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Class 26: Translation, Predicate Logic II (§8.1)

I. **Scope and Binding**

Compare:

$(x)(Px \supset Qx)$	Every P is Q
$(x)Px \supset Qx$	If everything is P, then x is Q

The scope of the quantifier is whatever term follows immediately.
If what follows the quantifier is a bracket, then whatever occurs until that bracket is closed is in the scope of the quantifier.
If what follows the quantifier is a negation, then everything negated is in the scope of the quantifier.
Quantifiers bind every instance of their variable in their scope.
So, in the first wff of this section, the 'x' in 'Qx' is bound.
In the second wff, the 'x' in 'Qx' is not bound.
Binding will become extremely important once we get to derivations, especially once we get to relational predicates (§8.6).
Examples of unbound terms

$(x)Px \vee Qx$	'Qx' is not in the scope of the quantifier, so is unbound.
$(\exists x)(Px \vee Qy)$	'Qy' is in the scope of the quantifier, but 'y' is not the quantifier variable, so is unbound.

II. **Many propositions contain more than two predicates**

Examples with more than one predicate in the subject part:

Some wooden desks are uncomfortable:	$(\exists x)[(Wx \cdot Dx) \cdot Ux]$
All wooden desks are uncomfortable:	$(x)[(Wx \cdot Dx) \supset Ux]$

Examples with more than one predicate in the attribute part:

Many applicants are untrained or inexperienced:	$(\exists x)[Ax \cdot (Ux \vee Ix)]$
All applicants are untrained or inexperienced:	$(x)[Ax \supset (\sim Tx \vee \sim Ix)]$

Start by asking whether the sentence is universal or existential.
Then, it is often helpful to think of a sentence in terms of the ordinary rules of subject-predicate grammar.
What are we talking about?
What are we saying about it?
The 'what we are talking about' goes as the antecedent in the universally quantified statement, and as the first conjunct in the existentially quantified statement.
The 'what we are saying about it' goes as the consequent or as the second conjunct.

Examples using 'only':
Note that 'only' inverts the antecedent and consequent.

Only men have been presidents: $(x)(Px \supset Mx)$
That is, if something has been a president, it must have been a man, since all presidents have been men.
Only famous men have been presidents: $(x)[(Px \cdot Mx) \supset Fx]$ or $(x)[(Px \supset (Mx \cdot Fx)]$
Only intelligent students understand Kant: $(x)[(Sx \cdot Kx) \supset Ix]$

III. **Some sentences contain more than one quantifier**

If anything is damaged, then everyone in the house complains: $(\exists x)Dx \supset (x)[(Ix \cdot Px) \supset Cx]$
Either all the gears are broken, or a cylinder is missing: $(x)(Gx \supset Bx) \vee (\exists x)(Cx \cdot Mx)$

IV. **Exercises A**. Translate each sentence into predicate logic.

1. Some jellybeans are tasty. (Jx, Tx)
2. Some black jellybeans are tasty. (Jx, Bx, Tx)
3. No green jellybeans are tasty. (Gx, Jx, Tx)
4. Some politicians are wealthy and educated. (Px, Wx, Ex)
5. All wealthy politicians are electable. (Wx, Px, Ex)
6. If all jellybeans are black then no jellybeans are red. (Jx, Bx, Rx)
7. If everything is physical then there are no ghosts. (Px, Gx)
8. Some one walked the dog, but no one washed the dishes. (Px, Wx, Dx)
9. Everyone can go home only if all the work is done. (Px, Gx, Wx, Dx)

V. **Exercises B**. Translate each sentence into predicate logic.

1. All mice are purple. (Mx, Px)
2. No mice are purple.
3. Some mice are purple.
4. Some mice are not purple.
5. Snakes are reptiles. (Sx, Rx)
6. Snakes are not all poisonous. (Sx, Px)
7. Children are present. (Cx, Px)
8. Executives all have secretaries. (Ex, Sx)
9. Only executives have secretaries.
10. All that glitters is not gold. (Gx, Ax)
11. Nothing in the house escaped destruction. (Hx, Ex)
12. Blessed is he that considers the poor. (Bx, Cx)
13. Some students are intelligent and hard working. (Sx, Ix, Hx)
14. He that hates dissembles with his lips, and lays up deceit within him. (Hx, Dx, Lx)
15. Everything enjoyable is either illegal, immoral, or fattening. (Ex, Lx, Mx, Fx)
16. Some medicines are dangerous if taken in excessive amounts. (Mx, Dx, Tx)
17. Some medicines are dangerous only if taken in excessive amounts.
18. Victorian houses are attractive (Vx, Hx, Ax)
19. Slow children are at play. (Sx, Cx, Px)
20. Any horse that is gentle has been well-trained. (Hx, Gx, Wx)
21. Only well-trained horses are gentle.
22. Only gentle horses have been well-trained.
23. A knowledgeable, inexpensive mechanic is hard to find. (Kx, Ex, Mx, Hx)
24. Dogs and cats chase birds and squirrels. (Dx, Cx, Bx, Sx)
25. If all survivors are women, then some women are fortunate. (Sx, Wx, Fx)
26. Some, but not all, of us got away. (Ux, Gx)
27. If all ripe bananas are yellow, then some yellow things are ripe. (Rx, Bx, Yx)
28. If any employees are lazy and some positions have no future, then some employees will not be successful. (Ex, Lx, Px, Fx, Sx)
29. No coat is waterproof unless it has been specially treated. (Cx, Wx, Sx)
30. A professor is a good lecturer if and only if he is both well-informed and entertaining. (Px, Gx, Wx, Ex)

VI. **Solutions**

Answers to Exercises A:

1) $(\exists x)(Jx \cdot Tx)$
2) $(\exists x)[(Bx \cdot Jx) \cdot Tx)]$
3) $(x)[(Gx \cdot Jx) \supset {\sim}Tx]$ or ${\sim}(\exists x)[(Gx \cdot Jx) \cdot Tx]$
4) $(\exists x)[Px \cdot (Wx \cdot Ex)]$
5) $(x)[(Wx \cdot Px) \supset Ex]$
6) $(x)(Jx \supset Bx) \supset (x)(Jx \supset {\sim}Rx)$ or $(x)(Jx \supset Bx) \supset {\sim}(\exists x)(Jx \cdot Rx)$
7) $(x)Px \supset {\sim}(\exists x)Gx$
8) $(\exists x)(Px \cdot Wx) \cdot {\sim}(\exists x)(Px \cdot Dx)$
9) $(x)(Px \supset Gx) \supset (x)(Wx \supset Dx)$

Answers to Exercises B:

1. $(x)(Mx \supset Px)$
2. $(x)(Mx \supset {\sim}Px)$
3. $(\exists x)(Mx \cdot Px)$
4. $(\exists x)(Mx \cdot {\sim}Px)$
5. $(x)(Sx \supset Rx)$
6. ${\sim}(x)(Sx \supset Px)$ or $(\exists x)(Sx \cdot {\sim}Px)$
7. $(\exists x)(Cx \cdot Px)$
8. $(x)(Ex \supset Sx)$
9. $(x)(Sx \supset Ex)$
10. ${\sim}(x)(Gx \supset Ax)$
11. $(x)(Hx \supset {\sim}Ex)$
12. $(x)(Cx \supset Bx)$
13. $(\exists x)[Sx \cdot (Ix \cdot Hx)]$
14. $(x)[Hx \supset (Dx \cdot Lx)]$
15. $(x)\{Ex \supset [({\sim}Lx \vee {\sim}Mx) \vee Fx]$
16. $(\exists x)[Mx \cdot (Tx \supset Dx)]$
17. $(\exists x)[Mx \cdot (Dx \supset Tx)]$
18. $(x)[(Hx \cdot Vx) \supset Ax]$
19. $(\exists x)[(Cx \cdot Sx) \cdot Px]$
20. $(x)[(Hx \cdot Gx) \supset Wx]$
21. $(x)[(Hx \cdot Gx) \supset Wx]$
22. $(x)[(Hx \cdot Wx) \supset Gx]$
23. $(x)\{[(Kx \cdot {\sim}Ex) \cdot Mx] \supset Hx\}$
24. $(x)[(Dx \vee Cx) \supset (Bx \cdot Sx)]$
25. $(x)(Sx \supset Wx) \supset (\exists x)(Wx \cdot Fx)$
26. $(\exists x)(Ux \cdot Gx) \cdot {\sim}(x)(Ux \supset Gx)$
27. $(x)[(Bx \cdot Rx) \supset Yx] \supset (\exists x)(Yx \cdot Rx)$
28. $[(\exists x)(Ex \cdot Lx) \cdot (\exists x)(Px \cdot {\sim}Fx)] \supset (\exists x)(Ex \cdot {\sim}Sx)$
29. $(x)[Cx \supset ({\sim}Wx \vee Sx)]$ or $(x)[Cx \supset ({\sim}Sx \supset {\sim}Wx)]$ or
    $(x)[(Cx \cdot Wx) \supset Sx]$ or ${\sim}(\exists x)(Cx \cdot Wx \cdot {\sim}Sx)$
30. $(x)\{Px \supset [Gx \equiv (Wx \cdot Ex)]\}$