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Changing Quantifiers, §8.3

I. **Four sets of equivalences**

Note that the two statements in each of the following pairs are equivalent.
1a. Everything is made of atoms.
1b. It's not the case that something is not made of atoms.

2a. Something is fishy.
2b. It's wrong to say that nothing is fishy.

3a. Nothing is perfect.
3b. It's false that something is perfect.

4a. At least one thing isn't blue.
4b. Not everything is blue.

Now look at the predicate logic regimentations of each.
1a. (x)Ax
1b. ~(∃x)~Ax

2a. (∃x)Fx
2b ~(x)~Fx

3a. (x)~Px
3b. ~(∃x)Px

4a. (∃x)~Bx
4b. ~(x)Bx

The rule of **Changing Quantifiers (CQ)**: Any place where you have an expression of one of the above forms, you may replace it with a statement of its logically equivalent form.
Like rules of replacement, (CQ) may be used on part of a line.

Another way to look at these four rules.
There are three spaces around each quantifier:
1. Directly before the quantifier
2. The quantifier itself
3. Directly following the quantifier
To change a quantifier, change each space.
Add or remove a negation directly before the quantifier.
Switch quantifiers: existential to universal or vice versa.
Add or remove a negation directly after the quantifier.

II. **Some transformations permitted by CQ**

'It's not the case that every P is Q' is equivalent to 'something is P and not Q'.

~(x)(Px ⊃ Qx)	
(∃x)~(Px ⊃ Qx)	CQ
(∃x)~(~Px ∨ Qx)	Impl
(∃x)(Px · ~Qx)	Dm, DN

'It's not the case that something is both P and Q' is equivalent to 'everything that's P is not Q' also, to 'everything that's Q is not P'.

~(∃x)(Px · Qx)	
(x)~(Px · Qx)	CQ
(x)(~Px ∨ ~Qx)	DM
(x)(Px ⊃ ~Qx)	Impl
(x)(Qx ⊃ ~Px)	Trans, DN

III. **Sample derivations using CQ**

1)

1. (∃x)Lx ⊃ (∃y)My		
2. (y)~My	/~La	
3. ~(∃y)My	2, CQ	
4. ~(∃x)Lx	1, 3, MT	
5. (x)~Lx	4, CQ	
6. ~La	5, UI	Note: You may not use EI to get this conclusion!

QED

2)

1. (x)[(Ax · Bx) ⊃ Ex]	
2. ~(x)(Ax ⊃ Ex)	/ ~(x)Bx
3. (∃x)~(Ax ⊃ Ex)	2, CQ
4. (∃x)~(~Ax ∨ Ex)	3, Impl
5. (∃x)(Ax · ~Ex)	4, DM, DN
6. Aa · ~Ea	5, EI
7. (Aa · Ba) ⊃ Ea	1, UI
8. ~Ea	6, Com, Simp
9. ~(Aa · Ba)	7, 8, MT
10. ~Aa ∨ ~Ba	9, DM
11. Aa	6, Simp
12. ~Ba	10, 11, DN, DS
13. (∃x)~Ba	12, EG
14. ~(x)Bx	13, CQ

QED

3)

1. (x)~Dx ⊃ (x)Ex	
2. (∃x)~Ex	/(∃x)Dx
3. ~(x)Ex	2, CQ
4. ~(x)~Dx	1, 3, MT
5. (∃x)Dx	4, CQ

QED — Note: No instantiation!

IV. **Exercises**. Derive the conclusions of each of the following arguments.

1)
1. ~(∃x)Hx
2. (x)~Hx ⊃ (z)Iz /Ia

2)
1. (∃x)(Hx · Gx) ⊃ (x)Ix
2. ~Ia /(x)(Hx ⊃ ~Gx)

3)
1. (∃x)(Ax ∨ Bx) ⊃ (x)Dx
2. (∃x)~Dx /~(∃x)Ax

4)
1. (x)~Fx ⊃ (x)~Gx / (∃x)Gx ⊃ (∃x)Fx

5)
1. (∃x)~Ax ⊃ (x)~Bx
2. (∃x)~Ax ⊃ (∃x)Bx
3. (x)(Ax ⊃ Fx) / (x)Fx

Solutions may vary.