Class 3: Translation and Wffs (§6.1, §6.2)

I. **Translation, ambiguous cases**
Consider: 'You may have salad or potatoes and carrots.'
Do we translate this as '(S ∨ P) ∨ C'?
Or as 'S ∨ (P ∨ C)'?
Look to commas and semicolons, and translate accordingly, using parentheses:
You may have salad, or potatoes and carrots: S ∨ (P ∨ C)
You may have salad or potatoes, and carrots: (S ∨ P) ∨ C
Commas are almost always located at the main connective.

II. **Wffs and Main Connectives**
A wff is a 'well-formed formula' and is pronounced "woof", as if you are barking.
Compare:'baker' and 'aebkr'.
One is a word and the other isn't.
We call statements of logic which are constructed properly 'wffs'.
Similarly, these are wffs:
P · Q
(~P v Q) ⊃ ~R
These are not wffs:
·P Q
PqvR~

Formation rules for wffs
1. A single capital English letter is a wff.
2. If α is a wff, so is ~α.
3. If α and β are wffs, then so are:
(α · β)
(α ∨ β)
(α ⊃ β)
(α ≡ β)
By convention, you may drop the outermost brackets.
4. These are the only ways to make wffs.

Main connectives
The last connective added according to the formation rules is called the main connective.

Analyze: (~M ⊃ P) · (~N ⊃ Q)
'M', 'P', 'N', and 'Q' are all wffs, by rule 1.
'~M' and '~N' are wffs by rule 2.
'(~M ⊃ P)' and '(~N ⊃ Q)' are then wffs by rule 3.
Finally, the whole formula is a wff also by rule 3, and the convention of dropping the outermost brackets.

III. **Exercises A**. Are the following formulas wffs? If so, find the main connective.
1. $(P \vee Q) \supset \sim R$
2. $\sim X(Y \vee Z)$
3. $(S \vee T \cdot U) \supset S$
4. $\sim(G \supset H)$
5. $\sim\{(P \supset Q) \cdot [P \equiv \sim(Q \vee R)]\}$
6. $\sim[A \cdot (B \vee C)] \equiv [(A \cdot B) \vee (A \cdot C)]$
7. $[(D \cdot E) \vee F] \cdot G$

IV. **Exercises B**. Translate these into propositional logic, using obvious letters:
1. Ford introduces a new model and either Chrysler raises prices or General Motors changes colors.
2. Both Toyota does not open a new plant and Ford does not introduce a new model.
3. Honda initiates an ad campaign if and only if Chrysler raises prices.
4. Either Saab increases salaries and Toyota opens a new plant or Honda initiates an ad campaign and General Motors changes colors.
5. Toyota's opening a new plant is a necessary condition for General Motors' changing colors, and Ford's introducing a new model is a sufficient condition for Chrysler's raising prices.
6. If Saab increases salaries, then if Toyota opens a new plant, then Honda initiates an ad campaign.
7. Audi lays off workers; however, if Chrysler raises prices then either General Motors does not change colors or Ford does not introduce a new model.

V. Translation from logic to english

Use the following key:
A: Bob owns an Audi
B: Bob owns a BMW
C: Bob owns a car
D: Bob drives
E: Ethel owns a BMW
F: Fred owns a BMW

Translate together

$B \cdot \sim(E \vee F)$	Bob owns a BMW, but neither Fred nor Ethel do.
$D \equiv C$	Bob owns a car just in case he drives

VI. **Exercises C**. Using the above key, translate each of the following sentences into English.
1. $C \supset (A \vee B)$
2. $E \cdot \sim F$
3. $\sim A \supset (\sim D \vee B)$
4. $\sim (A \vee B) \supset \sim C$
5. $\sim(A \cdot B) \cdot C$
6. $(F \cdot E) \equiv \sim B$

VII. **Solutions**

Answers to Exercises A

1. Yes, $\supset$
2. No
3. No
4. Yes, ~
5. Yes, ~
6. Yes, $\equiv$
7. Yes, $\cdot$

Answers to Exercises B

1. $F \cdot (C \vee G)$
2. $\sim T \cdot \sim F$
3. $H \equiv C$
4. $(S \cdot T) \vee (H \cdot G)$
5. $(G \supset T) \cdot (F \supset C)$
6. $S \supset (T \supset H)$
7. $A \cdot [C \supset (\sim G \vee \sim F)]$

Answers to Exercises C

1. If Bob owns a car, then it's either an Audi or a BMW
2. Ethel owns a BMW, but Fred doesn't
3. If Bob doesn't own an Audi, then either he doesn't drive, or he owns a BMW
4. If Bob owns neither an Audi nor a BMW, then he doesn't own a car.
5. Bob doesn't own both and Audi an a BMW, but he owns a car.
6. Fred and Ethel own BMW's if, and only if, Bob doesn't.

Note: alternate formulations are possible.