Philosophy 240: Symbolic Logic Hamilton College
Fall 2008 Russell Marcus
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Class 35: Relational Predicates, Translation II (§8.6)

I. **Introducing Relational Predicates**

Consider the argument:
Bob is taller than Charles. Andrew is taller than Bob. For any x, y and z, If x is taller than y and y is taller than z, then x is taller than z. So, Andrew is taller than Charles.

The conclusion should follow logically, but how do we translate the predicates?
If we only have monadic (1-place) predicates, like the ones we have so far considered, we have to translate the two first sentences with two different predicates:
Bob is taller than Charles: Tb
Andrew is taller than Bob: Ya

We really want a predicate that takes two objects. This is called a dyadic predicate. For examples:
Txy: x is taller than y
Kxy: x knows y
Bxy: x believes y
Dxy: x does y

We can have three-place predicates too, called triadic predicates:
Gxyz: x gives y to z
Kxyz: x kisses y in z
Bxyz: x is between y and z

Also, we can have four-place and higher level predicates. All predicates which take more than one object are called relational, or polyadic.

II. **Exercises A**. Translate each sentence into predicate logic.

1. John loves Mary
2. Tokyo isn't smaller than New York.
3. Marco was introduced to Erika by Paco.
4. America took California from Mexico.

III. **Quantifiers with relational predicates**

Consider again the original argument.
We can now translate the first two premises:

Bob is taller than Charles: Tbc
Andrew is taller than Bob: Tab

But what about the general statement?
We need to put quantifiers on the relations.
The following four sentences use ‘Bxy’ for ‘x is bigger than y’.

Joe is bigger than some thing : $(\exists x)Bjx$
Something is bigger than Joe: $(\exists x)Bxj$
Joe is bigger than everything: $(x)Bjx$
Everything is bigger than Joe: $(x)Bxj$

We can dispense with constants altogether, introducing overlapping quantifiers.
Consider: ‘Everything loves something’, using ‘Lxy’ for ‘x loves y’: $(x)(\exists y)Lxy$
Note the different quantifier letters: overlapping quantifiers must use different variables.
Also, the order of quantifiers matters
‘$(\exists x)(y)Lxy$’ means that something loves everything, which is different.

Here are some more complex examples:

1. Something taught Plato. (Txy: x taught y)
   $(\exists x)Txp$
2. Someone taught Plato.
   $(\exists x)(Px \cdot Txp)$
3. Plato taught everyone.
   $(x)(Px \supset Tpx)$
4. Everyone knows something. (Kxy: x knows y)
   $(x)[Px \supset (\exists y)Kxy]$
5. Everyone is wiser than someone. (Wxy: x is wiser than y)
   $(x)[Px \supset (\exists y)(Py \cdot Wxy)]$
6. Someone is wiser than everyone.
   $(\exists x)[Px \cdot (y)(Py \supset Wxy)]$
7. Some financier is richer than everyone. (Fx, Rxy: x is richer than y)
   $(\exists x)[Fx \cdot (y)(Py \supset Rxy)]$
8. No deity is weaker than some human. (Dx, Hx, Wxy: x is weaker than y)
   $\sim(\exists x)[Dx \cdot (\exists y)(Hy \cdot Wxy)]$ or $(x)[Dx \supset (y)(Hy \supset \sim Wxy)]$
9. Honest candidates are always defeated by dishonest candidates. (Hx, Cx, Dxy: x defeats y)
   $(x)\{(Cx \cdot Hx) \supset (\exists y)[(Cy \cdot \sim Hx) \cdot Dyx]\}$
10. No mouse is mightier than himself. (Mx, Mxy: x is mightier than y)
    $(x)(Mx \supset \sim Mxx)$
11. Everyone buys something from some store. (Px, Sx, Bxyz: x buys y from z)
    $(x)[Px \supset (\exists y)(\exists z)(Sz \cdot Bxyz)]$
12. There is a store from which everyone buys something.
    $(\exists x)\{Sx \cdot (y)[Py \supset (\exists z)Byzx]\}$
13. No store has everyone for a customer.
    $\sim(\exists x)\{Sx \cdot (y)[Py \supset (\exists z)Byzx]\}$ or $(x)\{Sx \supset (\exists y)[Py \cdot (z)\sim Byzx]\}$

IV. **Solutions**

Answers to Exercises A:
1. Ljm
2. ~Stn
3. Ipme
4. Tcam