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Class 32: Conditional and Indirect Proof, Predicate Versions (§8.4)

I. **A problem arising from using CP and IP in Predicate Logic**

With unrestricted CP we could construct the following derivation:

1. (x)Rx ⊃ (x)Bx	Premise
| 2. Rx	ACP
| 3. (x)Rx	2, UG
| 4. (x)Bx	1, 3, MP
| 5. Bx	4, UI
6. Rx ⊃ Bx	2-5, CP
7. (x)(Rx ⊃ Bx)	6, UG

This would mean that we could prove that everything red is blue (the conclusion) from 'If everything is red, then everything is blue' (the premise).
But that premise can be true while the conclusion is false.
So, the derivation should be invalid.

Moral of the story: we must restrict conditional proof.
The problem is in step 3.
We may not generalize on x within the assumption.
The assumption just means that a random thing is R, not that everything is R.
While variables retain their universal character in a proof, when they are used within an assumption (for CP or IP), they lose that universal character.
It is as if we are saying, "Imagine that some (particular) thing has the property ascribed in the assumption."
If if follows that the thing in the assumption also has other properties, we may generalize after we've discharged, as in line 7.
For, we have not made any specific claims about the thing, outside of the assumption.

The **Restriction on (CP) and (IP)**:

Never UG within an assumption on a variable that is free in the first line of the assumption.

II. **Examples of CP and IP in Predicate Logic**

One of two typical uses of CP:

1. $(x)[Ax \supset (Bx \vee Dx)]$		
2. $(x){\sim}Bx$	$/(x)(Ax \supset Dx)$	
	3. Ay	ACP	Pick a random object that has property A.
	4. $Ay \supset (By \vee Dy)$	1, UI	
	5. $By \vee Dy$	4, 3, MP	
	6. ${\sim}By$	2, UI	
	7. Dy	5, 6, DS	
8. $Ay \supset Dy$	3-7, CP	Given any object, if it has A, it provably has D.
9. $(x)(Ax \supset Dx)$	8, UG	Since we are no longer within the scope of the assumption, we may UG.

QED

So, to prove statements of the form $(x)(Px \supset Qx)$:

Assume Px.
Derive Qx.
Discharge $(Px \supset Qx)$.
Then use UG.

Another typical use of CP:

1. $(x)[Px \supset (Qx \cdot Rx)]$	
2. $(x)(Rx \supset Sx)$	$/ (\exists x)Px \supset (\exists x)Sx$
	3. $(\exists x)Px$	ACP
	4. Pa	3, EI
	5. $Pa \supset (Qa \cdot Ra)$	1, UI
	6. $Qa \cdot Ra$	5, 4, MP
	7. Ra	6, Com, Simp
	8. $Ra \supset Sa$	2, UI
	9. Sa	8, 7, MP
	10. $(\exists x) Sx$	9, EG
11. $(\exists x)Px \supset (\exists x)Sx$	3-10, CP

QED

Indirect Proof works basically in the same way as in propositional logic.
But the same restriction on CP holds for IP.

Typical use of IP:

1. (x)[(Ax ∨ Bx) ⊃ Ex]		
2. (x)[(Ex ∨ Dx) ⊃ ~Ax]	/(x)~Ax	
│3. ~(x)~Ax	AIP	Remember, you're looking for a contradiction.
│4. (∃x)Ax	3, CQ	
│5. Aa	4, EI	
│6. (Ea ∨ Da) ⊃ ~Aa	2, UI	
│7. ~(Ea ∨ Da)	6, 5, DN, MT	
│8. ~Ea · ~Da	7, DM	
│9. ~Ea	8, Simp	
│10. (Aa ∨ Ba) ⊃ Ea	1, UI	
│11. ~(Aa ∨ Ba)	10, 9, MT	
│12. ~Aa · ~Ba	11, DM	
│13. ~Aa	12, Simp	
│14. Aa · ~Aa	5, 13, Conj	
15. (x)~Ax	3-13, IP, DN	

QED

Note that with CP, sometimes you only assume part of a line, then generalize outside the assumption, but with IP, you almost always assume the negation of the whole conclusion.

III. **Exercises**. Derive the conclusions of the following arguments:

1.
    1. (x)(Fx ⊃ Gx)
    2. (x)(Fx ⊃ Hx) / (x)[Fx ⊃ (Gx · Hx)]

2.
    1. (x)(Jx ⊃ ~Kx) / ~(∃x)(Jx · Kx)

3.
    1. (x)(Rx ⊃ Bx) / (x)Rx ⊃ (x)Bx

4.
    1. (x)(Lx ⊃ Mx)
    2. ~(∃x)Lx ⊃ (∃x)Mx / ~(x)~Mx

Solutions may vary.

For Friday, check out Quine's 'On What There Is', as well as the Fisher, if you have time.