can stand for simple or complex formulae.
Another example of (MP):
$[(H \vee G) \supset I] \supset (K \cdot \sim L)$
$[(H \vee G) \supset I]$ / $(K \cdot \sim L)$

The following forms are also valid. (Check them, using indirect truth tables.)
1)
$P \supset Q$
$\sim Q$ / $\sim P$
This is called *Modus Tollens*, abbreviated (MT).
For example, "If I own a Toyota, I own a car. I don't own a car. So, I don't own a Toyota."

2)
$P \vee Q$
$\sim P$ / Q
This is called *Disjunctive Syllogism*, abbreviated (DS).
For example, "I will have soup or salad. I don't have soup. So, I will have salad."

3)
P ⊃ Q
Q ⊃ R / P ⊃ R
This is called *Hypothetical Syllogism*, abbreviated (HS).
For example, "If I own a Toyota I own a car. If I own a car, I have to pay for insurance. So, If I own a Toyota, I have to pay for insurance."

The following forms are invalid. (Check them, using indirect truth tables.)
1)
P ⊃ Q
Q / P
This is called the *Fallacy of Affirming the Consequent.*
For example, "If I own a Toyota, I own a car. I own a car. So I own a Toyota."
Note that the premises may be true while the conclusion is false.

2)
P ⊃ Q
~P / ~Q
This is called the *Fallacy of Denying the Antecedent.*
For example, "If I own a Toyota, I own a car. I don't own a Toyota. So, I don't own a car.

III. **Examples of deductions**

Show that the following argument is valid:
1. (X ⊃ Y) ⊃ T
2. S ∨ ~T
3. U ⊃ ~S
4. U / ~(X ⊃ Y)

We can use truth tables, but we can now also use the method of natural deduction.

1. (X ⊃ Y) ⊃ T		
2. S ∨ ~T		
3. U ⊃ ~S		
4. U	/ ~(X ⊃ Y)	
5. ~S	3, 4, MP	(taking 'U' for P and '~S' for Q)
6. ~T	2, 5, DS	(taking 'S' for P and '~T' for Q)
7. ~(X ⊃ Y)	1, 6, MT	(taking 'X ⊃ Y' for P and 'T' for Q)

QED

Notes on the above deduction:
All lines except the premises require justification, which includes the lines and rule of inference used to generate the new conclusion. For example, '3, 4, MP' means that the current line is derived directly from lines 3 and 4 by a use of the rule of Modus Ponens.
The conclusion, written after a single slash following the last premise is not technically part of the deduction.
Deductions are sometimes called proofs, and sometimes called derivations.
The explanations such as "taking 'U' for P and '~S' for Q" are not required elements of the derivation.
'QED' stands for 'Quod erat demonstratum', meaning 'Thus it has been shown', and serves as a logician's punctuation mark: "I'm done!" It is not required, but looks neat.

Another example:

1. ~G ⊃ [G ∨ (S ⊃ A)]	
2. (S ∨ L) ⊃ ~G	
3. S ∨ L	
4. A ⊃ G	/ L
5. ~G	2, 3, MP
6. G ∨ (S ⊃ A)	1, 5, MP
7. S ⊃ A	6, 5, DS
8. S ⊃ G	7, 4, HS
9. ~S	8, 5, MT
10. L	3, 9, DS

QED

Some hints for constructing a derivation:
Start with simple sentences, or negations of simple negations.
Plan ahead, work backwards on the side.
Don't worry about extraneous lines: not every line must be used.
Some lines may be used more than once.

IV. **Exercises**. Derive the conclusions of each of the following arguments using natural deduction.

1)

1. (A · B) ⊃ (E ∨ D)	
2. A · B	
3. ~E	/ D

2)

1. ~D ∨ (H ∨ F)	
2. H ⊃ G	
3. ~~D	
4. ~G	/ F

3)

1. X ⊃ Y	
2. ~Z	
3. Y ⊃ Z	
4. X ∨ W	/ W

4)

1. A ⊃ ~B	
2. A ∨ (D ⊃ E)	
3. ~B ⊃ E	
4. ~E	/ ~D

Solutions may vary.