Homework Handout #6: Functions

I. Use the given key to translate the following sentences into FF.

\begin{align*}
o & : \text{one} \\
t & : \text{two} \\
f(x) & : \text{the successor of } x \\
g(x) & : \text{the square of } x \\
f(x, y) & : \text{the product of } x \text{ and } y \\
g(x, y) & : \text{the sum of } x \text{ and } y \\
E & : x \text{ is even} \\
N & : x \text{ is a natural number (i.e. } 1, 2, 3...) \\
O & : x \text{ is odd} \\
P & : x \text{ is prime}
\end{align*}

Note: in the following sentences, take ‘number’ to mean ‘natural number’.

1. One is not prime, but its successor is.
2. Every number has a successor.
3. The successor of an odd number is even.
4. If the sum of a pair of numbers is even then either both members of the pair are even or both members are odd.
5. If the sum of a pair of numbers is odd, then one member of the pair is odd and the other member is even.
6. The product of a pair of prime numbers is not prime.
7. The square of an even number is even and the square of an odd number is odd.
8. The successor of the square of an even number is odd.
9. The sum of two and a prime number other than two is odd.
10. There is a pair of distinct prime numbers such that their product is the successor of their sum.

II. Derive the conclusions of each of the following arguments.

1. \begin{align*} & \neg \exists x \neg (A(x) \lor B(f(x))) \quad / \quad \exists x (A(f(x)) \lor B(f(f(x)))) \\
\end{align*}

2. \begin{align*} & \neg \exists x \neg (A(x) \lor B(f(x))) \\
\neg \forall x (A(x) \lor B(f(x))) & / \quad \exists x (A(f(x)) \lor B(f(f(x)))) \\
\end{align*}

3. \begin{align*} & \neg \exists x \neg (A(x) \lor B(f(x))) \\
\neg \forall x (A(x) \lor B(f(x))) & / \quad \exists x (A(f(x)) \lor B(f(f(x)))) \\
\end{align*}

\begin{align*} & \neg \exists x \neg (A(x) \lor B(f(x))) \\
\neg \forall x (A(x) \lor B(f(x))) & / \quad \exists x (A(f(x)) \lor B(f(f(x)))) \\
\end{align*}