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COUNTERPART THEORY AND QUANTIFIED MODAL LOGIC*

I. COUNTERPART THEORY

WE can conduct formalized discourse about most topics perfectly well by means of our all-purpose extensional logic, provided with predicates and a domain of quantification suited to the subject matter at hand. That is what we do when our topic is numbers, or sets, or wholes and parts, or strings of symbols. That is not what we do when our topic is modality: what might be and what must be, essence and accident. Then we introduce modal operators to create a special-purpose, nonextensional logic. Why this departure from our custom? Is it a historical accident, or was it forced on us somehow by the very nature of the topic of modality?

It was not forced on us. We have an alternative. Instead of formalizing our modal discourse by means of modal operators, we could follow our usual practice. We could stick to our standard logic (quantification theory with identity and without ineliminable singular terms) and provide it with predicates and a domain of quantification suited to the topic of modality. That done, certain expressions are available which take the place of modal operators. The new predicates required, together with postulates on them, constitute the system I call *Counterpart Theory.*

The primitive predicates of counterpart theory are these four:

Wx (x is a possible world)
Ixy (x is in possible world y)
Ax (x is actual)
Cxy (x is a counterpart of y).

* I am indebted to David Kaplan, whose criticisms have resulted in many important improvements. A. N. Prior has informed me that my theory resembles a treatment of *de re* modality communicated to him by P. T. Geach in 1964.

The domain of quantification is to contain every possible world and everything in every world. The primitives are to be understood according to their English readings and the following postulates:

P1: $\forall x \forall y(Ixy \supset Wy)$
(Nothing is in anything except a world)

P2: $\forall x \forall y \forall z(Ixy \;\&\; Ixz \;.\!\supset y = z)$
(Nothing is in two worlds)

P3: $\forall x \forall y(Cxy \supset \exists z Ixz)$
(Whatever is a counterpart is in a world)

P4: $\forall x \forall y(Cxy \supset \exists z Iyz)$
(Whatever has a counterpart is in a world)

P5: $\forall x \forall y \forall z(Ixy \;\&\; Izy \;\&\; Cxz \;.\!\supset x = z)$
(Nothing is a counterpart of anything else in its world)

P6: $\forall x \forall y(Ixy \supset Cxx)$
(Anything in a world is a counterpart of itself)

P7: $\exists x(Wx \;\&\; \forall y(Iyx \equiv Ay))$
(Some world contains all and only actual things)

P8: $\exists x Ax$
(Something is actual)

The world mentioned in P7 is unique, by P2 and P8. Let us abbreviate its description:

$$@ =_{df} \imath x \forall y(Iyx \equiv Ay) \quad \text{(the actual world)}$$

Unactualized possibles, things in worlds other than the actual world, have often been deemed "entia non grata",[1] largely because it is not clear when they are or are not identical. But identity literally understood is no problem for us. Within any one world, things of every category are individuated just as they are in the actual world; things in different worlds are *never* identical, by P2. The counterpart relation is our substitute for identity between things in different worlds.[2] Where some would say that you are in several worlds, in which you have somewhat different properties and somewhat different things happen to you, I prefer to say that you are in the actual world and no other, but you have counterparts in several other worlds. Your counterparts resemble you closely in content and context in important respects. They resemble you more closely than do the other things in their worlds. But they are not really you. For each of them is in his own world, and only you are here in the actual world. Indeed we might say, speaking casually, that your counter-

[1] W. V. Quine, *Word and Object* (Cambridge, Mass.: MIT Press, 1960), p. 245.

[2] Yet with this substitute in use, it would not matter if some things *were* identical with their counterparts after all! P2 serves only to rule out avoidable problems of individuation.

parts are you in other worlds, that they and you are the same; but this sameness is no more a literal identity than the sameness between you today and you tomorrow. It would be better to say that your counterparts are men you *would have been*, had the world been otherwise.[3]

The counterpart relation is a relation of similarity. So it is problematic in the way all relations of similarity are: it is the resultant of similarities and dissimilarities in a multitude of respects, weighted by the importances of the various respects[4] and by the degrees of the similarities.[5]

Carnap,[6] Kanger,[7] Hintikka,[8] Kripke,[9] Montague,[10] and others have proposed interpretations of quantified modal logic on which one thing is allowed to be in several worlds. A reader of this persuasion might suspect that he and I differ only verbally: that what I call a thing in a world is just what he would call a ⟨thing, world⟩ pair, and that what he calls the same thing in several worlds is just what I would call a class of mutual counterparts. But beware. Our difference is not just verbal, for I enjoy a generality he cannot match. The counterpart relation will not, in general, be an equivalence relation. So it will not hold just between those of his ⟨thing, world⟩ pairs with the same first term, no matter how he may choose to identify things between worlds.

It would not have been plausible to postulate that the counterpart relation was transitive. Suppose x_1 in world w_1 resembles you closely in many respects, far more closely than anything else in w_1 does. And suppose x_2 in world w_2 resembles x_1 closely, far more closely than anything else in w_2 does. So x_2 is a counterpart of your counterpart x_1. Yet x_2 might not resemble you very closely, and something else in w_2 might resemble you more closely. If so, x_2 is not your counterpart.

[3] This way of describing counterparts is due to L. Sprague de Camp, "The Wheels of If," in *Unknown Fantasy Fiction*, October, 1940.

[4] As discussed in Michael A. Slote, "The Theory of Important Criteria," this JOURNAL, LXIII, 8 (Apr. 14, 1966): 211–224.

[5] The counterpart relation is very like the relation of intersubjective correspondence discussed in Rudolf Carnap, *Der Logische Aufbau der Welt* (Berlin-Schlactensee: Weltkreis-Verlag, 1928), sec. 146.

[6] "Modalities and Quantification," *Journal of Symbolic Logic*, XI, 2 (June 1946): 33–64.

[7] *Provability in Logic* (Stockholm: Almqvist and Wiksell, 1957).

[8] "Modality as Referential Multiplicity," *Ajatus*, XX (1957): 49–64.

[9] "A Completeness Theorem in Modal Logic," *Journal of Symbolic Logic*, XXIV, 1 (March 1959): 1–14; "Semantical Considerations on Modal Logic," *Acta Philosophica Fennica*, XVI (1963): 83–94.

[10] "Logical Necessity, Physical Necessity, Ethics, and Quantifiers," *Inquiry*, III (1960): 259–269.

It would not have been plausible to postulate that the counterpart relation was symmetric. Suppose x_3 in world w_3 is a sort of blend of you and your brother; x_3 resembles both of you closely, far more closely than anything else in w_3 resembles either one of you. So x_3 is your counterpart. But suppose also that the resemblance between x_3 and your brother is far closer than that between x_3 and you. If so, you are not a counterpart of x_3.

It would not have been plausible to postulate that nothing in any world had more than one counterpart in any other world. Suppose x_{4a} and x_{4b} in world w_4 are twins; both resemble you closely; both resemble you far more closely than anything else in w_4 does; both resemble you equally. If so, both are your counterparts.

It would not have been plausible to postulate that no two things in any world had a common counterpart in any other world. Suppose you resemble both the twins x_{4a} and x_{4b} far more closely than anything else in the actual world does. If so, you are a counterpart of both.

It would not have been plausible to postulate that, for any two worlds, anything in one was a counterpart of something in the other. Suppose there is something x_5 in world w_5—say, Batman—which does not much resemble anything actual. If so, x_5 is not a counterpart of anything in the actual world.

It would not have been plausible to postulate that, for any two worlds, anything in one had some counterpart in the other. Suppose whatever thing x_6 in world w_6 it is that resembles you more closely than anything else in w_6 is nevertheless quite unlike you; nothing in w_6 resembles you at all closely. If so, you have no counterpart in w_6.

II. TRANSLATION

Counterpart theory and quantified modal logic seem to have the same subject matter; seem to provide two rival ways of formalizing our modal discourse. In that case they should be intertranslatable; indeed they are. Hence I need not give directions for formalizing modal discourse directly by means of counterpart theory; I can assume the reader is accustomed to formalizing modal discourse by means of modal operators, so I need only give directions for translating sentences of quantified modal logic into sentences of counterpart theory.

Counterpart theory has at least three advantages over quantified modal logic as a vehicle for formalized discourse about modality. (1) Counterpart theory is a theory, not a special-purpose intensional logic. (2) Whereas the obscurity of quantified modal logic has proved intractable, that of counterpart theory is at least divided, if not conquered. We can trace it to its two independent sources. There is our uncertainty about analyticity, and, hence, about whether certain descriptions describe possible worlds; and there is our uncertainty about the relative importance of different respects of similarity and dissimilarity, and, hence, about which things are counterparts of which. (3) If the translation scheme I am about to propose is correct, every sentence of quantified modal logic has the same meaning as a sentence of counterpart theory, its translation; but not every sentence of counterpart theory is, or is equivalent to, the translation of any sentence of quantified modal logic. Therefore, starting with a fixed stock of predicates other than those of counterpart theory, we can say more by adding counterpart theory than we can by adding modal operators.

Now let us examine my proposed translation scheme.[11] We begin with some important special cases, leading up to a general definition.

First consider a closed (0-place) sentence with a single, initial modal operator: $\Box\phi$ or $\Diamond\phi$. It is given the familiar translation: $\forall\beta(W\beta \supset \phi^{\beta})$ (ϕ holds in any possible world β) or $\exists\beta(W\beta \mathbin{\&} \phi^{\beta})$ (ϕ holds in some possible world β). To form the sentence ϕ^{β} (ϕ holds in world β) from the given sentence ϕ, we need only restrict the range of each quantifier in ϕ to the domain of things in the world denoted by β; that is, we replace $\forall\alpha$ by $\forall\alpha(I\alpha\beta \supset \cdots)$ and $\exists\alpha$ by $\exists\alpha(I\alpha\beta \mathbin{\&} \cdots)$ throughout ϕ.

Next consider a 1-place open sentence with a single, initial modal operator: $\Box\phi\alpha$ or $\Diamond\phi\alpha$. It is given the translation $\forall\beta\forall\gamma(W\beta \mathbin{\&} I\gamma\beta \mathbin{\&} C\gamma\alpha \mathbin{.\supset} \phi^{\beta}\gamma)$ (ϕ holds of every counterpart γ of α in any world β) or $\exists\beta\exists\gamma(W\beta \mathbin{\&} I\gamma\beta \mathbin{\&} C\gamma\alpha \mathbin{\&} \phi^{\beta}\gamma)$ (ϕ holds of some counterpart γ of α in some world β). Likewise for an open sentence with any number of places.

If the modal operator is not initial, we translate the subsentence it governs. And if there are quantifiers that do not lie within the scope of any modal operator, we must restrict their range to the domain of things in the actual world; for that is their range in quantified modal logic, whereas an unrestricted quantifier in counterpart theory would range at least over all the worlds and everything in any of them. A sentence of quantified modal logic that contains *no* modal operator—a nonmodal sentence in a modal context—is therefore

[11] NOTATION: Sentences are mentioned by means of the Greek letters 'ϕ', 'ψ',. . .; variables by means of 'α', 'β', 'γ', 'δ',. . .. If ϕ is any n-place sentence and $\alpha_1 \ldots \alpha_n$ are any n different variables, then $\phi\alpha_1 \cdots \alpha_n$ is the sentence obtained by substituting α_1 uniformly for the alphabetically first free variable in ϕ, α_2 for the second, and so on. Variables introduced in translation are to be chosen in some systematic way that prevents confusion of bound variables. Symbolic expressions are used autonymously.

translated simply by restricting its quantifiers to things in the actual world.

Finally, consider a sentence in which there are modal operators within the scopes of other modal operators. Then we must work inward; to obtain ϕ^β from ϕ we must not only restrict quantifiers in ϕ but also translate any subsentences of ϕ with initial modal operators.

The general translation scheme can best be presented as a direct definition of the translation of a sentence ϕ of quantified modal logic:

T1: The translation of ϕ is $\phi^@$ (ϕ holds in the actual world); that is, in primitive notation, $\exists\beta(\forall\alpha(I\alpha\beta \equiv A\alpha)\ \&\ \phi^\beta)$

followed by a recursive definition of ϕ^β (ϕ holds in world β)

T2a: ϕ^β is ϕ, if ϕ is atomic
T2b: $(\sim\phi)^\beta$ is $\sim\phi^\beta$
T2c: $(\phi\ \&\ \psi)^\beta$ is $\phi^\beta\ \&\ \psi^\beta$
T2d: $(\phi \vee \psi)^\beta$ is $\phi^\beta \vee \psi^\beta$
T2e: $(\phi \supset \psi)^\beta$ is $\phi^\beta \supset \psi^\beta$
T2f: $(\phi \equiv \psi)^\beta$ is $\phi^\beta \equiv \psi^\beta$
T2g: $(\forall\alpha\phi)^\beta$ is $\forall\alpha(\mathrm{I}\alpha\beta \supset \phi^\beta)$
T2h: $(\exists\alpha\phi)^\beta$ is $\exists\alpha(\mathrm{I}\alpha\beta\ \&\ \phi^\beta)$
T2i: $(\Box\phi\alpha_1\ldots\alpha_n)^\beta$ is $\forall\beta_1\forall\gamma_1\cdots\forall\gamma_n$
$(W\beta_1\ \&\ I\gamma_1\beta_1\ \&\ C\gamma_1\alpha_1\ \&\cdots\&\ I\gamma_n\beta_1\ \&\ C\gamma_n\alpha_n\ .\supset \phi^{\beta_1}\gamma_1\ldots\gamma_n)$
T2j: $(\Diamond\phi\alpha_1\ldots\alpha_n)^\beta$ is $\exists\beta_1\exists\gamma_1\cdots\exists\gamma_n$
$(\mathrm{W}\beta_1\ \&\ \mathrm{I}\gamma_1\beta_1\ \&\ \mathrm{C}\gamma_1\alpha_1\ \&\cdots\&\ \mathrm{I}\gamma_n\beta_1\ \&\ C\gamma_n\alpha_n\ \&\ \phi^{\beta_1}\gamma_1\ldots\gamma_n)$

Using these two definitions, we find, for example, that

$$\forall xFx$$
$$\Diamond\exists xFx$$
$$\Box Fx$$
$$\forall x(Fx \supset \Box Fx)$$
$$\Box\Diamond Fx$$

are translated, respectively, as

$\forall x(Ix@ \supset Fx)$
(Everything actual is an F)

$\exists y(Wy\ \&\ \exists x(Ixy\ \&\ Fx))$
(Some possible world contains an F)

$\forall y_1\forall x_1(Wy_1\ \&\ Ix_1y_1\ \&\ Cx_1x\ .\supset Fx_1)$
(Every counterpart of x, in any world, is an F)

$\forall x(Ix@ \supset .\ Fx \supset \forall y_1\forall x_1(Wy_1\ \&\ Ix_1y_1\ \&\ Cx_1x\ .\supset Fx_1))$
(If anything is a counterpart of an actual F, then it is an F)

$\forall y_1\forall x_1(Wy_1\ \&\ Ix_1y_1\ \&\ Cx_1x\ .\supset \exists y_2\exists x_2(Wy_2\ \&\ Ix_2y_2\ \&\ Cx_2x_1\ \&\ Fx_2))$
(Every counterpart of x has a counterpart which is an F)

The reverse translation, from sentences of counterpart theory to sentences of quantified modal logic, can be done by finite search whenever it can be done at all. For if a modal sentence ψ is the translation of a sentence ϕ of counterpart theory, then ψ must be shorter than ϕ and ψ must contain no predicates or variables not in ϕ. But not every sentence of counterpart theory is the translation of a modal sentence, or even an equivalent of the translation of a modal sentence. For instance, our postulates P1–P7 are not.

It may disturb us that the translation of $\forall x \Box \exists y(x = y)$ (everything actual necessarily exists) comes out true even if something actual lacks a counterpart in some world. To avoid this, we might be tempted to adopt the alternative translation scheme, brought to my attention by David Kaplan, in which T2i and T2j are replaced by

$$\text{T2i}': (\Box\phi\alpha_1 \cdots \alpha_n)^\beta \text{ is } \forall\beta_1(W\beta_1 \supset \exists\gamma_1 \cdots \exists\gamma_n(I\gamma_1\beta_1 \;\&\; C\gamma_1\alpha_1 \;\&\cdots \\ \&\; I\gamma_n\beta_1 \;\&\; C\gamma_n\alpha_n \;\&\; \phi^{\beta_1}\gamma_1 \cdots \gamma_n))$$

$$\text{T2j}': (\Diamond\phi\alpha_1 \cdots \alpha_n)^\beta \text{ is } \exists\beta_1(W\beta_1 \;\&\; \forall\gamma_1 \ldots \forall\gamma_n(I\gamma_1\beta_1 \;\&\; C\gamma_1\alpha_1 \;\&\cdots \\ \&\; I\gamma_n\beta_1 \;\&\; C\gamma_n\alpha_n \;.\!\supset \phi^{\beta_1}\gamma_1 \ldots \gamma_n))$$

with heterogeneous rather than homogeneous quantifiers. Out of the frying pan, into the fire: with T2j′, $\exists x \Diamond (x \neq x)$ (something actual is possibly non-self-identical) comes out true unless everything actual has a counterpart in every world! We might compromise by taking T2i′ and T2j, but at the price of sacrificing the ordinary duality of necessity and possibility.[12] So I chose to take T2i and T2j.

III. ESSENTIALISM

Quine has often warned us that by quantifying past modal operators we commit ourselves to the view that "an object, of itself and by whatever name or none, must be seen as having some of its traits necessarily and others contingently, despite the fact that the latter traits follow just as analytically from some ways of specifying the object as the former traits do from other ways of specifying it."[13] This so-called "Aristotelian essentialism"–the doctrine of essences not relative to specifications–"should be every bit as congenial to [the champion of quantified modal logic] as quantified modal logic itself."[14]

Agreed. Essentialism is congenial. We do have a way of saying

[12] If we also postulate that the counterpart relation is an equivalence relation, we get an interpretation like that of Føllesdal in "Referential Opacity in Modal Logic" (unpublished Ph.D. dissertation, Harvard, 1961), sec. 20, and in "A Model-Theoretic Approach to Causal Logic," forthcoming in *Det Kongeliger Norske Videnskabers Selskabs Forhandlinger.*

[13] "Reference and Modality," in *From a Logical Point of View*, 2d ed. (Cambridge, Mass.: Harvard, 1961), p. 155.

[14] "Reply to Professor Marcus," in *The Ways of Paradox* (New York: Random House, 1966), p. 182.

that an attribute is an essential attribute of an object—essential regardless of how the object happens to have been specified and regardless of whether the attribute follows analytically from any or all specifications of the object.

Consider the attribute expressed by a 1-place sentence ϕ and the object denoted by a singular term[15] ζ. To say that this attribute is an essential attribute of this object is to assert the translation of $\Box\phi\zeta$.

But we have not yet considered how to translate a modal sentence containing a singular term. For we know that any singular term ζ may be treated as a description $\iota\alpha(\psi\alpha)$ (although often only by letting ψ contain some artificial predicate made from a proper name); and we know that any description may be eliminated by Russell's contextual definition. Our translation scheme did not take account of singular terms because they need never occur in the primitive notation of quantified modal logic. We must always eliminate singular terms before translating; afterwards, if we like, we can restore them.

There is just one hitch: before eliminating a description, we must assign it a scope. Different choices of scope will, in general, lead to nonequivalent translations. This is so even if the eliminated description denotes precisely one thing in the actual world and in every possible world.[16]

Taking ζ as a description $\iota\alpha(\psi\alpha)$ and assigning it narrow scope, our sentence $\Box\phi\zeta$ is interpreted as

$$\Box\exists\alpha(\forall\delta(\psi\delta \equiv \delta = \alpha) \,\&\, \phi\alpha)$$

Its translation under this interpretation is

$$\forall\beta(W\beta \supset \exists\alpha(I\alpha\beta \,\&\, \forall\delta(I\delta\beta \supset .\, \psi^{\beta}\delta \equiv \delta = \alpha) \,\&\, \phi^{\beta}\alpha))$$

(Any possible world β contains a unique α such that $\psi^{\beta}\alpha$; and for any such α, $\phi^{\beta}\alpha$)

This is an interpretation *de dicto*: the modal operator attaches to the already closed sentence $\phi\zeta$. It is referentially opaque: the translation of an ostensible use of Leibniz's Law

$$\begin{array}{rl} & \Box\phi\zeta \\ & \underline{\eta = \zeta} \\ \therefore & \Box\phi\eta \end{array}$$

[15] NOTATION: Terms are mentioned by means of the Greek letters 'ζ', 'η',.... The sentence $\phi\zeta$ is that obtained by substituting the term ζ uniformly into the 1-place sentence ϕ.

[16] I follow Arthur Smullyan's treatment of scope ambiguity in modal sentences, given in "Modality and Description," *Journal of Symbolic Logic*, XIII, 1 (March 1948): 31–37, as qualified by Wilson's objection, in *The Concept of Language* (Toronto: University Press, 1959), p. 43, that some ostensible uses of Leibniz's law on modal sentences are invalid under *any* choice of scope in the conclusion.

or of an ostensible existential generalization

$$\frac{\Box\phi\zeta}{\therefore\ \exists\alpha\,\Box\phi\alpha}$$

is an invalid argument if the terms involved are taken as descriptions with narrow scope.

Taking ζ as a description with wide scope, $\Box\phi\zeta$ is interepreted as

$$\exists\alpha(\forall\delta(\psi\delta \equiv \delta = \alpha)\ \&\ \Box\phi\alpha)$$

and translated as

$$\exists\alpha(I\alpha@\ \&\ \forall\delta(I\delta@ \supset .\, \psi^{@}\delta \equiv \delta = \alpha)\ \&\ \forall\beta\forall\gamma(W\beta\ \&\ I\gamma\beta\ \&\ C\gamma\alpha \mathbin{.\supset} \phi^{\beta}\gamma))$$

(The actual world contains a unique α such that $\psi^{@}\alpha$; and for any counterpart γ thereof, in any world β, $\phi^{\beta}\gamma$)

This is an interpretation *de re*: the modal operator attaches to the open sentence ϕ to form a new open modal sentence $\Box\phi$, and the attribute expressed by $\Box\phi$ is then predicated of the actual thing denoted by ζ. This interpretation is referentially transparent: the translation of an ostensible use of Leibniz's law or of an ostensible existential generalization is a valid argument if the terms involved are taken as descriptions with wide scope.

How are we to choose between the two interpretations of $\Box\phi\zeta$? Often we cannot, unless by fiat; there is a genuine ambiguity. But there are several conditions that tend to favor the wide-scope interpretation as the more natural: (1) whenever ζ is a description formed by turning a proper name into an artificial predicate; (2) whenever the description ζ has what Donellan calls its referential use;[17] (3) whenever we are prepared to accept

ζ is something α such that necessarily $\phi\alpha$

as one possible English reading of $\Box\phi\zeta$. (The force of the third condition is due to the fact that $\exists\alpha(\zeta = \alpha\ \&\ \Box\phi\alpha)$ is unambiguously equivalent to $\Box\phi\zeta$ with ζ given wide scope.[18])

The translations of $\Box\phi\zeta$ under its two interpretations are logically independent. Neither follows from the other just by itself. But with the aid of suitable auxiliary premises we can go in both directions. The inference from the narrow-scope translation to the wide-scope translation (exportation[19]) requires the further premise

[17] "Reference and Definite Descriptions," *Philosophical Review*, LXXV, 3 (July 1966): 281–304.

[18] Cf. Hintikka, *Knowledge and Belief* (Ithaca, N.Y.: Cornell, 1962), pp. 156–157.

[19] I follow Quine's use of this term in "Quantifiers and Propositional Attitudes," in *The Ways of Paradox*, p. 188.

$\exists\alpha(I\alpha@\ \&\ \forall\beta\forall\gamma(I\gamma\beta\ \&\ C\gamma\alpha\ .\supset\ \forall\delta(I\delta\beta\supset.\ \psi^{\beta}\delta\equiv\delta=\gamma)))$
(There is something α in the actual world, any counterpart γ of which is the only thing δ in its world β such that $\psi^{\beta}\delta$)

which is a simplified equivalent of the translation of $\exists\alpha\Box(\zeta=\alpha)$ with ζ given narrow scope.[20] The inference from the wide-scope translation to the narrow-scope translation (importation) requires the same auxiliary premise, and another as well:

$\exists\alpha(I\alpha@\ \&\ \forall\delta(I\delta@\supset.\ \psi^{@}\delta\equiv\delta=\alpha)\ \&\ \forall\beta(W\beta\supset\exists\gamma(I\gamma\beta\ \&\ C\gamma\alpha)))$
(The unique α in the actual world such that $\psi^{@}\alpha$, has at least one counterpart γ in any world β)

This second auxiliary premise is not equivalent to the translation of any modal sentence.[21]

In general, of course, there will be more than two ways to assign scopes. Consider $\Box\Diamond(\eta=\zeta)$. Each description may be given narrow, medium, or wide scope; so there are nine nonequivalent translations.

It is the wide-scope, *de re*, transparent translation of $\Box\phi\zeta$ which says that the attribute expressed by ϕ is an essential attribute of the thing denoted by ζ. In short, an essential attribute of something is an attribute it shares with all its counterparts. All your counterparts are probably human; if so, you are essentially human. All your counterparts are even more probably corporeal; if so, you are essentially corporeal.

An attribute that something shares with all its counterparts is an essential attribute of that thing, part of its essence. The whole of its essence is the intersection of its essential attributes, the attribute it shares with all and only its counterparts. (*The* attribute, because there is no need to distinguish attributes that are coextensive not only in the actual world but also in every possible world.) There may or may not be an open sentence that expresses the attribute that is the essence of something; to assert that the attribute expressed by ϕ is the essence of the thing denoted by ζ is to assert

$\exists\alpha(I\alpha@\ \&\ \forall\delta(I\delta@\supset.\ \psi^{@}\delta\equiv\delta=\alpha)\ \&\ \forall\beta\forall\gamma(I\gamma\beta\supset.\ C\gamma\alpha\equiv\phi^{\beta}\gamma))$
(The actual world contains a unique α such that $\psi^{@}\alpha$; and for anything γ in any world β, γ is a counterpart of α if and only if $\phi^{\beta}\gamma$)

This sentence is not equivalent to the translation of any modal sentence.

Essence and counterpart are interdefinable. We have just defined the essence of something as the attribute it shares with all and only

[20] Cf. Hintikka, *op. cit.*, pp. 138–155.

[21] But under any variant translation in which T2i is replaced by T2i′, it would be equivalent to the translation of $\Box\exists\alpha(\zeta=\alpha)$ (ζ necessarily exists) with ζ given wide scope.

its counterparts; a counterpart of something is anything having the attribute which is its essence. (This is not to say that that attribute is the *counterpart's* essence, or even an essential attribute of the counterpart.)

Perhaps there are certain attributes that can only be essential attributes of things, never accidents. Perhaps every human must be essentially human; more likely, perhaps everything corporeal must be essentially corporeal. The attribute expressed by ϕ is of this sort, incapable of being an accident, just in case it is closed under the counterpart relation; that is, just in case

$\forall\alpha\forall\beta\forall\gamma\forall\beta_1(I\alpha\beta \;\&\; I\gamma\beta_1 \;\&\; C\gamma\alpha \;\&\; \phi^{\beta}\alpha \,.\!\supset \phi^{\beta_1}\gamma)$
(For any counterpart γ in any world β_1 of anything α in any world β, if $\phi^{\beta}\alpha$ then $\phi^{\beta_1}\gamma$)

This is a simplified equivalent of the translation of

$$\Box\forall\alpha(\phi\alpha \supset \Box\phi\alpha)$$

We might wonder whether these attributes incapable of being accidents are what we call "natural kinds." But notice first that we must disregard the necessarily universal attribute, expressed, for instance, by the open sentence $\alpha = \alpha$, since it is an essential attribute of everything. And notice second that arbitrary unions of attributes incapable of being accidents are themselves attributes incapable of being accidents; so to exclude gerrymanders we must confine ourselves to *minimal* attributes incapable of being accidents. All of these may indeed be natural kinds; but these cannot be the only natural kinds, since some unions and all intersections of natural kinds are themselves natural kinds.

IV. MODAL PRINCIPLES

Translation into counterpart theory can settle disputed questions in quantified modal logic. We can test a suggested modal principle by seeing whether its translation is a theorem of counterpart theory; or, if not, whether the extra postulates that would make it a theorem are plausible. We shall consider eight principles and find only one that should be accepted.

$$\Box\phi \prec \Box\Box\phi \quad \text{(Becker's principle)}$$

The translation is not a theorem unless ϕ is a closed sentence, but would have been a theorem in general under the rejected postulate that the counterpart relation was transitive.

$$\phi \prec \Box\Diamond\phi \quad \text{(Brouwer's principle)}$$

The translation is not a theorem unless ϕ is a closed sentence, but would have been a theorem in general under the rejected postulate that the counterpart relation was symmetric.

$$\alpha_1 = \alpha_2 \dashv \Box \alpha_1 = \alpha_2 \quad (\alpha_1 \text{ and } \alpha_2 \text{ not the same variable})$$

The translation is not a theorem, but would have been under the rejected postulate that nothing in any world had more than one counterpart in any other world.

$$\alpha_1 \neq \alpha_2 \dashv \Box \alpha_1 \neq \alpha_2 \quad (\alpha_1 \text{ and } \alpha_2 \text{ not the same variable})$$

The translation is not a theorem, but would have been under the rejected postulate that no two things in any world had a common counterpart in any other world.

$$\forall\alpha \Box \phi\alpha \dashv \Box \forall\alpha\phi\alpha \quad \text{(Barcan's principle)}$$

The translation is not a theorem, but would have been under the rejected postulate that, for any two worlds, anything in one was a counterpart of something in the other.

$$\exists\alpha \Box \phi\alpha \dashv \Box \exists\alpha\phi\alpha$$

The translation is not a theorem, but would have been under the rejected postulate that, for any two worlds, anything in one had some counterpart in the other.

$$\Box \forall\alpha\phi\alpha \dashv \forall\alpha \Box \phi\alpha \quad \text{(Converse of Barcan's principle)}$$

The translation is a theorem.

$$\Box \exists\alpha\phi\alpha \dashv \exists\alpha \Box \phi\alpha$$

The translation is not a theorem, nor would it have been under any extra postulates with even the slightest plausibility.

V. RELATIVE MODALITIES

Just as a sentence ϕ is necessary if it holds in all worlds, so ϕ is causally necessary if it holds in all worlds compatible with the laws of nature; obligatory for you if it holds in all worlds in which you act rightly; implicitly known, believed, hoped, asserted, or perceived by you if it holds in all worlds compatible with the content of your knowledge, beliefs, hopes, assertions, or perceptions. These, and many more, are *relative* modalities, expressible by quantifications over restricted ranges of worlds. We can write any dual pair of relative modalities as

$$\Box^i \delta_1 \cdots \delta_m$$
$$\Diamond^i \delta_1 \cdots \delta_m$$

where the index i indicates how the restriction of worlds is to be made and the m arguments $\delta_1, \ldots, \delta_m$, with $m \geq 0$, denote things to be considered in making the restriction (say, the person whose implicit knowledge we are talking about). To every dual pair of relative modalities there corresponds a characteristic relation

$R^i xyz_1 \cdots z_m$ (world x is i-related to world y and $z_1, \ldots, z_m$ therein)

governed by the postulate

P9: $\forall x \forall y \forall z_1 \cdots \forall z_m (R^i xyz_1 \cdots z_m \supset. Wx \;\&\; Wy \;\&\; Iz_1 y \;\&\cdots\&\; Iz_m y)$

The characteristic relation gives the appropriate restriction: we are to consider only worlds i-related to whatever world we are in (and certain things in it). Necessity and possibility themselves are that pair of relative modalities whose characteristic relation is just the 2-place universal relation between worlds.[22]

We can easily extend our translation scheme to handle sentences containing miscellaneous modal operators. We will treat them just as we do necessity and possibility, except that quantifiers over worlds will range over only those worlds which bear the appropriate characteristic relation to some world and perhaps some things in it. The translation of ϕ remains $\phi^@$; we need only add two new clauses to the recursive definition of ϕ:

T2i*: $(\Box^i \delta_1 \cdots \delta_m \phi \alpha_1 \cdots \alpha_n)^\beta$ is $\forall \beta_1 \forall \gamma_1 \cdots \forall \gamma_n$
$(R^i \beta_1 \beta \delta_1 \cdots \delta_m \;\&\; I\gamma_1\beta_1 \;\&\; C\gamma_1\alpha_1 \;\&\cdots\&\; I\gamma_n\beta_1 \;\&\; C\gamma_n\alpha_n \;.\!\supset \phi^{\beta_1}\gamma_1 \cdots \gamma_n)$
T2j*: $(\Diamond^i \delta_1 \cdots \delta_m \phi \alpha_1 \cdots \alpha_n)^\beta$ is $\exists \beta_1 \exists \gamma_1 \cdots \exists \gamma_n$
$(R^i \beta_1 \beta \delta_1 \cdots \delta_m \;\&\; I\gamma_1\beta_1 \;\&\; C\gamma_1\alpha_1 \;\&\cdots\&\; I\gamma_n\beta_1 \;\&\; C\gamma_n\alpha_n \;\&\; \phi^{\beta_1}\gamma_1 \cdots \gamma_n)$

(since necessity and possibility are relative modalities, we no longer need T2i and T2j). For example, our translations of

$$\Box^i \phi$$
$$\Box^j \delta \psi \alpha$$
$$\Box^i \Box^j \delta \phi$$

where ϕ is a 0-place sentence, ψ is a 1-place sentence, $\Box^i$ is a 0-place relative modality, and $\Box^j$ is a 1-place relative modality, are, respectively,

$\forall \beta (R^i \beta @ \supset \phi^\beta)$
(ϕ holds in any world i-related to the actual world)

$\forall \beta \forall \gamma (R^j \beta @ \delta \;\&\; I\gamma\beta \;\&\; C\gamma\alpha \;.\!\supset \psi^\beta \alpha)$
(ψ holds of any counterpart γ of α in any world β j-related to the actual world and δ therein)

[22] Cf. Hintikka, "Quantifiers in Deontic Logic," *Societas Scientiarum Fennica, Commentationes Humanarum Litterarum*, XXIII, 4; Kanger, *op. cit.*; Kripke, *op. cit.*; Montague, *op. cit.*; Prior, "Possible Worlds," *Philosophical Quarterly*, XII, 46 (January 1962): 36–43; Hintikka, *Knowledge and Belief*, pp. 42–49; Føllesdal, "Quantification into Causal Contexts," in *Boston Studies in the Philosophy of Science*, II (New York: Humanities Press, 1965), pp. 263–274; Hintikka, "The Logic of Perception," presented at the 1967 Oberlin Colloquium in Philosophy.

$\forall\beta_1\forall\gamma(R^i\beta @ \;\&\; I\gamma\beta_1 \;\&\; C\gamma\delta \;.\!\supset \forall\beta_2(R^j\beta_2\beta_1\gamma \supset \phi^{\beta_2}))$

(ϕ holds in any world β_2 such that, for some world β_1 that is i-related to the actual world and for some counterpart γ in β_1 of δ, β_2 is j-related to β_1 and γ)

The third example illustrates the fact that free variables occurring as arguments of relative modal operators may need to be handled by means of the counterpart relation.

Our previous discussion of singular terms as eliminable descriptions subject to ambiguity of scope carries over, with one change: in general, the auxiliary premise for exportation (and the first of two auxiliary premises for importation) must be the translation of $\Box^i\delta_1\cdots\delta_m(\zeta = \zeta)$ with one occurrence of ζ given wide scope and the other given narrow scope. The translation of $\exists\alpha\Box^i\delta_1\cdots\delta_m(\zeta = \alpha)$ will do only for those relative modalities, like necessity, for which $R^i@@\delta_1\cdots\delta_m$—and, hence, the translation of $\Box^i\delta_1\cdots\delta_m\phi \supset \phi$—are theorems under the appropriate postulates on the i-relation. More generally, the argument

$$\begin{array}{ll} & \Box^i\delta_1\cdots\delta_m\phi\eta \\ & \underline{\Box^i\delta_1\cdots\delta_m(\eta = \zeta)} \\ \therefore & \Box^1\delta_1\cdots\delta_m\phi\zeta \end{array}$$

where ϕ is a 1-place sentence, has a valid translation if ζ is given wide scope and η is given narrow scope throughout.

Principles corresponding to those discussed in section IV can be formulated for any relative modality (or, in the case of Becker's and Brouwer's principles, for any mixture of relative modalities). The acceptability of such principles will depend, in general, not just on the logical properties of the counterpart relation and the i-relations involved, but on the logical relations *between* the counterpart relation and the i-relations. For example, consider a relative necessity without arguments, so that its characteristic i-relation will be 2-place. (Such an i-relation is often called an *accessibility* relation between worlds.) And consider Becker's principle for this relative necessity (but with '$\dashv$' still defined in terms of necessity itself): $\Box^i\phi \dashv \Box^i\Box^i\phi$; that is, $\Box(\Box^i\phi \supset \Box^i\Box^i\phi)$. It is often said that Becker's principle holds just in case accessibility is transitive, which is correct if ϕ is a closed sentence. But for open ϕ, Becker's principle holds just in case

$$\forall x_1\forall y_1\forall x_2\forall y_2\forall x_3\forall y_3(Ix_1y_1 \;\&\; Ix_2y_2 \;\&\; Ix_3y_3 \;\&\; Cx_2x_1 \;\&\; Cx_3x_2 \;\&\; R^iy_2y_1 \;\&\; R^iy_3y_2 \;.\!\supset\!.\; Cx_3x_1 \;\&\; R^iy_3y_1)$$

even if neither accessibility nor the counterpart relation is transitive.

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