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Class 25: Translation, Predicate Logic I (§8.1)

I. **Introduction**

Consider the following argument:

> All philosophers are happy.
> Emily is a philosopher.
> So, Emily is happy.

The conclusion follows logically from the premises, but not when regimented into propositional logic:

> P
> Q / R

Propositional Logic is insufficient to derive all logical consequences.
We need a stronger logic, one that explores the logic inside the sentences.
This is called predicate logic.

In Propositional Logic, we have the following elements:
- Terms for statements , simple letters
- Five connectives
- Punctuation (brackets)

In Predicate Logic, we have the following elements:
- Complex Terms for statements, made of objects and predicates
- Quantifiers
- The same five connectives as in propositional logic.
- The same punctuation as in propositional logic.

II. **Objects and Predicates**

We represent objects using lower case letters.
- 'a, b, c,...w' stand for specific objects, and are called constants.
- 'x, y, z' are used as variables.

We represent predicates using capital letters.
These stand for properties of the objects, and are placed in front of the object letters.

> Pa: means object a has property P, and is said "P of a"
> Pe: Emily is a philosopher
> He: Emily is happy

III. **Exercises A**. Translate each sentence into predicate logic.

1. Alice is clever.
2. Bobby works hard.
3. Chuck plays tennis regularly.
4. Dan will see Erika on Tuesday at noon in the gym.

IV. **Quantifiers**

Consider: ‘All philosophers are happy’
The subject of this sentence is not a specific philosopher, no specific object.
Similarly for, ‘Something is made in the USA’
There is no a specific thing to which the sentence refers.
For sentences like these, we use quantifiers.

There are two quantifiers.
1. The existential quantifier: (∃x).
It is used with any of the following expressions:

There exists an x, such that
For some x
There is an x
For at least one x
Something

2. The universal quantifier: (x).
It is used with:

For all x
Everything

Some terms, like ‘anything’, can indicate either quantifier, depending on the context:
In ‘If anything is missing, you’ll be sorry’, we use an existential quantifier.
In ‘Anything goes’, we use a universal quantifier.

Examples of simple translations using quantifiers:

Something is made in the USA: (∃x)Ux
Everything is made in the USA: (x)Ux
Nothing is made in the USA: (x)~Ux or ~(∃x)Ux

Note that all statements with quantifiers and negations can be translated in at least two different ways.

Most sentences get translated as (at least) two predicates:
One is used for the subject of the sentence.
One is used for the attribute of the sentence.
Universals tend to use conditionals (as main connective) to separate the subject from the attribute.
Existentials usually use conjunctions between the subject predicate and the attribute predicate.

More sample translations:

All persons are mortal:	$(x)(Px \supset Mx)$		
Some actors are vain:	$(\exists x)(Ax \cdot Vx)$		
Some gods aren't mortal:	$(\exists x)(Gx \cdot {\sim}Mx)$		
No frogs are people:	$(x)(Fx \supset {\sim}Px)$	or	${\sim}(\exists x)(Fx \cdot Px)$

V. **Exercises B**. Translate each sentence into predicate logic.

1. All roads lead to Rome. (Rx, Lx)
2. Beasts eat their young. (Bx, Ex)
3. Everything worthwhile requires effort. (Wx, Rx)
4. Some jellybeans are black. (Jx, Bx)
5. Some jellybeans are not black.

XI. **Solutions**

Answers to Exercises A:

1. Ca
2. Wb
3. Pc
4. Sd

Answers to Exercises B:

1) $(x)(Rx \supset Lx)$
2) $(x)(Bx \supset Ex)$
3) $(x)(Wx \supset Rx)$
4) $(\exists x)(Jx \cdot Bx)$
5) $(\exists x)(Jx \cdot {\sim}Bx)$