Philosophy 240: Symbolic Logic Hamilton College
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Class 8: Truth Tables for Propositions (§6.3)

I. **Classifying propositions using truth tables**

Compare the following propositions:

1. I exist.
2. I am here, now.
3. I am in New York.
4. I am in Canada.
5. 2+2=4
6. 2+2=5

1-3, and 5, are true; 4 and 6 are false.
Still, the philosopher wants to distinguish between necessary truths (1, 2, and 5), and merely contingent ones (3); and between necessary falsehoods (6) and merely contingent ones (4).
Furthermore, some compound propositions are necessarily true or false, independently of the truth values of their component propositions.
We can use truth tables to make these distinctions among tautologies, contingencies, and contradictions.

Consider again the truth table for ‘P ⊃ P’

P	⊃	P
T	**T**	T
F	**T**	F

This is a *tautology*: statement that is always true.
We saw a couple of other tautologies in our last class.

The Law of the Excluded Middle: P ∨ ~P
The Law of Non-Contradiction: ~(P • ~P)

Tautologies are the theorems of propositional logic.
They are sometimes call logical truths.
Here is a tautology in English: ‘Either the Mets win the World Series this year, or they don’t.’

Here's a longer tautology: '[(P ⊃ Q) · (Q ⊃ R)] ⊃ (P ⊃ R)'

[(P	⊃	Q)	·	(Q	⊃	R)]	⊃	(P	⊃	R)
T	T	T	T	T	T	T	**T**	T	T	T
T	T	T	F	T	F	F	**T**	T	F	F
T	F	F	F	F	T	T	**T**	T	T	T
T	F	F	F	F	T	F	**T**	T	F	F
F	T	T	T	T	T	T	**T**	F	T	T
F	T	T	F	T	F	F	**T**	F	T	F
F	T	F	T	F	T	T	**T**	F	T	T
F	T	F	T	F	T	F	**T**	F	T	F

Some philosophers argue that tautologies like this are not true in themselves.
Rather, they say, they are true on all substitutions of propositions for the variables.

Only a small portion of the sentences of propositional logic are tautologies.
Consider the truth table for 'P ∨ ~Q'

P	∨	~	Q
T	**T**	F	T
T	**T**	T	F
F	**F**	F	T
F	**T**	T	F

This is a *contingency*: statement that may or may not be true.
It is true in at least one row of the truth table.
It is false in at least one row.
The truth of the complex proposition is contingent (depends) on the values of the component premises.
Most wffs will be contingent.

Consider the truth table for ‘P · ~P’

P	·	~	P
T	F	F	T
F	F	T	F

This is a *self-contradiction*: statement that is never true.
Here is another: ‘(~P ⊃ Q) ≡ ~(Q ∨ P)’

(~	P	⊃	Q)	≡	~	(Q	∨	P)
F	T	T	T	**F**	F	T	T	T
F	T	T	F	**F**	F	F	T	T
T	F	T	T	**F**	F	T	T	F
T	F	F	F	**F**	T	F	F	F

II. **Exercises A**. Classify each proposition as tautologous, contingent, or self-contradictory.

1. ~A ⊃ ~A
2. B · (B ∨ F)
3. (~D · E) · (E ⊃ D)

III. **Classifying pairs of sentences using truth tables**
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Consider ‘(A ∨ B) ≡ (~B ⊃ A)’.

(A	∨	B)	≡	(~	B	⊃	A)
T	T	T	**T**	F	T	T	T
T	T	F	**T**	T	F	T	T
F	T	T	**T**	F	T	T	F
F	F	F	**T**	T	F	F	F

It is a tautology.
Eliminate the biconditional, and consider the two remaining halves as separate statements:

A	∨	B		~	B	⊃	A
T	**T**	T		F	T	**T**	T
T	**T**	F		T	F	**T**	T
F	**T**	T		F	T	**T**	F
F	**F**	F		T	F	**F**	F

These two statements are *logically equivalent*: Two or more statements with identical truth values in every row of the truth table.
This concept will help us understand some left-over issues about translation.

Consider ‘A ∨ ~B’ and ‘B · ~A’.

A	∨	~	B		B	·	~	A
T	**T**	F	T		T	**F**	F	T
T	**T**	T	F		F	**F**	F	T
F	**F**	F	T		T	**T**	T	F
F	**T**	T	F		F	**F**	T	F

These statements form a *contradiction*: Two statements with opposite truth values in all rows of the truth table.
Note that the biconditional connecting the two statements of a contradiction is self-contradictory.
‘P · ~P’ is a simple contradiction, with common use.
In English: “It’s raining. It’s not raining.”
A person who makes both statements together has to be wrong about at least one of them.

Consider ‘E ⊃ D’ and ‘~E · D’.

E	⊃	D		~	E	·	D
T	**T**	T		F	T	**F**	T
T	**F**	F		F	T	**F**	F
F	**T**	T		T	F	**T**	T
F	**F**	F		T	F	**F**	F

These statements are neither contradictory (see rows 2, 3, and 4) nor logically equivalent (see row 1).
But a person who makes both statements can be making true statements. (See row 3).
It depends on what the substitutions are (for E and D).

If two statements are neither logically equivalent nor contradictory, they may be consistent or inconsistent.
Consistent: Can be true together, for at least one valuation (one row of the table).
Inconsistent: Not consistent. I.e. there is no row of the truth table in which both statements are true.
An inconsistent pair: 'E · F' and '~(E ⊃ F)'

E	·	F		~	(E	⊃	F)
T	**T**	T		**F**	T	T	T
T	**F**	F		**T**	T	F	F
F	**F**	T		**F**	F	T	T
F	**F**	F		**F**	F	T	F

Note that the conjunction of two inconsistent statements is a self-contradiction.
When comparing two propositions, first look for the stronger conditions: logical equivalence and contradiction.
Then, if these fail, look for the weaker conditions: consistency and inconsistency.

IV. **Exercises B**. Are the statements logically equivalent or contradictory? If neither, are they consistent or inconsistent?

1. A ⊃ ~B ~(B · A)
2. A · ~B B · ~A
3. B · A A ⊃ ~B
4. A ≡ B ~(A ∨ B)
5. A ∨ (B · D) ~A · ~(B ∨ ~D)

V. **The Biconditional**

The biconditional is superfluous, since it is logically equivalent to a statement which uses only other connectives: '(P ⊃ Q) · (Q ⊃ P)'

P	≡	Q		(P	⊃	Q)	·	(Q	⊃	P)
T	**T**	T		T	T	T	**T**	T	T	T
T	**F**	F		T	F	F	**F**	F	T	T
F	**F**	T		F	T	T	**F**	T	F	F
F	**T**	F		F	T	F	**T**	F	T	F

Other connectives can be shown to be superfluous, in similar ways.
We will look at this topic in depth on October 31.

VI. **Solutions**

Answers to Exercises A

1. Tautologous
2. Contingent
3. Contradictory

Answers to Exercises B

1. Logically equivalent
2. Inconsistent
3. Contradictory
4. Consistent
5. Inconsistent