Philosophy 240: Symbolic Logic Hamilton College
Fall 2008 Russell Marcus
Mondays, Wednesdays, Fridays: 9am - 9:50am rmarcus1@hamilton.edu

Class 29: Quantifier Introduction and Elimination II (§8.2)

I. **More complex derivations**

Sometimes terms can be unbound:

1.	(x)(Dx · Ex)	
2.	(x)Dx ⊃ Fa	/ (∃x)Fx
3.	Dx · Ex	1, UI
4.	Dx	3, Simp
5.	(x)Dx	4, UG
6.	Fa	2, 5, MP
7.	(∃x)Fx	6, EG

QED

Two quantifiers on a line:

1.	(x)(Jx ∨ Kx) ⊃ (∃y)Ly	
2.	(x)(Jx ∨ Lx)	
3.	(x)(~Lx ∨ Kx)	/ (∃y)Ly
4.	Jx ∨ Lx	2, UI
5.	~Jx ⊃ Lx	4, DN, Impl
6.	~Lx ∨ Kx	3, UI
7.	Lx ⊃ Kx	6, Impl
8.	~Jx ⊃ Kx	5, 7, HS
9.	Jx ∨ Kx	8, Impl, DN
10.	(x)(Jx ∨ Kx)	9, UG
11.	(∃y)Ly	1, 10, MP

QED

Instantiating twice from the same statement:

1.	(x)(Mx ⊃ Nx)	
2.	(x)(Nx ⊃ Ox)	
3.	Ma · Mb	/ Na · Ob
4.	Ma ⊃ Na	1, UI
5.	Ma	3, Simp
6.	Na	4, 5, MP
7.	Mb ⊃ Nb	1, UI
8.	Mb	3, Com, Simp
9.	Nb	7, 8, MP
10.	Nb ⊃ Ob	2, UI
11.	Ob	10, 9, MP
12.	Na · Ob	6, 11, Conj

QED

II. **Exercises B**. Derive the conclusions of each of the following arguments.

1. 1. $(x)(Mx \supset Nx)$
   2. $\sim Na$ / $\sim Ma$

2. 1. $(x)(Ox \supset \sim Px)$
   2. $(\exists x)(Rx \cdot Px)$ / $(\exists x)(Rx \cdot \sim Ox)$

3. 1. $(\exists x)Sx \supset (x)Tx$
   2. $(\exists x)Ux \supset (\exists x)Wx$
   3. $Sb \cdot Ub$ / $(\exists x)(Wx \cdot Tx)$

4. 1. $(\exists x)Gx \supset (y)(\sim Hy \vee Iy)$
   2. Gc
   3. $\sim If$ / $(\exists x)\sim Hx$

5. 1. $(\exists x)Ax \supset (x)(Bx \supset Ex)$
   2. $(\exists x)Dx \supset (\exists x)\sim Ex$
   3. $(\exists x)(Ax \cdot Dx)$ / $(\exists x)\sim Bx$