Class 9: Truth Tables for Arguments (§6.4)

I. **Clearing up some translation issues**

In our last class, we saw that the biconditional was superfluous, since any statement made with the biconditional could be made, in slightly more complex form, with a conjunction of two conditionals. The notion of logical equivalence can help us clear up some translation questions.

Unless and Exclusive Disjunction:
Previously, we translated 'unless; with a 'v'.
Consider the complex proposition: 'A car will not run unless there is gas in the tank.'

The car does not run	The car runs	**unless**	The car has gas
F	T	**T**	T
F	T	**F**	F
T	F	**T**	T
T	F	**T**	F

In the first row, the car runs and has gas, so the complex proposition should be true.
In the second row, the car runs, but does not have gas, and so the complex proposition should be false.
In the third row, the car does not run, but has gas.
This does not falsify the complex proposition, which does not say what else the car needs in order to run.
The complex proposition indicates a necessary condition (having gas) but not sufficient conditions for a car to run.
Thus the statement should be considered true.
In the fourth row, the car does not run, but does not have gas, and so the proposition should be true.
The following truth table thus appropriately represents the complex proposition, translating 'unless' as '∨', since it is logically equivalent to the one we want.

~	R	∨	G
F	T	T	T
F	T	F	F
T	F	T	T
T	F	T	F

But now, consider: ‘Carol will attend school full time unless she gets a job’

Carol attends school	unless	Carol gets a job
T	?	T
T	T	F
F	T	T
F	F	F

In the second row, she attends school but doesn’t get a job, and so the proposition should be true.
In the third row, she gets the job, and doesn’t go to school, and so the proposition should be true.
In the last row, she doesn’t get the job but doesn’t go to school, and so the proposition should be false.
What about the first row?
Here she gets a job but attends school anyway.
Is the proposition true or false?
If we say it’s false, we arrive at a truth table for exclusive disjunction.
Unless is as ambiguous as ‘or’, and in the same way: there’s an inclusive and exclusive ‘unless’.
The following translation will suffice for exclusive disjunction and exclusive unless: $\sim A \equiv J$, since it is logically equivalent to the one we want.

~	A	≡	J
F	T	**F**	T
F	T	**T**	F
T	F	**T**	T
T	F	**F**	F

We thus think of the exclusive unless as a biconditional: Carol will not attend school if, and only if, she gets a job.
So, when translating unless, use the wedge for inclusive senses, and as the default translation.
Use the biconditional (with one element negated) for exclusive senses.

Given two variables, there are 16 possible distributions of truth values.
We have labels for four.
We can define the other 12, using combinations of the five connectives.
(This is kind of a fun exercise. You might try it.)
As long as we can define all possibilities, it doesn’t matter which we take to be basic.
We just have to be careful to translate correctly.

Here is another logically equivalent formulation for exclusive disjunction: '$(P \vee Q) \cdot \sim(P \cdot Q)$'.

(P	∨	Q)	·	~	(P	·	Q)
T	T	T	**F**	F	T	T	T
T	T	F	**T**	T	T	F	F
F	T	T	**T**	T	F	F	T
F	F	F	**F**	T	F	F	F

On the first test, you were asked to translate, 'If the weather is warm but the sky isn't clear, then we will go hiking unless we play cards.
Tim read the 'unless' as exclusive disjunction, and so translated the sentence as:

$$(W \bullet \sim C) \supset [(H \vee P) \bullet \sim(H \bullet P)]$$

Of course, one could also have used, for the exclusive 'unless':

$$(W \bullet \sim C) \supset (\sim H \equiv P)$$

II. **Valid and Invalid Arguments**
Compare the following arguments:

A. 1. If God exists then every effect has a cause.
2. God exists.
∴ Every effect has a cause.

B. 1. If God exists then every effect has a cause.
2. Every effect has a cause.
∴ God exists.

A is valid, and has the following form:

$P \supset Q$
$P \qquad / \; Q$

This form is known as Modus Ponens.
Note that we write the premises on sequential lines, and the conclusion on the same line as the final premise, following a single slash.
B is invalid, and has the following form:

$P \supset Q$
$Q \qquad / \; P$

Arguments of the form B commit the Fallacy of Affirming the Consequent.

Recall: In a **valid argument**, if the premises are true then the conclusion must be true.
Note, this definition says nothing about what happens if the premises are false.
An **invalid argument** is one in which it is possible for true premises to yield a false conclusion.
By focusing on valid arguments, we can make sure that if all our premises are true, so must our conclusions be.

III. **A method for determining if an argument is valid**

Step 1: Line up premises and conclusion horizontally, separating premises with a single slash and separating the premises from the conclusion with a double slash.
Step 2: Construct truth tables for each premise and the conclusion, using consistent assignments to component variables.
Step 3: Look for a counter-example: a row in which all premises are true and the conclusion is false.
If there is a counter-example, the argument is invalid. Specify a counter-example.
If there is no counterexample, the argument is valid.

A valid argument:

P	⊃	Q	/	P	//	Q
T	**T**	T		T		T
T	**F**	F		T		F
F	**T**	T		F		T
F	**T**	F		F		F

Note: On no line are the premises true and the conclusion false. There is no counter-example.

An invalid argument:

P	⊃	Q	/	Q	//	P
T	**T**	T		T		T
T	**F**	F		F		T
F	**T**	T		T		F
F	**T**	F		F		F

This third row is the *counterexample*:
The argument is invalid when P is false and Q is true.

Consider: Is this a valid argument?

1. P ⊃ (Q ⊃ P)
2. ~P / Q

P	⊃	(Q	⊃	P)	/	~P	//	Q
T	**T**	T	T	T		**F**		**T**
T	**T**	F	T	T		**F**		**F**
F	**T**	T	F	F		**T**		**T**
F	**T**	F	T	F		**T**		**F**

Row 4 is a counter-example.
The argument is shown invalid when P is false and Q is false.

IV. **Exercises**. Determine whether each argument is valid. If invalid, specify the counter-example.

1. A ⊃ B
   ~A / ~B

2. C ∨ D
   ~D / C

3. E ≡ F / ~E ∨ F

4. G · H
   H ⊃ I / ~(G · I)

5. J ⊃ K
   K ⊃ ~J
   ~J ⊃ K / J ∨ ~K

V. **Solutions**

1. Invalid, when A is false, and B is true
2. Valid
3. Valid
4. Invalid, when G, H, and I are all true
5. Invalid, when J is false and K is true