

Exercise 7.1

Part I

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|-------------------|----------|--|----------|
| 1. $\sim G$ | 1, 2, MT | 11. A | 2, 4, MP |
| 2. M | 1, 2, MP | 12. S | 2, 3, DS |
| 3. $E \supset D$ | 1, 2, HS | 13. $\sim\sim C$ | 1, 3, MT |
| 4. C | 1, 2, DS | 14. N | 2, 4, DS |
| 5. K | 1, 3, MP | 15. $C \supset \sim T$ | 1, 3, HS |
| 6. $\sim S$ | 2, 3, MT | 16. $\sim P$ | 1, 2, MP |
| 7. $F \supset D$ | 1, 3, HS | 17. $A \supset C$ | 1, 4, DS |
| 8. N | 2, 3, DS | 18. $(R \supset M) \supset (M \vee E)$ | 1, 3, HS |
| 9. $\sim N$ | 2, 4, MT | 19. $\sim(S \vee C)$ | 1, 3, MT |
| 10. $G \supset A$ | 1, 4, HS | 20. $A \vee N$ | 2, 4, MP |

Part II

- | | | | |
|------------------|----------|-----------------------------|----------|
| 1. $\sim B$ | 1, 2, DS | 11. $F \supset G$ | 2, 4, HS |
| 2. N | 1, 2, MP | 12. $\sim M$ | 2, 4, DS |
| 3. $\sim T$ | 1, 2, MT | 13. $\sim\sim S$ | 3, 4, MT |
| 4. $R \supset C$ | 1, 2, HS | 14. $\sim F \supset \sim J$ | 2, 4, HS |
| 5. $\sim N$ | 1, 3, MT | 15. $\sim\sim S$ | 1, 4, DS |
| 6. $W \supset T$ | 2, 3, HS | 16. $\sim Z$ | 3, 4, MP |
| 7. Q | 2, 3, MP | 17. $H \supset (E \cdot R)$ | 3, 4, HS |
| 8. $\sim C$ | 1, 3, DS | 18. $\sim(M \cdot G)$ | 2, 4, MT |
| 9. S | 3, 4, MP | 19. $H \vee G$ | 2, 4, MP |
| 10. $\sim A$ | 1, 4, MT | 20. $\sim(H \cdot A)$ | 1, 4, DS |

Exercise 7.1

Part III

- (1) 1. $\sim C \supset (A \supset C)$
 2. $\sim C$ / $\sim A$
 3. $A \supset C$ 1, 2, MP
 4. $\sim A$ 2, 3, MT
- (2) 1. $F \vee (D \supset T)$
 2. $\sim F$
 3. D / T
 4. $D \supset T$ 1, 2, DS
 5. T 3, 4, MP
- (3) 1. $(K \cdot B) \vee (L \supset E)$
 2. $\sim(K \cdot B)$
 3. $\sim E$ / $\sim L$
 4. $L \supset E$ 1, 2, DS
 5. $\sim L$ 3, 4, MT
- (4) 1. $P \supset (G \supset T)$
 2. $Q \supset (T \supset E)$
 3. P
 4. Q / $G \supset E$
 5. $G \supset T$ 1, 3, MP
 6. $T \supset E$ 2, 4, MP
 7. $G \supset E$ 5, 6, HS
- (5) 1. $\sim W \supset [\sim W \supset (X \supset W)]$
 2. $\sim W$ / $\sim X$
 3. $\sim W \supset (X \supset W)$ 1, 2, MP
 4. $X \supset W$ 2, 3, MP
 5. $\sim X$ 2, 4, MT
- (6) 1. $J \supset (K \supset L)$
 2. $L \vee J$
 3. $\sim L$ / $\sim K$
 4. J 2, 3, DS
 5. $K \supset L$ 1, 4, MP
 6. $\sim K$ 3, 5, MT
- (7) 1. $\sim S \supset D$
 2. $\sim S \vee (\sim D \supset K)$
 3. $\sim D$ / K
 4. $\sim\sim S$ 1, 3, MT
 5. $\sim D \supset K$ 2, 4, DS
 6. K 3, 5, MP

- (8)
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|----|--------------------------------|-----------------|
| 1. | $A \supset (E \supset \sim F)$ | |
| 2. | $H \vee (\sim F \supset M)$ | |
| 3. | A | |
| 4. | $\sim H$ | / $E \supset M$ |
| 5. | $E \supset \sim F$ | 1, 3, MP |
| 6. | $\sim F \supset M$ | 2, 4, DS |
| 7. | $E \supset M$ | 5, 6, HS |
- (9)
- | | | |
|----|---------------------------------------|----------|
| 1. | $N \supset (J \supset P)$ | |
| 2. | $(J \supset P) \supset (N \supset J)$ | |
| 3. | N | / P |
| 4. | $J \supset P$ | 1, 3, MP |
| 5. | $N \supset J$ | 2, 4, MP |
| 6. | $N \supset P$ | 4, 5, HS |
| 7. | P | 3, 6, MP |
- (10)
- | | | |
|----|--|----------|
| 1. | $G \supset [\sim O \supset (G \supset D)]$ | |
| 2. | $O \vee G$ | |
| 3. | $\sim O$ | / D |
| 4. | G | 2, 3, DS |
| 5. | $\sim O \supset (G \supset D)$ | 1, 4, MP |
| 6. | $G \supset D$ | 3, 5, MP |
| 7. | D | 4, 6, MP |
- (11)
- | | | |
|----|-------------------------------|------------|
| 1. | $\sim M \vee (B \vee \sim T)$ | |
| 2. | $B \supset W$ | |
| 3. | $\sim \sim M$ | |
| 4. | $\sim W$ | / $\sim T$ |
| 5. | $B \vee \sim T$ | 1, 3, DS |
| 6. | $\sim B$ | 2, 4, MT |
| 7. | $\sim T$ | 5, 6, DS |
- (12)
- | | | |
|----|--|------------|
| 1. | $(L \equiv N) \supset C$ | |
| 2. | $(L \equiv N) \vee (P \supset \sim E)$ | |
| 3. | $\sim E \supset C$ | |
| 4. | $\sim C$ | / $\sim P$ |
| 5. | $\sim(L \equiv N)$ | 1, 4, MT |
| 6. | $P \supset \sim E$ | 2, 5, DS |
| 7. | $\sim \sim E$ | 3, 4, MT |
| 8. | $\sim P$ | 6, 7, MT |

Exercise 7.1

- (13) 1. $\sim J \supset [\sim A \supset (D \supset A)]$
 2. $J \vee \sim A$
 3. $\sim J$ / $\sim D$
 4. $\sim A \supset (D \supset A)$ 1, 3, MP
 5. $\sim A$ 2, 3, DS
 6. $D \supset A$ 4, 5, MP
 7. $\sim D$ 5, 6, MT

- (14) 1. $(B \supset \sim M) \supset (T \supset \sim S)$
 2. $B \supset K$
 3. $K \supset \sim M$
 4. $\sim S \supset N$ / $T \supset N$
 5. $B \supset \sim M$ 2, 3, HS
 6. $T \supset \sim S$ 1, 5, MP
 7. $T \supset N$ 4, 6, HS

- (15) 1. $\sim A \supset (B \supset \sim C)$
 2. $\sim D \supset (\sim C \supset A)$
 3. $D \vee \sim A$
 4. $\sim D$ / $\sim B$
 5. $\sim A$ 3, 4, DS
 6. $B \supset \sim C$ 1, 5, MP
 7. $\sim C \supset A$ 2, 4, MP
 8. $B \supset A$ 6, 7, HS
 9. $\sim B$ 5, 8, MT

- (16) 1. $\sim G \supset [G \vee (S \supset G)]$
 2. $(S \vee L) \supset \sim G$
 3. $S \vee L$ / L
 4. $\sim G$ 2, 3, MP
 5. $G \vee (S \supset G)$ 1, 4, MP
 6. $S \supset G$ 4, 5, DS
 7. $\sim S$ 4, 6, MT
 8. L 3, 7, DS

- (17) 1. $H \supset [\sim E \supset (C \supset \sim D)]$
 2. $\sim D \supset E$
 3. $E \vee H$
 4. $\sim E$ / $\sim C$
 5. H 3, 4, DS
 6. $\sim E \supset (C \supset \sim D)$ 1, 5, MP
 7. $C \supset \sim D$ 4, 6, MP
 8. $C \supset E$ 2, 7, HS
 9. $\sim C$ 4, 8, MT

- (18) 1. $(R \supset F) \supset [(R \supset \sim G) \supset (S \supset Q)]$
 2. $(Q \supset F) \supset (R \supset Q)$
 3. $\sim G \supset F$
 4. $Q \supset \sim G$ / $S \supset F$
 5. $Q \supset F$ 3, 4, HS
 6. $R \supset Q$ 2, 5, MP
 7. $R \supset F$ 5, 6, HS
 8. $(R \supset \sim G) \supset (S \supset Q)$ 1, 7, MP
 9. $R \supset \sim G$ 4, 6, HS
 10. $S \supset Q$ 8, 9, MP
 11. $S \supset F$ 5, 10, HS

- (19) 1. $\sim A \supset [A \vee (T \supset R)]$
 2. $\sim R \supset [R \vee (A \supset R)]$
 3. $(T \vee D) \supset \sim R$
 4. $T \vee D$ / D
 5. $\sim R$ 3, 4, MP
 6. $R \vee (A \supset R)$ 2, 5, MP
 7. $A \supset R$ 5, 6, DS
 8. $\sim A$ 5, 7, MT
 9. $A \vee (T \supset R)$ 1, 8, MP
 10. $T \supset R$ 8, 9, DS
 11. $\sim T$ 5, 10, MT
 12. D 4, 11, DS

- (20) 1. $\sim N \supset [(B \supset D) \supset (N \vee \sim E)]$
 2. $(B \supset E) \supset \sim N$
 3. $B \supset D$
 4. $D \supset E$ / $\sim D$
 5. $B \supset E$ 3, 4, HS
 6. $\sim N$ 2, 5, MP
 7. $(B \supset D) \supset (N \vee \sim E)$ 1, 6, MP
 8. $N \vee \sim E$ 3, 7, MP
 9. $\sim E$ 6, 8, DS
 10. $\sim D$ 4, 9, MT

Part IV

- (1) 1. $W \supset (P \vee C)$
 2. $\sim P$
 3. W / C
 4. $P \vee C$ 1, 3, MP
 5. C 2, 4, DS

Exercise 7.1

- (2) 1. $E \supset O$
 2. $R \vee \sim O$
 3. $\sim R$ / $\sim E$
 4. $\sim O$ 2, 3, DS
 5. $\sim E$ 1, 4, MT
- (3) 1. $(Q \supset \sim J) \supset (M \supset \sim D)$
 2. $Q \supset M$
 3. $M \supset \sim J$ / $Q \supset \sim D$
 4. $Q \supset \sim J$ 2, 3, HS
 5. $M \supset \sim D$ 1, 4, MP
 6. $Q \supset \sim D$ 2, 5, HS
- (4) 1. $(R \supset L) \supset (L \supset \sim F)$
 2. $\sim F \vee (R \supset L)$
 3. $\sim \sim F$ / $\sim R$
 4. $R \supset L$ 2, 3, DS
 5. $L \supset \sim F$ 1, 4, MP
 6. $\sim L$ 3, 5, MT
 7. $\sim R$ 4, 6, MT
- (5) 1. $C \vee (H \supset R)$
 2. $S \vee (R \supset E)$
 3. $\sim C$
 4. $\sim S$ / $H \supset E$
 5. $H \supset R$ 1, 3, DS
 6. $R \supset E$ 2, 4, DS
 7. $H \supset E$ 5, 6, HS
- (6) 1. $\sim D \vee (L \supset \sim F)$
 2. $\sim D \supset \sim F$
 3. $\sim \sim F$ / $\sim L$
 4. $\sim \sim D$ 2, 3, MT
 5. $L \supset \sim F$ 1, 4, DS
 6. $\sim L$ 2, 5, MT
- (7) 1. $H \supset (D \equiv A)$
 2. $\vee \vee (R \supset V)$
 3. $R \vee H$
 4. $\sim V$ / $D \equiv A$
 5. $R \supset \vee$ 2, 4, DS
 6. $\sim R$ 4, 5, MT
 7. H 3, 6, DS
 8. $D \equiv A$ 1, 7, MP

- (8)
- | | | |
|----|--------------------------------|----------|
| 1. | $R \vee B$ | |
| 2. | $(I \supset H) \supset \sim R$ | |
| 3. | $L \supset (I \supset T)$ | |
| 4. | $T \supset H$ | |
| 5. | L | / B |
| 6. | $I \supset T$ | 3, 5, MP |
| 7. | $I \supset H$ | 4, 6, HS |
| 8. | $\sim R$ | 2, 7, MP |
| 9. | B | 1, 8, DS |

- (9)
- | | | |
|----|------------------------------------|----------|
| 1. | $(D \supset C) \supset (N \vee W)$ | |
| 2. | $D \supset S$ | |
| 3. | $S \supset C$ | |
| 4. | $\sim N$ | / W |
| 5. | $D \supset C$ | 2, 3, HS |
| 6. | $N \vee W$ | 1, 5, MP |
| 7. | W | 4, 6, DS |

- (10)
- | | | |
|----|---|------------|
| 1. | $\sim C \supset [C \vee (J \supset D)]$ | |
| 2. | $C \supset (C \cdot U)$ | |
| 3. | $\sim(C \cdot U)$ | |
| 4. | $\sim D$ | / $\sim J$ |
| 5. | $\sim C$ | 2, 3, MT |
| 6. | $C \vee (J \supset D)$ | 1, 5, MP |
| 7. | $J \supset D$ | 5, 6, DS |
| 8. | $\sim J$ | 4, 7, MT |

Exercise 7.2

Part I

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|----|-------------|------|-----|---------------------------|------|
| 1. | B | 1 | 6. | $A \vee \sim N$ | 1, 3 |
| 2. | $T \vee Q$ | 1, 3 | 7. | $Q \vee K$ | 1 |
| 3. | $D \cdot W$ | 1, 2 | 8. | $(E \cdot G) \vee T$ | 1 |
| 4. | $H \vee F$ | 1 | 9. | $\sim B \cdot (F \vee N)$ | 1, 2 |
| 5. | R | 1 | 10. | $\sim L \vee M$ | 1, 2 |

Part II

- | | | | | | |
|----|-----|-------------------|----|-----|------------------------|
| 1. | G | 2, Simp
3, Add | 2. | E | 1, 2, DS
1, 3, Conj |
|----|-----|-------------------|----|-----|------------------------|

Exercise 7.2

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|--|------------------------|------------------|------------------------|
| 3. $(B \supset N) \cdot (K \supset R)$ | 1, 3, Conj
2, 4, CD | 7. $\sim F$ | 2, 3, MT
1, 4, Conj |
| 4. $T \vee U$ | 1, Add
3, 4, MP | 8. $E \supset B$ | 1, Simp
3, 4, HS |
| 5. $S \cdot P$ | 2, 3, DS
4, Simp | 9. $H \cdot R$ | 2, 3, MP
Simp |
| 6. $N \vee F$ | 1, Add
3, 4, CD | 10. $M \cdot E$ | 1, 3, Conj
2, 4, MP |

Part III

- (1) 1. $\sim M \supset Q$
 2. $R \supset \sim T$
 3. $\sim M \vee R$ / $Q \vee \sim T$
 4. $(\sim M \supset Q) \cdot (R \supset \sim T)$ 1, 2, Conj
 5. $Q \vee \sim T$ 3, 4, CD

- (2) 1. $E \supset (A \cdot C)$
 2. $A \supset (F \cdot E)$
 3. E / F
 4. $A \cdot C$ 1, 3, MP
 5. A 4, Simp
 6. $F \cdot E$ 2, 5, MP
 7. F 6, Simp

- (3) 1. $G \supset (S \cdot T)$
 2. $(S \vee T) \supset J$
 3. G / J
 4. $S \cdot T$ 1, 3, MP
 5. S 4, Simp
 6. $S \vee T$ 5, Add
 7. J 2, 6, MP

- (4) 1. $(L \vee T) \supset (B \cdot G)$
 2. $L \cdot (K \equiv R)$ / $L \cdot B$
 3. L 2, Simp
 4. $L \vee T$ 3, Add
 5. $B \cdot G$ 1, 4, MP
 6. B 5, Simp
 7. $L \cdot B$ 3, 6, Conj

- (5) 1. $(\sim F \vee X) \supset (P \vee T)$
 2. $F \supset P$
 3. $\sim P$ / T
 4. $\sim F$ 2, 3, MT
 5. $\sim F \vee X$ 4, Add
 6. $P \vee T$ 1, 5, MP
 7. T 3, 6, DS
- (6) 1. $(N \supset B) \cdot (O \supset C)$
 2. $Q \supset (N \vee O)$
 3. Q / $B \vee C$
 4. $N \vee O$ 2, 3, MP
 5. $B \vee C$ 1, 4, CD
- (7) 1. $(U \vee W) \supset (T \supset R)$
 2. $U \cdot H$
 3. $\sim R \cdot \sim J$ / $U \cdot \sim T$
 4. U 2, Simp
 5. $U \vee W$ 4, Add
 6. $T \supset R$ 1, 5, MP
 7. $\sim R$ 3, Simp
 8. $\sim T$ 6, 7, MT
 9. $U \cdot \sim T$ 4, 8, Conj
- (8) 1. $(D \vee E) \supset (G \cdot H)$
 2. $G \supset \sim D$
 3. $D \cdot F$ / M
 4. D 3, Simp
 5. $D \vee E$ 4, Add
 6. $G \cdot H$ 1, 5, MP
 7. G 6, Simp
 8. $\sim D$ 2, 7, MP
 9. $D \vee M$ 4, Add
 10. M 8, 9, DS
- (9) 1. $(B \vee F) \supset (A \supset G)$
 2. $(B \vee F) \supset (G \supset K)$
 3. $B \cdot \sim H$ / $A \supset K$
 4. B 3, Simp
 5. $B \vee F$ 4, Add
 6. $A \supset G$ 1, 5, MP
 7. $B \vee E$ 4, Add
 8. $G \supset K$ 2, 7, MP
 9. $A \supset K$ 6, 8, HS

Exercise 7.2

- (10) 1. $(P \supset R) \supset (M \supset P)$
 2. $(P \vee M) \supset (P \supset R)$
 3. $P \vee M$ / $R \vee P$
 4. $P \supset R$ 2, 3, MP
 5. $M \supset P$ 1, 4, MP
 6. $(P \supset R) \cdot (M \supset P)$ 4, 5, Conj
 7. $R \vee P$ 3, 6, CD

- (11) 1. $(C \supset N) \cdot E$
 2. $D \vee (N \supset D)$
 3. $\sim D$ / $\sim C \vee P$
 4. $N \supset D$ 2, 3, DS
 5. $\sim N$ 3, 4, MT
 6. $C \supset N$ 1, Simp
 7. $\sim C$ 5, 6, MT
 8. $\sim C \vee P$ 7, Add

- (12) 1. $[A \vee (K \cdot J)] \supset (\sim E \cdot \sim F)$
 2. $M \supset [A \cdot (P \vee R)]$
 3. $M \cdot U$ / $\sim E \cdot A$
 4. M 3, Simp
 5. $A \cdot (P \vee R)$ 2, 4, MP
 6. A 5, Simp
 7. $A \vee (K \cdot J)$ 6, Add
 8. $\sim E \cdot \sim F$ 1, 7, MP
 9. $\sim E$ 8, Simp
 10. $\sim E \cdot A$ 6, 9, Conj

- (13) 1. $\sim H \supset (\sim T \supset R)$
 2. $H \vee (E \supset F)$
 3. $\sim T \vee E$
 4. $\sim H \cdot D$ / $R \vee F$
 5. $\sim H$ 4, Simp
 6. $\sim T \supset R$ 1, 5, MP
 7. $E \supset F$ 2, 5, DS
 8. $(\sim T \supset R) \cdot (E \supset F)$ 6, 7, Conj
 9. $R \vee F$ 3, 8, CD

- (14) 1. $(U \cdot \sim\sim P) \supset Q$
 2. $\sim O \supset U$
 3. $\sim P \supset O$
 4. $\sim O \cdot T$ / Q
 5. $\sim O$ 4, Simp
 6. U 2, 5, MP
 7. $\sim\sim P$ 3, 5, MT

8. $U \cdot \sim P$ 6, 7, Conj
 9. Q 1, 8, MP

- (15) 1. $(M \vee N) \supset (F \supset G)$
 2. $D \supset \sim C$
 3. $\sim C \supset B$
 4. $M \cdot H$
 5. $D \vee F$ / $B \vee G$
 6. $D \supset B$ 2, 3, HS
 7. M 4, Simp
 8. $M \vee N$ 7, Add
 9. $F \supset G$ 1, 8, MP
 10. $(D \supset B) \cdot (F \supset G)$ 6, 9, Conj
 11. $B \vee G$ 5, 10, CD

- (16) 1. $(F \cdot M) \supset (S \vee T)$
 2. $(\sim S \vee A) \supset F$
 3. $(\sim S \vee B) \supset M$
 4. $\sim S \cdot G$ / T
 5. $\sim S$ 4, Simp
 6. $\sim S \vee A$ 5, Add
 7. F 2, 6, MP
 8. $\sim S \vee B$ 5, Add
 9. M 3, 8, MP
 10. $F \cdot M$ 7, 9, Conj
 11. $S \vee T$ 1, 10, MP
 12. T 5, 11, DS

- (17) 1. $(\sim K \cdot \sim N) \supset [(\sim P \supset K) \cdot (\sim R \supset G)]$
 2. $K \supset N$
 3. $\sim N \cdot B$
 4. $\sim P \vee \sim R$ / G
 5. $\sim N$ 3, Simp
 6. $\sim K$ 2, 5, MT
 7. $\sim K \cdot \sim N$ 5, 6, Conj
 8. $(\sim P \supset K) \cdot (\sim R \supset G)$ 1, 7, MP
 9. $K \vee G$ 4, 8, CD
 10. G 6, 9, DS

- (18) 1. $(\sim A \vee D) \supset (B \supset F)$
 2. $(B \vee C) \supset (A \supset E)$
 3. $A \vee B$
 4. $\sim A$ / $E \vee F$
 5. $\sim A \vee D$ 4, Add
 6. $B \supset F$ 1, 5, MP

Exercise 7.2

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|---|------------|
| 7. B | 3, 4, DS |
| 8. $B \vee C$ | 7, Add |
| 9. $A \supset E$ | 2, 8, MP |
| 10. $(A \supset E) \cdot (B \supset F)$ | 8, 9, Conj |
| 11. $E \vee F$ | 3, 10, CD |

- (19)
- | | |
|--|------------|
| 1. $(J \supset K) \cdot (\sim O \supset \sim P)$ | |
| 2. $(L \supset J) \cdot (\sim M \supset \sim O)$ | |
| 3. $\sim K \supset (L \vee \sim M)$ | |
| 4. $\sim K \cdot G$ | / $\sim P$ |
| 5. $\sim K$ | 4, Simp |
| 6. $L \vee \sim M$ | 3, 5, MP |
| 7. $J \vee \sim O$ | 2, 6, CD |
| 8. $K \vee \sim P$ | 1, 7, CD |
| 9. $\sim P$ | 5, 8, DS |

- (20)
- | | |
|--|--------------------|
| 1. $(\sim M \cdot \sim N) \supset [(\sim M \vee H) \supset (K \cdot L)]$ | |
| 2. $\sim M \cdot (C \supset D)$ | |
| 3. $\sim N \cdot (F \equiv G)$ | / $K \cdot \sim N$ |
| 4. $\sim M$ | 2, Simp |
| 5. $\sim N$ | 3, Simp |
| 6. $\sim M \cdot \sim N$ | 4, 5, Conj |
| 7. $(\sim M \vee H) \supset (K \cdot L)$ | 1, 6, MP |
| 8. $\sim M \vee H$ | 4, Add |
| 9. $K \cdot L$ | 7, 8, MP |
| 10. K | 9, Simp |
| 11. $K \cdot \sim N$ | 5, 10, Conj |

- (21)
- | | |
|---|--------------|
| 1. $(P \vee S) \supset (E \supset F)$ | |
| 2. $(P \vee T) \supset (G \supset H)$ | |
| 3. $(P \vee U) \supset (E \vee G)$ | |
| 4. P | / $F \vee H$ |
| 5. $P \vee S$ | 4, Add |
| 6. $E \supset F$ | 1, 5, MP |
| 7. $P \vee T$ | 4, Add |
| 8. $G \supset H$ | 2, 7, MP |
| 9. $P \vee U$ | 4, Add |
| 10. $E \vee G$ | 3, 9, MP |
| 11. $(E \supset F) \cdot (G \supset H)$ | 6, 8, Conj |
| 12. $F \vee H$ | 10, 11, CD |

- (22)
- | | |
|---|---------------|
| 1. $(S \supset Q) \cdot (Q \supset \sim S)$ | |
| 2. $S \vee Q$ | |
| 3. $\sim Q$ | / $P \cdot R$ |
| 4. $Q \vee \sim S$ | 1, 2, CD |

- | | |
|-------------------------|----------|
| 5. $\sim S$ | 3, 4, DS |
| 6. Q | 2, 5, DS |
| 7. $Q \vee (P \cdot R)$ | 6, Add |
| 8. $P \cdot R$ | 3, 7, DS |

- (23)
- | | |
|--|--------------|
| 1. $(D \supset B) \cdot (C \supset D)$ | |
| 2. $(B \supset D) \cdot (E \supset C)$ | |
| 3. $B \vee E$ | / $D \vee B$ |
| 4. $D \vee C$ | 2, 3, CD |
| 5. $B \vee D$ | 1, 4, CD |
| 6. $B \supset D$ | 2, Simp |
| 7. $D \supset B$ | 1, Simp |
| 8. $(B \supset D) \cdot (D \supset B)$ | 6, 7, Conj |
| 9. $D \vee B$ | 5, 8, CD |

- (24)
- | | |
|---|--------------|
| 1. $(R \supset H) \cdot (S \supset I)$ | |
| 2. $(\sim H \cdot \sim L) \supset (R \vee S)$ | |
| 3. $\sim H \cdot (K \supset T)$ | |
| 4. $H \vee \sim L$ | / $I \vee M$ |
| 5. $\sim H$ | 3, Simp |
| 6. $\sim L$ | 4, 5, DS |
| 7. $\sim H \cdot \sim L$ | 5, 6, Conj |
| 8. $R \vee S$ | 2, 7, MP |
| 9. $H \vee I$ | 1, 8, CD |
| 10. I | 5, 9, DS |
| 11. $I \vee M$ | 10, Add |

- (25)
- | | |
|--|--------------|
| 1. $(W \cdot X) \supset (Q \vee R)$ | |
| 2. $(S \vee F) \supset (Q \vee W)$ | |
| 3. $(S \vee G) \supset (\sim Q \supset X)$ | |
| 4. $Q \vee S$ | |
| 5. $\sim Q \cdot H$ | / R |
| 6. $\sim Q$ | 5, Simp |
| 7. S | 4, 6, DS |
| 8. $S \vee F$ | 7, Add |
| 9. $Q \vee W$ | 2, 8, MP |
| 10. W | 6, 9, DS |
| 11. $S \vee G$ | 7, Add |
| 12. $\sim Q \supset X$ | 3, 11, MP |
| 13. X | 6, 12, MP |
| 14. $W \cdot X$ | 10, 13, Conj |
| 15. $Q \vee R$ | 1, 14, MP |
| 16. R | 6, 15, DS |

Exercise 7.2

Part IV

(1) 1. $T \supset (Q \cdot F)$

2. $T \cdot C$ / $Q \vee O$
 3. T 2, Simp
 4. $Q \cdot F$ 1, 3, MP
 5. Q 4, Simp
 6. $Q \vee O$ 5, Add

(2) 1. $(C \cdot \sim E) \supset (E \vee T)$

2. C
 3. $\sim E$ / T
 4. $C \cdot \sim E$ 2, 3, Conj
 5. $E \vee T$ 1, 4, MP
 6. T 3, 5, DS

(3) 1. $(R \vee P) \supset (G \vee M)$

2. $R \cdot U$
 3. $(G \supset E) \cdot (M \supset F)$
 4. $(E \vee F) \supset C$ / C
 5. R 2, Simp
 6. $R \vee P$ 5, Add
 7. $G \vee M$ 1, 6, MP
 8. $E \vee F$ 3, 7, CD
 9. C 4, 8, MP

(4) 1. $M \vee P$

2. $(P \vee S) \supset (R \cdot D)$
 3. $\sim M$ / R
 4. P 1, 3, DS
 5. $P \vee S$ 4, Add
 6. $R \cdot D$ 2, 5, MP
 7. R 6, Simp

(5) 1. $H \supset [(I \vee P) \supset E]$

2. $(H \vee W) \supset \sim E$
 3. H / $\sim(I \vee P)$
 4. $H \vee W$ 3, Add
 5. $\sim E$ 2, 4, MP
 6. $(I \vee P) \supset E$ 1, 3, MP
 7. $\sim(I \vee P)$ 5, 6, MT

(6) 1. $(P \vee S) \supset [(L \supset R) \cdot (I \supset M)]$

2. $(P \vee N) \supset (L \vee I)$
 3. $P \cdot W$ / $R \vee M$
 4. P 3, Simp

- | | |
|--|----------|
| 5. $P \vee N$ | 4, Add |
| 6. $L \vee I$ | 2, 5, MP |
| 7. $P \vee S$ | 4, Add |
| 8. $(L \supset R) \cdot (I \supset M)$ | 1, 7, MP |
| 9. $R \vee M$ | 6, 8, CD |

(7) 1. $(\sim C \vee \sim M) \supset (\sim C \supset T)$

- | | |
|-------------------------|----------|
| 2. $C \vee \sim T$ | |
| 3. $\sim C$ | / B |
| 4. $\sim C \vee \sim M$ | 3, Add |
| 5. $\sim C \supset T$ | 1, 4, MP |
| 6. T | 3, 5, MP |
| 7. $T \vee B$ | 6, Add |
| 8. $\sim T$ | 2, 3, DS |
| 9. B | 7, 8, DS |

(8) 1. $(F \vee A) \supset (H \cdot P)$

- | | |
|--|------------|
| 2. $L \supset A$ | |
| 3. $D \supset F$ | |
| 4. $(D \vee L) \cdot I$ | / H |
| 5. $(D \supset F) \cdot (L \supset A)$ | 2, 3, Conj |
| 6. $D \vee L$ | 4, Simp |
| 7. $F \vee A$ | 5, 6, CD |
| 8. $H \cdot P$ | 1, 7, MP |
| 9. H | 8, Simp |

(9) 1. $L \supset G$

- | | |
|---|------------|
| 2. $C \supset A$ | |
| 3. $(G \vee A) \supset (\sim L \cdot \sim M)$ | |
| 4. $L \vee C$ | |
| 5. $C \supset I$ | / I |
| 6. $(L \supset G) \cdot (C \supset A)$ | 1, 2, Conj |
| 7. $G \vee A$ | 4, 6, CD |
| 8. $\sim L \cdot \sim M$ | 3, 7, MP |
| 9. $\sim L$ | 8, Simp |
| 10. C | 4, 9, DS |
| 11. I | 5, 10, MP |

(10) 1. $(V \cdot \sim E) \supset (P \supset E)$

- | | |
|--------------------------------|--------------------|
| 2. $V \supset \sim E$ | |
| 3. $V \cdot I$ | |
| 4. $\sim E \supset (P \vee J)$ | / $J \cdot \sim E$ |
| 5. V | 3, Simp |
| 6. $\sim E$ | 2, 5, MP |
| 7. $V \cdot \sim E$ | 5, 6, Conj |
| 8. $P \supset E$ | 1, 7, MP |

Exercise 7.3

9. $\sim P$	6, 8, MT
10. $P \vee J$	4, 6, MP
11. J	9, 10, DS
12. $J \cdot \sim E$	6, 11, Conj

Exercise 7.3

Part I

1. $\sim N \cdot \sim G$	2	9. $E \cdot (H \cdot Q)$	3
2. $(S \cdot T) \vee (S \cdot U)$	3	10. $\sim(R \cdot P)$	1
3. $(M \vee G) \vee T$	1	11. $\sim B \vee \sim \sim E$	1
4. $A \cdot S$	3	12. $(\sim Q \vee T) \cdot (\sim Q \vee R)$	3
5. $R \vee T$	1	13. $H \supset \sim(L \vee D)$	2
6. $H \cdot (Z \vee W)$	2	14. $(M \cdot R) \cdot \sim T$	3
7. $\sim G \cdot \sim \sim Q$	1	15. $D \vee (\sim K \cdot \sim W)$	1
8. $J \supset (\sim N \vee S)$	3		

Part II

1. $C \vee K$	1, Com 2, 3, DS	9. $\sim \sim L$	2, DN 1, 3, DS
2. $\sim(R \cdot N)$	2, DM 1, 3, MT	10. $(D \vee N) \cdot (D \vee H)$	1, Dist 2, Simp
3. $T \cdot H$	1, Com 2, Simp	11. $K \vee (E \cdot G)$	1, Dist 2, 3, DS
4. $L \cdot (S \cdot F)$	1, Assoc 2, Simp	12. $N \vee F$	2, Com 1, 3, CD
5. $\sim B \vee \sim \sim K$	1, DN 2, DM	13. $M \vee (G \vee T)$	1, Assoc 2, 3, DS
6. $\sim \sim A$	2, DN 1, 3, MT	14. $\sim A \vee \sim S$	2, DM 1, 3, CD
7. $D \cdot (M \vee N)$	1, Dist 2, Simp	15. $\sim R \vee \sim T$	1, Add 2, DM
8. $U \vee T$	2, Com 1, 3, MP		

Part III

- (1) 1. $(\sim M \supset P) \cdot (\sim N \supset Q)$
 2. $\sim(M \cdot N)$ / $P \vee Q$
 3. $\sim M \vee \sim N$ 2, DM
 4. $P \vee Q$ 1, 3, CD
- (2) 1. $J \vee (K \cdot L)$
 2. $\sim K$ / J
 3. $(J \vee K) \cdot (J \vee L)$ 1, Dist
 4. $J \vee K$ 3, Simp
 5. $K \vee J$ 4, Com
 6. J 2, 5, DS
- (3) 1. $R \supset \sim B$
 2. $D \vee R$
 3. B / D
 4. $\sim\sim B$ 3, DN
 5. $\sim R$ 1, 4, MT
 6. $R \vee D$ 2, Com
 7. D 5, 6, DS
- (4) 1. $(O \vee M) \supset S$
 2. $\sim S$ / $\sim M$
 3. $\sim(O \vee M)$ 1, 2, MT
 4. $\sim O \cdot \sim M$ 3, DM
 5. $\sim M \cdot \sim O$ 4, Com
 6. $\sim M$ 5, Simp
- (5) 1. $Q \vee (L \vee C)$
 2. $\sim C$ / $L \vee Q$
 3. $(Q \vee L) \vee C$ 1, Assoc
 4. $C \vee (Q \vee L)$ 3, Com
 5. $Q \vee L$ 2, 4, DS
 6. $L \vee Q$ 5, Com
- (6) 1. $\sim(\sim E \cdot \sim N) \supset T$
 2. $G \supset (N \vee E)$ / $G \supset T$
 3. $(\sim\sim E \vee \sim\sim N) \supset T$ 1, DM
 4. $(E \vee N) \supset T$ 3, DN
 5. $(N \vee E) \supset T$ 4, Com
 6. $G \supset T$ 2, 5, HS
- (7) 1. $H \cdot (C \cdot T)$
 2. $\sim(\sim F \cdot T)$ / F

Exercise 7.3

- | | |
|-----------------------------|----------|
| 3. $\sim\sim F \vee \sim T$ | 2, DM |
| 4. $F \vee \sim T$ | 3, DN |
| 5. $\sim T \vee F$ | 4, Com |
| 6. $(H \cdot C) \cdot T$ | 1, Assoc |
| 7. $T \cdot (H \cdot C)$ | 6, Com |
| 8. T | 7, Simp |
| 9. $\sim\sim T$ | 8, DN |
| 10. F | 5, 9, DS |

- (8)
- | | |
|-----------------------------------|----------|
| 1. $(E \cdot I) \vee (M \cdot U)$ | |
| 2. $\sim E$ | / M |
| 3. $\sim E \vee \sim I$ | 2, Add |
| 4. $\sim(E \cdot I)$ | 3, DM |
| 5. $M \cdot U$ | 1, 4, DS |
| 6. M | 5, Simp |

- (9)
- | | |
|---------------------------|-------------------------|
| 1. $\sim(J \vee K)$ | |
| 2. $B \supset K$ | |
| 3. $S \supset B$ | / $\sim S \cdot \sim J$ |
| 4. $\sim J \cdot \sim K$ | 1, DM |
| 5. $\sim K \cdot \sim J$ | 4, Com |
| 6. $\sim K$ | 5, Simp |
| 7. $\sim B$ | 2, 6, MT |
| 8. $\sim S$ | 3, 7, MT |
| 9. $\sim J$ | 4, Simp |
| 10. $\sim S \cdot \sim J$ | 8, 9, Conj |

- (10)
- | | |
|-----------------------------------|----------|
| 1. $(G \cdot H) \vee (M \cdot G)$ | |
| 2. $G \supset (T \cdot A)$ | / A |
| 3. $(G \cdot H) \vee (G \cdot M)$ | 1, Com |
| 4. $G \cdot (H \vee M)$ | 3, Dist |
| 5. G | 4, Simp |
| 6. $T \cdot A$ | 2, 5, MP |
| 7. $A \cdot T$ | 6, Com |
| 8. A | 7, Simp |

- (11)
- | | |
|-----------------------------------|----------|
| 1. $(D \cdot S) \vee (C \cdot T)$ | |
| 2. $\sim T$ | / S |
| 3. $\sim T \vee \sim C$ | 2, Add |
| 4. $\sim C \vee \sim T$ | 3, Com |
| 5. $\sim(C \cdot T)$ | 4, DM |
| 6. $(C \cdot T) \vee (D \cdot S)$ | 1, Com |
| 7. $D \cdot S$ | 5, 6, DS |
| 8. $S \cdot D$ | 7, Com |
| 9. S | 8, Simp |

- (12) 1. $\sim(U \vee R)$
 2. $(\sim R \vee N) \supset (P \cdot H)$
 3. $Q \supset \sim H$
 4. $\sim U \cdot \sim R$
 5. $\sim R \cdot \sim U$
 6. $\sim R$
 7. $\sim R \vee N$
 8. $P \cdot H$
 9. $H \cdot P$
 10. H
 11. $\sim \sim H$
 12. $\sim Q$
- / $\sim Q$
 1, DM
 4, Com
 5, Simp
 6, Add
 2, 7, MP
 8, Com
 9, Simp
 10, DN
 3, 11, MT
- (13) 1. $\sim(F \cdot A)$
 2. $\sim(L \vee \sim A)$
 3. $D \supset (F \vee L)$
 4. $\sim L \cdot \sim \sim A$
 5. $\sim \sim A \cdot \sim L$
 6. $\sim \sim A$
 7. $\sim F \vee \sim A$
 8. $\sim A \vee \sim F$
 9. $\sim F$
 10. $\sim L$
 11. $\sim F \cdot \sim L$
 12. $\sim(F \vee L)$
 13. $\sim D$
- / $\sim D$
 2, DM
 4, Com
 5, Simp
 1, DM
 7, Com
 6, 8, DS
 4, Simp
 9, 10, Conj
 11, DM
 3, 12, MT
- (14) 1. $[(I \vee M) \vee G] \supset \sim G$
 2. $M \vee G$
 3. $(M \vee G) \vee I$
 4. $I \vee (M \vee G)$
 5. $(I \vee M) \vee G$
 6. $\sim G$
 7. $G \vee M$
 8. M
- / M
 2, Add
 3, Com
 4, Assoc
 1, 5, MP
 2, Com
 6, 7, DS
- (15) 1. $E \supset \sim B$
 2. $U \supset \sim C$
 3. $\sim(\sim E \cdot \sim U)$
 4. $(E \supset \sim B) \cdot (U \supset \sim C)$
 5. $\sim \sim E \vee \sim \sim U$
 6. $E \vee U$
 7. $\sim B \vee \sim C$
 8. $\sim(B \cdot C)$
- / $\sim(B \cdot C)$
 1, 2, Conj
 3, DM
 5, DN
 4, 6, CD
 7, DM

Exercise 7.3

- (16)
- | | |
|--------------------------------|--------------------|
| 1. $\sim(K \vee F)$ | |
| 2. $\sim F \supset (K \vee C)$ | |
| 3. $(G \vee C) \supset \sim H$ | $/ \sim(K \vee H)$ |
| 4. $\sim K \cdot \sim F$ | 1, DM |
| 5. $\sim F \cdot \sim K$ | 4, Com |
| 6. $\sim F$ | 5, Simp |
| 7. $K \vee C$ | 2, 6, MP |
| 8. $\sim K$ | 4, Simp |
| 9. C | 7, 8, DS |
| 10. $C \vee G$ | 9, Add |
| 11. $G \vee C$ | 10, Com |
| 12. $\sim H$ | 3, 11, MP |
| 13. $\sim K \cdot \sim H$ | 8, 12, Conj |
| 14. $\sim(K \vee H)$ | 13, DM |

- (17)
- | | |
|---|---------------------|
| 1. $S \vee (I \cdot \sim J)$ | |
| 2. $S \supset \sim R$ | |
| 3. $\sim J \supset \sim Q$ | $/ \sim(R \cdot Q)$ |
| 4. $(S \vee I) \cdot (S \vee \sim J)$ | 1, Dist |
| 5. $(S \vee \sim J) \cdot (S \vee I)$ | 4, Com |
| 6. $S \vee \sim J$ | 5, Simp |
| 7. $(S \supset \sim R) \cdot (\sim J \supset \sim Q)$ | 2, 3, Conj |
| 8. $\sim R \vee \sim Q$ | 6, 7, CD |
| 9. $\sim(R \cdot Q)$ | 8, DM |

- (18)
- | | |
|--------------------------------|--------------------|
| 1. $(J \vee F) \vee M$ | |
| 2. $(J \vee M) \supset \sim P$ | |
| 3. $\sim F$ | $/ \sim(F \vee P)$ |
| 4. $(F \vee J) \vee M$ | 1, Com |
| 5. $F \vee (J \vee M)$ | 4, Assoc |
| 6. $J \vee M$ | 3, 5, DS |
| 7. $\sim P$ | 2, 6, MP |
| 8. $\sim F \cdot \sim P$ | 3, 7, Conj |
| 9. $\sim(F \vee P)$ | 8, DM |

- (19)
- | | |
|-----------------------------------|--------------|
| 1. $(K \cdot P) \vee (K \cdot Q)$ | |
| 2. $P \supset \sim K$ | $/ Q \vee T$ |
| 3. $K \cdot (P \vee Q)$ | 1, Dist |
| 4. K | 3, Simp |
| 5. $\sim \sim K$ | 4, DN |
| 6. $\sim P$ | 2, 5, MT |
| 7. $(P \vee Q) \cdot K$ | 3, Com |
| 8. $P \vee Q$ | 7, Simp |
| 9. Q | 6, 8, DS |
| 10. $Q \vee T$ | 9, Add |

- (20) 1. $E \vee \sim(D \vee C)$
 2. $(E \vee \sim D) \supset C$ / E
 3. $E \vee (\sim D \cdot \sim C)$ 1, DM
 4. $(E \vee \sim D) \cdot (E \vee \sim C)$ 3, Dist
 5. $E \vee \sim D$ 4, Simp
 6. C 2, 5, MP
 7. $(E \vee \sim C) \cdot (E \vee \sim D)$ 4, Com
 8. $E \vee \sim C$ 7, Simp
 9. $\sim C \vee E$ 8, Com
 10. $\sim \sim C$ 6, DN
 11. E 9, 10, DS
- (21) 1. $A \cdot (F \cdot L)$
 2. $A \supset (U \vee W)$
 3. $F \supset (U \vee X)$ / $U \vee (W \cdot X)$
 4. $(A \cdot F) \cdot L$ 1, Assoc
 5. $A \cdot F$ 4, Simp
 6. A 5, Simp
 7. $U \vee W$ 2, 6, MP
 8. $F \cdot A$ 5, Com
 9. F 8, Simp
 10. $U \vee X$ 3, 9, MP
 11. $(U \vee W) \cdot (U \vee X)$ 7, 10, Conj
 12. $U \vee (W \cdot X)$ 11, Dist
- (22) 1. $(T \cdot R) \supset P$
 2. $(\sim P \cdot R) \cdot G$
 3. $(\sim T \vee N) \supset H$ / H
 4. $\sim P \cdot (R \cdot G)$ 2, Assoc
 5. $\sim P$ 4, Simp
 6. $\sim(T \cdot R)$ 1, 5, MT
 7. $\sim T \vee \sim R$ 6, DM
 8. $\sim R \vee \sim T$ 7, Com
 9. $(R \cdot \sim P) \cdot G$ 2, Com
 10. $R \cdot (\sim P \cdot G)$ 9, Assoc
 11. R 10, Simp
 12. $\sim \sim R$ 11, DN
 13. $\sim T$ 8, 12, DS
 14. $\sim T \vee N$ 13, Add
 15. H 3, 14, MP
- (23) 1. $P \vee (I \cdot L)$
 2. $(P \vee I) \supset \sim(L \vee C)$
 3. $(P \cdot \sim C) \supset (E \cdot F)$ / $F \vee D$
 4. $(P \vee I) \cdot (P \vee L)$ 1, Dist

Exercise 7.3

- | | |
|----------------------------------|--------------|
| 5. $P \vee I$ | 4, Simp |
| 6. $\sim(L \vee C)$ | 2, 5, MP |
| 7. $\sim L \cdot \sim C$ | 6, DM |
| 8. $\sim L$ | 7, Simp |
| 9. $(P \vee L) \cdot (P \vee I)$ | 4, Com |
| 10. $P \vee L$ | 9, Simp |
| 11. $L \vee P$ | 10, Com |
| 12. P | 8, 11, DS |
| 13. $\sim C \cdot \sim L$ | 7, Com |
| 14. $\sim C$ | 13, Simp |
| 15. $P \cdot \sim C$ | 12, 14, Conj |
| 16. $E \cdot F$ | 3, 15, MP |
| 17. $F \cdot E$ | 16, Com |
| 18. F | 17, Simp |
| 19. $F \vee D$ | 18, Add |

- (24)
- | | |
|--|--------------|
| 1. $B \vee (S \cdot N)$ | |
| 2. $B \supset \sim S$ | |
| 3. $S \supset \sim N$ | $/ B \vee W$ |
| 4. $(B \vee S) \cdot (B \vee N)$ | 1, Dist |
| 5. $B \vee S$ | 4, Simp |
| 6. $(B \supset \sim S) \cdot (S \supset \sim N)$ | 2, 3, Conj |
| 7. $\sim S \vee \sim N$ | 5, 6, CD |
| 8. $\sim(S \cdot N)$ | 7, DM |
| 9. $(S \cdot N) \vee B$ | 1, Com |
| 10. B | 8, 9, DS |
| 11. $B \vee W$ | 10, Add |

- (25)
- | | |
|--|-----------------|
| 1. $(\sim M \vee E) \supset (S \supset U)$ | |
| 2. $(\sim Q \vee E) \supset (U \supset H)$ | |
| 3. $\sim(M \vee Q)$ | $/ S \supset H$ |
| 4. $\sim M \cdot \sim Q$ | 3, DM |
| 5. $\sim M$ | 4, Simp |
| 6. $\sim M \vee E$ | 5, Add |
| 7. $S \supset U$ | 1, 6, MP |
| 8. $\sim Q \cdot \sim M$ | 4, Com |
| 9. $\sim Q$ | 8, Simp |
| 10. $\sim Q \vee E$ | 9, Add |
| 11. $U \supset H$ | 2, 10, MP |
| 12. $S \supset H$ | 7, 11, HS |

- (26)
- | | |
|--|--------------------|
| 1. $(\sim R \vee D) \supset \sim(F \cdot G)$ | |
| 2. $(F \cdot R) \supset S$ | |
| 3. $F \cdot \sim S$ | $/ \sim(S \vee G)$ |
| 4. $\sim S \cdot F$ | 3, Com |

5. $\sim S$	4, Simp
6. $\sim(F \cdot R)$	2, 5, MT
7. $\sim F \vee \sim R$	6, DM
8. F	3, Simp
9. $\sim\sim F$	8, DN
10. $\sim R$	7, 9, DS
11. $\sim R \vee D$	10, Add
12. $\sim(F \cdot G)$	1, 11, MP
13. $\sim F \vee \sim G$	12, DM
14. $\sim G$	9, 13, DS
15. $\sim S \cdot \sim G$	5, 14, Conj
16. $\sim(S \vee G)$	15, DM

(27) 1. $\sim Q \supset (C \cdot B)$	
2. $\sim T \supset (B \cdot H)$	
3. $\sim(Q \cdot T)$	/ B
4. $\sim Q \vee \sim T$	3, DM
5. $[\sim Q \supset (C \cdot B)] \cdot [\sim T \supset (B \cdot H)]$	1, 2, Conj
6. $(C \cdot B) \vee (B \cdot H)$	4, 5, CD
7. $(B \cdot C) \vee (B \cdot H)$	6, Com
8. $B \cdot (C \vee H)$	7, Dist
9. B	8, Simp

(28) 1. $\sim(A \cdot G)$	
2. $\sim(A \cdot E)$	
3. $G \vee E$	/ $\sim(A \cdot F)$
4. $\sim A \vee \sim G$	1, DM
5. $\sim A \vee \sim E$	2, DM
6. $(\sim A \vee \sim G) \cdot (\sim A \vee \sim E)$	4, 5, Conj
7. $\sim A \vee (\sim G \cdot \sim E)$	6, Dist
8. $\sim A \vee \sim(G \vee E)$	7, DM
9. $\sim(G \vee E) \vee \sim A$	8, Com
10. $\sim\sim(G \vee E)$	3, DN
11. $\sim A$	9, 10, DS
12. $\sim A \vee \sim F$	11, Add
13. $\sim(A \cdot F)$	12, DM

(29) 1. $(M \cdot N) \vee (O \cdot P)$	
2. $(N \vee O) \supset \sim P$	/ N
3. $[(M \cdot N) \vee O] \cdot [(M \cdot N) \vee P]$	1, Dist
4. $(M \cdot N) \vee O$	3, Simp
5. $O \vee (M \cdot N)$	4, Com
6. $(O \vee M) \cdot (O \vee N)$	5, Dist
7. $(O \vee N) \cdot (O \vee M)$	6, Com
8. $O \vee N$	7, Simp

Exercise 7.3

9. $N \vee O$	8, Com
10. $\sim P$	2, 9, MP
11. $[(M \cdot N) \vee P] \cdot [(M \cdot N) \vee O]$	3, Com
12. $(M \cdot N) \vee P$	11, Simp
13. $P \vee (M \cdot N)$	12, Com
14. $M \cdot N$	10, 13, DS
15. $N \cdot M$	14, Com
16. N	15, Simp

(30)

1. $(T \cdot K) \vee (C \cdot E)$	
2. $K \supset \sim E$	
3. $E \supset \sim C$	/ $T \cdot K$
4. $[(T \cdot K) \vee C] \cdot [(T \cdot K) \vee E]$	1, Dist
5. $[(T \vee C) \cdot (K \vee C)] \cdot [(T \vee E) \cdot (K \vee E)]$	4, Com, Dist
6. $K \vee E$	5, Com, Assoc, Simp
7. $(K \supset \sim E) \cdot (E \supset \sim C)$	2, 3, Conj
8. $\sim E \vee \sim C$	6, 7, CD
9. $\sim C \vee \sim E$	8, Com
10. $\sim(C \cdot E)$	9, DM
11. $(C \cdot E) \vee (T \cdot K)$	1, Com
12. $T \cdot K$	10, 11, DS

Part IV

(1)

1. $(S \cdot D) \vee (S \cdot H)$	
2. $S \supset (I \cdot R)$	/ $S \cdot R$
3. $S \cdot (D \vee H)$	1, Dist
4. S	3, Simp
5. $I \cdot R$	2, 4, MP
6. $R \cdot I$	5, Com
7. R	6, Simp
8. $S \cdot R$	4, 7, Conj

(2)

1. $(C \cdot I) \vee (H \cdot I)$	
2. $B \supset \sim I$	/ $\sim B$
3. $(I \cdot C) \vee (I \cdot H)$	1, Com
4. $I \cdot (C \vee H)$	3, Dist
5. I	4, Simp
6. $\sim \sim I$	5, DN
7. $\sim B$	2, 6, MT

(3)

1. $\sim(S \vee C)$	
2. $\sim(S \cdot R) \supset (C \vee D)$	/ D
3. $\sim S \cdot \sim C$	1, DM
4. $\sim S$	3, Simp

- | | |
|---|---------------|
| 5. $\sim S \vee \sim R$ | 4, Add |
| 6. $\sim(S \cdot R)$ | 5, DM |
| 7. $C \vee D$ | 2, 6, MP |
| 8. $\sim C$ | 3, Com, Simp |
| 9. D | 7, 8, DS |
|
 | |
| (4) 1. $G \vee (R \cdot E)$ | |
| 2. $(G \vee E) \supset \sim R$ | / $G \vee M$ |
| 3. $(G \vee R) \cdot (G \vee E)$ | 1, Dist |
| 4. $(G \vee E) \cdot (G \vee R)$ | 3, Com |
| 5. $G \vee E$ | 4, Simp |
| 6. $\sim R$ | 2, 5, MP |
| 7. $G \vee R$ | 3, Simp |
| 8. $R \vee G$ | 7, Com |
| 9. G | 6, 8, DS |
| 10. $G \vee M$ | 9, Add |
|
 | |
| (5) 1. $E \cdot (P \cdot B)$ | |
| 2. $(E \cdot B) \supset \sim(P \cdot \sim M)$ | / $E \cdot M$ |
| 3. $E \cdot (B \cdot P)$ | 1, Com |
| 4. $(E \cdot B) \cdot P$ | 3, Assoc |
| 5. $E \cdot B$ | 4, Simp |
| 6. $\sim(P \cdot \sim M)$ | 2, 5, MP |
| 7. $\sim P \vee \sim \sim M$ | 6, DM |
| 8. $P \cdot (E \cdot B)$ | 4, Com |
| 9. P | 8, Simp |
| 10. $\sim \sim P$ | 9, DN |
| 11. $\sim \sim M$ | 7, 10, DS |
| 12. M | 11, DN |
| 13. E | 3, Simp |
| 14. $E \cdot M$ | 12, 13, Conj |
|
 | |
| (6) 1. $F \supset (U \cdot R)$ | |
| 2. $\sim F \supset (P \cdot J)$ | |
| 3. $F \vee \sim F$ | / $U \vee P$ |
| 4. $[F \supset (U \cdot R)] \cdot [\sim F \supset (P \cdot J)]$ | 1, 2, Conj |
| 5. $(U \cdot R) \vee (P \cdot S)$ | 3, 4, CD |
| 6. $[(U \cdot R) \vee P] \cdot [(U \cdot R) \vee S]$ | 5, Dist |
| 7. $(U \cdot R) \vee P$ | 6, Simp |
| 8. $P \vee (U \cdot R)$ | 7, Com |
| 9. $(P \vee U) \cdot (P \vee R)$ | 8, Dist |
| 10. $P \vee U$ | 9, Simp |
| 11. $U \vee P$ | 10, Com |

Exercise 7.3

- (7)
- | | |
|---------------------------|------------|
| 1. $R \supset (C \vee M)$ | |
| 2. $\sim(I \vee C)$ | |
| 3. $\sim(A \vee M)$ | $/ \sim R$ |
| 4. $\sim I \cdot \sim C$ | 2, DM |
| 5. $\sim A \cdot \sim M$ | 3, DM |
| 6. $\sim C \cdot \sim I$ | 4, Com |
| 7. $\sim C$ | 6, Simp |
| 8. $\sim M \cdot \sim A$ | 5, Com |
| 9. $\sim M$ | 8, Simp |
| 10. $\sim C \cdot \sim M$ | 7, 9, Conj |
| 11. $\sim(C \vee M)$ | 10, DM |
| 12. $\sim R$ | 1, 11, MT |

- (8)
- | | |
|--------------------------------|--------------------|
| 1. $(P \vee U) \supset \sim L$ | |
| 2. $(I \vee W) \supset \sim K$ | |
| 3. $L \cdot K$ | $/ \sim(U \vee W)$ |
| 4. L | 3, Simp |
| 5. $\sim \sim L$ | 4, DN |
| 6. $\sim(P \vee U)$ | 1, 5, MT |
| 7. $\sim P \cdot \sim U$ | 6, DM |
| 8. $\sim U \cdot \sim P$ | 7, Com |
| 9. $\sim U$ | 8, Simp |
| 10. K | 3, Com, Simp |
| 11. $\sim \sim K$ | 10, DN |
| 12. $\sim(I \vee W)$ | 2, 11, MT |
| 13. $\sim I \cdot \sim W$ | 12, DM |
| 14. $\sim W$ | 13, Com, Simp |
| 15. $\sim U \cdot \sim W$ | 9, 14, Conj |
| 16. $\sim(U \vee W)$ | 15, DM |

- (9)
- | | |
|---------------------------------|------------|
| 1. $\sim(S \vee E)$ | |
| 2. $(\sim S \cdot M) \supset E$ | $/ \sim M$ |
| 3. $\sim S \cdot \sim E$ | 1, DM |
| 4. $\sim E \cdot \sim S$ | 3, Com |
| 5. $\sim E$ | 4, Simp |
| 6. $\sim(\sim S \cdot M)$ | 2, 5, MT |
| 7. $\sim \sim S \vee \sim M$ | 6, DM |
| 8. $S \vee \sim M$ | 7, DN |
| 9. $\sim S$ | 3, Simp |
| 10. $\sim M$ | 8, 9, DS |

2. $\sim E \vee (G \cdot W)$	
3. $\sim P \vee \sim W$	/ $\sim E$
4. $(\sim E \vee B) \cdot (\sim E \vee P)$	1, Dist
5. $(\sim E \vee P) \cdot (\sim E \vee B)$	4, Com
6. $\sim E \vee P$	5, Simp
7. $(\sim E \vee G) \cdot (\sim E \vee W)$	2, Dist
8. $(\sim E \vee W) \cdot (\sim E \vee G)$	7, Com
9. $\sim E \vee W$	8, Simp
10. $(\sim E \vee P) \cdot (\sim E \vee W)$	6, 9, Conj
11. $\sim E \vee (P \cdot W)$	10, Dist
12. $(P \cdot W) \vee \sim E$	11, Com
13. $\sim(P \cdot W)$	3, DM
14. $\sim E$	12, 13, DS

Exercise 7.4

Part I

1. $G \supset Q$	3	9. $D \equiv H$	1
2. $(R \cdot S) \supset N$	1	10. $S \supset G$	3
3. $\sim P \supset \sim H$	2	11. $J \supset F$	2
4. $B \equiv N$	1	12. $(C \cdot H) \supset A$	3
5. A	2	13. $W \equiv \sim T$	2
6. $\sim Q \supset \sim L$	3	14. $(K \supset M) \equiv S$	1
7. $\sim C \vee F$	1	15. $S \supset D$	1
8. $G \supset (N \supset Z)$	2		

Part II

1. $J \supset M$	1, Impl 2, 3, HS	5. $(G \supset B) \cdot (H \supset C)$	1, Trans 2, 3, CD
2. $J \supset (F \supset N)$	1, Exp 2, 3, MP	6. $(J \cdot M) \supset Q$	1, Exp 2, 3, MP
3. $(C \supset A) \cdot (A \supset C)$	1, 2, Conj 3, Equiv	7. $H \supset (C \supset R)$	1, Impl 2, Exp
4. $K \vee K$	1, 2, CD 3, Taut	8. $T \supset G$	1, Trans 2, 3, HS