Philosophy 240: Symbolic Logic Hamilton College
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Class 33: Quine and Ontological Commitment
Fisher 59-69

Re HW: Don't copy from key, please!

Quine and Quantification

I. The riddle of non-being

Two basic philosophical questions are:

Q1. What exists?
Q2. How do we know?

The first question starts us on the road to metaphysics.
Are there minds? Are there laws of nature?
The objects on our list of what we think exists are called our ontology, or our ontological commitments.
The second question starts us on the road to epistemology.

Some things obviously exist: trees and houses and people.
Others are debatable: numbers, souls, quarks, James Brown.
In his article "On What There Is" (OWTI), Quine worries about Pegasus.
Consider the sentence:

A: There is no such thing as Pegasus.

Part of Quine's worry is semantic: How can I state N, or any equivalent, without committing myself to the existence of Pegasus?
If we analyze this sentence the way that we have been analyzing sentences in predicate logic, it might seem that A says that there is some thing, Pegasus, that lacks the property of existence.
I can not say something about nothing!
So, if Pegasus does not exist, then it seems a bit puzzling how I can deny that it exists.
I am talking about a particular thing, so it has to have some sort of being.

One option, which Quine ascribes to an imaginary philosopher McX, appeals to the idea of Pegasus as the referent of my term.
'Pegasus' refers to my idea; A claims that the idea is not instantiated.
McX's solution, as Quine points out, demonstrates a basic confusion of ideas and objects.
'Benedict is a warm building' refers to an object, not an idea.
'Pegasus is a winged horse' seems to have the same structure.
Why would it refer to an idea, rather than an object?
"McX would sooner be deceived by the crudest and most flagrant counterfeit than grant the nonbeing of Pegasus" (2)!

Another option, which Quine ascribes to the imaginary Wyman, who represents early Russell or Meinong, distinguishes between existence and subsistence.

Only some names refer to existent objects.
All names of possible objects refer to subsistent objects.
"Wyman, by the way, is one of those philosophers who have united in ruining the good old word 'exist'" (3)!
There might also be impossible objects, like a round square cupola.
Wyman claims that terms for impossible objects are meaningless.
Quine: "Certainly the doctrine has no intrinsic appeal..." (5)
Note that if we take 'round square' to be meaningless, even though 'round' and 'square' are meaningful, we have to abandon the compositionality of meaning, that the meanings of longer strings of our language are built out of the meanings of their component parts.
Quine's main argument against Wyman, though, consists of his positive account of how to deal with names which lack referents, and how to deal with debates about existence claims, generally.

Answers to Q1 are tied to answers to Q2.
If I claim that electrons exist, I should be able to demonstrate how I discovered them, or how I posited them, or how their existence was revealed to me.
If you deny my claim that the tooth fairy exists, you will appeal the fact that we never see such a thing, for example.
To resolve disputes about what exists, we should have a method to determine what exists.
At least, we should agree on a way to debate what exists.

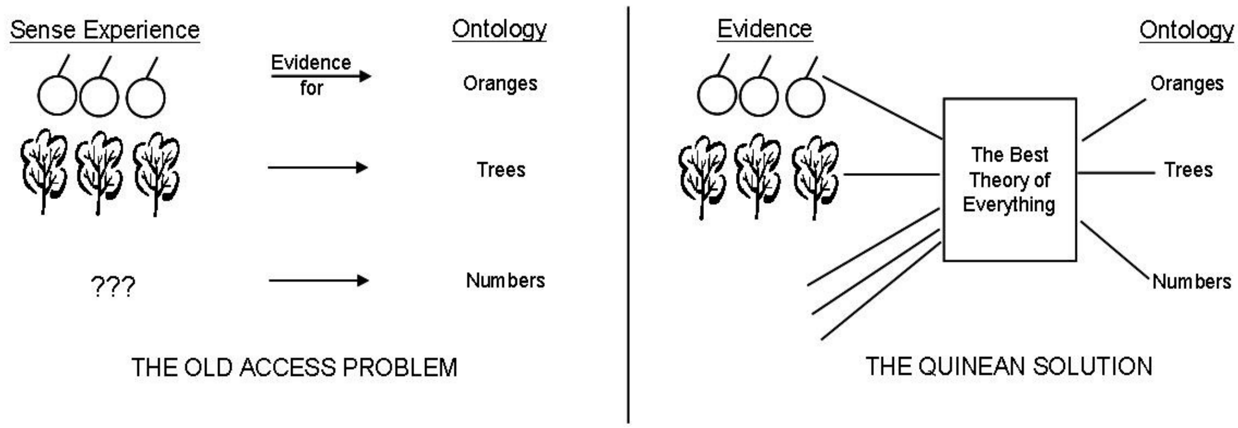
II. Quine's method

One method for determining what we think exists, that favored by Locke and Hume and Quine's mentor Rudolf Carnap, relies on sense experience.
For these philosophers, all claims about what exists must be derived from some kind of sense experience.
These empiricists had difficulty explaining our knowledge of numbers and atoms, for example.
Another method, favored by Descartes and the great logician Kurt Gödel, relies on human reasoning as well as sense experience.
The rationalists have an account of numbers, but are often accused of mysticism.
A seemingly magical ability to know something independently of experience can be used to try to justify beliefs in ghosts and spirits, as well as numbers and electrons.

Quine's method uses the tools of first-order logic.
"To be is to be the value of a variable (15)".
I will attempt to answer two questions about Quine's method.
First, what variables are relevant to the question of what exists?
Second, what does it mean to be a value of a variable?

The answer to the first question is fairly straightforward.
Quine is concerned with the best theories for explaining our sense experience.
Quine is thus much like his empiricist predecessors in narrowing his focus on sense experience.
But, he is unlike traditional empiricists in that he does not reduce all claims of existence directly to sense experiences.

Instead, Quine constructs a theory of our sense experience.
Then, he looks at the theory, and decides what it presupposes, or what it posits.
Our best ontology will be derived from our best theory.



There may be competing best theories.
Thus, at the end of OWTI, Quine seems agnostic about whether to commit to phenomenalism or physicalism.
Should we commit only to the experiences we have, or to the physical world which we ordinarily think causes our experience?
But, the best theory will have to have some relation to the best science we can muster.

Still, there are questions about how to read a theory.
The question of how to formulate and read a theory is a main point of dispute between McX and Wyman.
Quine urges that the least controversial and most effective way of formulating a theory is to put it in the language of first-order logic.
He motivates his appeal to first-order logic with a discussion of Russell's theory of definite descriptions.
We will look at Russell's theory in greater depth in §8.7.
Consider, 'The King of America is bald'.
If we regiment 'the king of America' as a name, a constant, then we are led to the following paradox:

P: ~Bk • ~ ~Bk

We assert '~Bk' because the sentence 'the king of America is bald' seems false.
We assert '~ ~Bk' because '~Bk' seems to entail that the king of America has hair, and that claim must be false, too.
If we regiment the sentence as a definite description, the paradox disappears.
'The king of America is bald' becomes: (∃x)[Kx • (y)(Ky ⊃ y=x) • Bx]
'The king of America is not bald' becomes: (∃x)[Kx • (y)(Ky ⊃ y=x) • ~Bx]
Conjoining their negations, as we did in P, leads to no paradox.
You can derive the non-existence of a unique king of America, though, which is a desired result.

As Quine notes, we have to turn 'Pegasus' into a definite description in order to use Russell's technique on it.
Quine mentions the equivalence of 'Pegasus' and 'the winged horse captured by Bellerophon' in both

OWTI and ‘Designation and Existence’ (DE).
Only in OWTI, though, does he introduce the predicate ‘pegasizes’.
We can regiment ‘Pegasus does not exist’ as ‘~(∃x)Px’.
To this point, all we have done is write the awkward claim in first-order logic.
Quine further thinks that we have solved a problem, that we no longer have any temptation to think that there is a Pegasus in order to claim ‘~(∃x)Px’.
“The singular noun in question can always be expanded into a singular description, trivially or otherwise, and then analyzed out *á la* Russell” (8).
That is, Quine claims that a name can be meaningful, even if it has no bearer.
The distinction between meaning, or sense, and reference derives, as Quine notes, from Frege.
Frege used the example of the morning star (classically known as ‘Phosphorus’) and the evening star (‘Hesperus’) which both turned out to be the planet Venus.
The two terms referred to the same thing, despite having different meanings.
Compare: ‘Clark Kent’ and ‘Superman’.

To defend his claim that we can have meaningful terms without referents, that we can use terms like ‘Pegasus’ without committing to the existence of something named by ‘Pegasus’, Quine appeals to his method of determining our commitments by looking at interpretations of first-order logic.
Reading the commitments of first-order logic is fairly straightforward, if a bit technical.
Discussion of the details of the case will take us into a bit of technical machinery.

III. Formal systems, an introduction to metalogic

A formal theory is a set of sentences of a formal language.
A formal language may be identified with its wffs.
Similarly, English may be identified with its sentences.
To establish a formal language, we start with specifying the syntax of that language, its alphabet and some formation rules.
In propositional logic, our alphabet is:

Language PL_H (Hurley’s propositional logic)
a. Capital English letters
b. Punctuation marks
c. ~
d. •, ∨, ⊃, ≡

Aside:
There is a question in the philosophy of language whether the sentences of a language, or its words, are primary.
Quine argues, elsewhere, that we start with sentences.
We might think that a language is identified with its words if we identify language with dictionaries.
Quine believes that words are derived out of sentences by breaking up wholes into parts.
We need not concern ourselves with this question here.
The central role of logic is to deal with assertions, so we will take them as primary.

We specified formation rules at the beginning of the course:

Formation rules for wffs of PL_H

1. A single capital English letter is a wff.
2. If α is a wff, so is '~α'.
3. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
   By convention, you may drop the outermost brackets.
4. These are the only ways to make wffs.

In predicate, or quantificational, logic, we add to the alphabet.

Language QL_H (Hurley's quantificational logic)

a-d of PL_H
e. lower-case english letters: variables are x, y, and z; the rest are constants
f. ∃
g. =

The formation rules for predicate logic are a bit more complicated.
The following rules approximate the rules in Hurley.

Formation rules for wffs of QL_H

1. A single capital English letter followed by one or more lower-case letters is a wff
2. If α is a wff, then '(x)α', '(y)α', '(z)α' are wffs
3. If α is a wff, then '(∃x)α', '(∃y)α', '(∃z)α' are wffs
4. If α is a wff, so is '~α'.
5. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
6. If μ and ν are variables or constants, then 'μ=ν' is a wff

Sometimes, a first-order theory will explicitly include functions, which are specific kinds of relations.
The addition of functions would be important once we tried to extend the theory, and include mathematical axioms.

Once we have specified the wffs of a language, we can do two things:

1. Proof theory, which specifies a deductive apparatus for the language.
In a proof theory, we specify axioms and rules of transformation.
Hurley's book uses a system called natural deduction, which means that he does not take any axioms.
Instead, he loads up on rules of inference.
This is the ordinary route taken in contemporary introductory logic courses.
Systems of natural deduction seem to mirror ordinary reasoning, since the rules of inference are often

intuitive.
Also, natural deduction systems make proofs shorter than they would be in axiomatic systems of logic.
Natural deduction systems have one main drawback: metalogical reasoning about them is more complicated.
When we start to reason about the system of logic we have chosen, we ordinarily choose a more austere system.
If we want to show that a system of natural deduction is legitimate, we can show that it is equivalent to a more austere system.
Once we have specified logical axioms and rules of inference, we have turned our language into a formal system, or a theory of logic.

Here is an example of an axiomatic system, I'll call PS_R in the language of propositional logic:

Formal system PS_R

Language and wffs: those of PL_H[1]

Axioms:

For any wffs α, β, and γ

Axiom 1: $\alpha \supset (\beta \supset \alpha)$

Ax. 2: $(\alpha \supset (\beta \supset \gamma)) \supset ((\alpha \supset \beta) \supset (\alpha \supset \gamma))$

Ax. 3: $(\sim\alpha \supset \sim\beta) \supset (\beta \supset \alpha)$

Rule of inference:

Modus ponens

PS_R and Hurley's system of natural deduction are provably equivalent, since they are both complete.[2]
Completeness, for the logician, means approximately that all the true wffs are provable.
Both systems are also sound, which means approximately that everything we can prove is also true.
Intuitively, we know what truth is.
But, we need to specify what we mean by 'true' for a formal system.
To do so, we engage in model theory.

2. Model theory specifies an interpretation of the language.
Until this point, we have not considered the meanings of any of the objects of our system.
We have not even considered the meanings of the operators.
The system PS_R is completely uninterpreted, just a system of manipulating formal symbols.
It is an empty game.

An interpretation of the language assign meanings to the various particles.
We use the truth tables to instruct us how to combine terms.
These truth tables provide the semantics for the operators of the language.

To specify an interpretation of the entire language, we also assign T or F to each simple term of the language.
For propositional logic, defining an interpretation is simple.

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[1] We do not need to use any of the wffs which use ∨, •, and ≡.

[2] I am assuming completeness and soundness for Hurley's system.

For Hurley's system, we only have 26 simple terms, the capital English letters.
Thus, there are only 2^{26} possible interpretations.
That is a large number, but it is a finite number.
A more useful system will have infinitely many simple terms.
(We can create infinitely many formulas by allowing formulas like A′, A′′, A′′′, etc.)
A system with infinitely many formulas will have an even greater infinitely many interpretations.

We have not yet defined a system for quantificational logic.
Here is one:

Formal system $QS{=}_R$

Language and wffs: those of QL_H

Axioms:

Ax.1-Ax3 of PS_R

Ax. 4: $(x)\alpha \supset \alpha\ a/x$ (with appropriate restrictions on free variables)

Ax. 5: $\alpha \supset (x)\alpha$, if x is not free in α

Ax. 6: $(x)(\alpha \supset \beta) \supset ((x)\alpha \supset (x)\beta)$

Ax. 7: If α is an axiom, then $(x)\alpha$ is also an axiom

Ax. 8: $(\mu)\mu=\mu$

Ax. 9: $\mu=\nu \supset (\mathscr{F}\mu \supset \mathscr{F}\nu)$, where $\mathscr{F}\mu$ and $\mathscr{F}\nu$ are any formulae containing μ or ν.

Rule of Inference:

Modus ponens

To define an interpretation in predicate logic, we have to specify how to handle quantifications and relations.
This is where Quine's doctrine of ontological commitment comes in.
We call a an interpretation on which all of a set of sentences come out true a model of that set.
A logically valid formula is one which is true on every interpretation.
When Quine says that to be is to be the value of a variable, he means that when we interpret our formal best theory, we need certain objects to model our theories.
Only certain kinds of objects will model the theory.
Any objects which appear in a model of the theory are said, by that theory, to exist.

I mentioned that PS_R and Hurley's system are sound and complete.
We can refine the definitions a bit.
Soundness means that every provable formula is true under all interpretations.
Completeness means that any formula which is true under all interpretations is provable.
The formulas which are true under all interpretations are the tautologies, or logical truths.

If we add non-logical axioms, we create a first-order theory of whatever that axiom concerns.
If we add mathematical axioms, we can create a first-order mathematical theory.
If we add axioms of physics, we can create a first-order physical theory.

We can turn $QS{=}_R$ into a system of set theory by adding one element to the language: $\in$.
We would also add axioms for set theory.
There are different axiomatizations of set theory.
Quine developed a set theory, called New Foundations (NF) in response to set-theoretic paradoxes.

Axioms for NF

NF. 1. $\forall x \forall y[x=y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y)]$

NF. 2. $\exists x \forall y(y \in x \leftrightarrow \Phi)$, where ‘x’ is not free in Φ and Φ is (weakly) stratified.

We could also add the axioms of Peano arithmetic

Peano axioms for number theory, informally:

PA1: 0 is a natural number.
PA2: If a is a natural number then so is a+1.
PA3: If you can prove something about a and that implies that you can prove it for a+1, and if you can prove the very same thing for 0 , then will this hold for all natural numbers.
PA4: If a+1 = b+1 then a=b.
PA5: You can not add 1 to a natural number to get 0.

I spare you physical axioms, like those of Newtonian gravitational theory, or relativity.
The central point is that all of these formal systems regiment the complete theories.
We can add such axioms to different logical systems.
In particular, we can extend predicate logic into second-order logic by allowing quantification over properties.
Quine believes that first-order logic is the canonical language for any theory which we could call our best, for any theory in which we find our real commitments.

Still, we have not talked about how to find those commitments.
This is where things get interesting.

IV. Existence and quantification

Our goal is to interpret predicate logic, $QS{=}_R$.
To interpret a first-order theory, we must use some set theory.
We need not add set theory to our formal language.
We are using it in our metalanguage, the language in which we are doing our model theory.
In contrast, $QS{=}_R$ will be called the object language.

We interpret a first-order theory in four steps.

Step 1. Specify a set to serve as a domain of interpretation, or domain of quantification.
We will specify domains of interpretation in §8.5, in order to show arguments invalid.
A valid argument will have to be valid under any interpretation.
So, consider a universe of three objects: U={1, 2, 3}; or U={Barack Obama, John McCain, and George Bush}

Step 2. Assign a member of the domain to each constant.

Step 3. Assign some set of, or relation on, the objects in the domain to each predicate.
That is, we interpret predicates as sets of objects in the domain, sets of which that predicate holds.
If we use a predicate ‘Ex’ to stand for ‘x has been elected president’, then the interpretation of that predicate will be the set of things that were elected president.

In our examples, the interpretation of 'Ex' might be {Barack Obama, George Bush}.
It might also be {Barack Obama, John McCain}.
Any set can be used as an interpretation, whether true or false.
We can an interpretation on which all statements come out true a model.
The former, though not the latter, interpretation could serve to model a theory which included the predicate 'E'.
A relation is interpreted by an ordered n-tuple.
A two-place predicate is assigned an ordered pair, a three-place predicate is assigned a three-place relation, etc.
So, the relation 'Gxy', which could be understood as meaning 'is greater than' would be modeled, in the universe described above, with {<2,1>, <3,1>, <3, 2>}

Step 4. Use the ordinary truth tables for the interpretation of the connectives.

Ordinarily, in order to determine the truth of sentenes of our formal theory we first define satisfaction, and then truth for an interpretation.
Objects in the domain may satisfy predicates; ordered n-tuples may satisfy relations.
A wff will be true iff there are objects or ordered n-tuples which satisfy it, that is if there are objects in the domain of quantification, which stand in the relations indicated in the wff.

V. Pegasus

So, consider again Quine's original worry about Pegasus.
The problem that embroiled McX and Wyman in systems of idealism and subsistence was that names seemed unavoidably referential.
But, Quine urges us to take names as constituent substituends of variables.
We regiment our best theory.
It will include, or entail, a sentence like:

$NR_{\exists}$: ~(∃x)Px

NR is logically equivalent to:

$NR_{\forall}$: (x)~Px

If we want to know whether this sentence is true, we look inside the domain of quantification.
The domain of quantification is just a set of objects.
If there is no object with the property of being Pegasus, we call this sentence true in the interpretation.
We construct our best theory so that everything in the world is in our domain of quantification, and nothing else is.

VI. Universals

Universals are among the entities whose existence philosophers debate.
In DE, Quine discusses appendicitis.
In OWTI, Quine focuses on redness.
In both cases, the profligate ontologist thinks there are abstract objects in addition to the concrete objects which have their properties.
There is appendicitis in addition to people and their appendixes.
There is redness in addition to fire engines and apples.
McX accepts that there is a distinction between meaning and naming, but points out that meanings are also universals.
Quine insists that just as we can have red fire engines without redness, we can have meaningful statements without meanings.

The issues concerning universals lead directly into Quine's discussion of three schools of philosophy of mathematics: logicism, intuitionism, and formalism.
We can discuss these more, if you wish.

VII. Paper Topics

1. What is the ontological status of abstract objects, like numbers or appendicitis? How can we characterize the debate between nominalists and realists? How does Quine's method facilitate the debate? Discuss the role of contextual definition Quine mentions at the end of DE.

2. Are there universals? What is the relationship between the distinction between singular and general statements and the distinction between abstract and concrete terms. Does that relationship help us understand the problem of unviersals? How does Quine's criterion facilitate the debate? Why does Quine reject meanings, in OWTI, and how does the rejection of meanings relate to the problem of universals?

3. What is the problem of non-existence? Consider the solutions provided by McX and Wyman. How does Quine's approach differ? How does Quine's approach relate to Russell's theory of definite descriptions?

4. What is a name? What is the relationship between naming and quantification? Discuss Quine's dictum, that to be is to be the value of a variable.

Check out the quiz on OWTI:
http://www.jcu.edu/philosophy/gensler/ap/quine-00.htm