

Class 2: Validity and Translation (§1.4, §6.1, §1.2)

I. Validity and Soundness

Consider the following three arguments. Are they good?

1. All persons are mortal.
Socrates is a person.
 \therefore Socrates is mortal.

This is a good argument.

2. All men are fish.
Joe is a man.
 \therefore Joe is a fish.

The conclusion follows from the premises, but the premises are false.

3. All Toyotas are cars.
I own a car.
 \therefore I own a Toyota.

The conclusion does not follow from the premises.

Note that the last two are bad for different reasons: 3) is invalid, 2) is valid, but unsound.

The validity of an argument depends on its form.

An argument is valid if the conclusion follows logically from the premises.

Certain forms are valid.

Certain forms are invalid

The soundness of a valid argument depends on truth of its premises.

A valid argument is sound if its premises are true.

Only valid arguments can be sound.

The most important sentence of this course:

In deductive logic, if the form of an argument is valid and the premises are all true, then the conclusion must be true.

In invalid arguments, the premises can be true at the same time that the conclusion is false, though all can be true.

Validity is independent of truth.

Validity is related to possibility, while soundness is related to truth.

II. Exercises A. Are the following valid? If so, are they sound?

1. If it snows more than two feet, there will be no classes at Hamilton. It snowed more than two feet last Monday. Therefore, there were no classes at Hamilton.
2. The Mets are a professional baseball team. Professional baseball teams are sports businesses. So, the Mets are a sports business.
3. If police departments improve their effectiveness, crime rates go down. Crime rates have gone down. So, police departments have improved their effectiveness.
4. Since the sun is pink, and made of cheese, it follows that some cheese is pink.
5. Some cars are green. Some cars are Toyotas. So, some cars are green Toyotas.
6. All great singers have strong voices. Celine Dion does not have a strong voice. So Celine Dion is not a great singer.

III. The form of an argument

Consider each of the following arguments:

1. Either McCain will win Ohio or Obama will.
McCain won't win Ohio.
So, Obama will.
2. You will get either rice or beans.
You don't get the rice.
So, you'll have the beans.
3. The square root of two is either rational or irrational.
It's not rational.
So, it's irrational.

They have the same form:

Either p or q
not-p
So, q.

This form is called 'Disjunctive Syllogism'

We'll study it later.

Just as an architect, when building a building, looks only at the essential structures, so a logician looks only at the form of an argument.

'p' and 'q', above, are like variables, standing for statements;
'either p or q' is a compound sentence, made of simple ones

The language of propositional logic uses capital letters to stand for simple, positive propositions. Simple propositions are often of subject-predicate form, but not necessarily. They are the shortest examples of statements; they can not be decomposed further in propositional logic. In predicate logic, we go beneath the surface, at end of term.

IV. Connectives

From simple propositions, we can construct more complex ones of any length using any of five connectives:

Negation: \sim
Conjunction: \cdot
Disjunction: \vee
Material Implication: \supset
Biconditional: \equiv

What follows is a more detailed explication of each of the five connectives.

Negation

Some English indicators of negation: Not, it is not the case that p, p is not true, it is false that p

Examples:

John will take the train: J

John won't take the train: $\sim J$

It's not the case that John will take the train: $\sim J$

John takes the train...not!: $\sim J$

In symbols, all of the following are negations:

$\sim R$

$\sim(P \cdot Q)$

$\sim\{(A \vee B) \supset C\} \cdot \sim D\}$

Conjunction

Some English indicators of conjunction: and, but, also, however, yet, still, moreover, although, nevertheless, both.

Examples:

Angelina walks the dog and Brad cleans the floors: $M \cdot P$

Although Angelina walks the dog, Brad cleans the floors: $M \cdot P$

Bob and Ray are comedians: $B \cdot R$

Carolyn is nice, but Emily is really nice: $C \cdot E$

In symbols, all of the following are conjunctions:

$P \cdot \sim Q$

$(A \supset B) \cdot (B \supset A)$

$(P \vee \sim Q) \cdot \sim[P \equiv (Q \cdot R)]$

Disjunction

Some English indicators of disjunction: or, either, unless

Examples:

Either Paco makes the Website, or Matt does: $P \vee M$

Jared or Rene will go to the party: $J \vee R$

Justin doesn't feed the kids unless Carolyn asks him to: $J \vee C$

In symbols, all of the following are conjunctions:

$\sim P \vee Q$

$(A \supset B) \vee (B \supset A)$

$(P \vee \sim Q) \vee \sim[P \equiv (Q \cdot R)]$

We'll discuss 'unless' in more detail after we are familiar with truth conditions.

The Conditional

Some English indicators of a conditional: if, only if, only when, is a necessary condition for, is a sufficient condition for, implies, entails, provided that, given that, on the condition that, in case.

The conditional is also called 'material implication', or just 'implication'.

In ' $A \supset B$ ', A is called the antecedent, B is called the consequent.

Examples, using 'A' to stand for 'you join me' and 'B' to stand for 'I go to the movies'.

- | | | |
|---|----------------------------|------------------|
| 1. If you join me, then I go to the movies. | 1. If A then B | 1. $A \supset B$ |
| 2. You join me if I go to the movies. | 2. If B then A | 2. $B \supset A$ |
| 3. You join me only if (only when) I go to the movies. | 3. A only if (only when) B | 3. $A \supset B$ |
| 4. Your joining me is a necessary condition for my going. | 4. A is necessary for B | 4. $B \supset A$ |
| 5. Your joining me is a sufficient condition for my going. | 5. A is sufficient for B | 5. $A \supset B$ |
| 6. A necessary condition of your joining me is my going. | 6. B is necessary for A | 6. $A \supset B$ |
| 7. A sufficient condition for your joining me is my going. | 7. B is sufficient for A | 7. $B \supset A$ |
| 8. Your joining me entails (implies) that I go to the movies. | 8. A entails (implies) B | 8. $A \supset B$ |
| 9. You join me given (provided, on the condition) that I go. | 9. A given B | 9. $B \supset A$ |

Note that necessary conditions are consequents, while sufficient conditions are antecedents.

If A is necessary for B, then if B is true, we can infer that A must also be true.

We use the mnemonic 'SUN' to remember this, changing the 'U' to a ' \supset ' we get ' $S \supset N$ '.

In symbols, all of the following are conditionals:

$\sim P \supset Q$

$(A \supset B) \supset (B \supset A)$

$(P \vee \sim Q) \supset \sim[P \equiv (Q \cdot R)]$

The Biconditional

Some English indicators of a biconditional: if and only if, is a necessary and sufficient condition for, just in case.

The biconditional is short for ' $(A \supset B) \cdot (B \supset A)$ ', to which we will return, once we are familiar with truth conditions.

An example:

You'll be successful just in case you work hard and are lucky: $S \equiv L$

In symbols, all of the following are biconditionals:

$$\sim P \equiv Q$$

$$(A \supset B) \equiv (B \supset A)$$

$$(P \vee \sim Q) \equiv \sim [P \equiv (Q \cdot R)]$$

V. **Exercises B.** Translate to Propositional Logic, using obvious letters for the legend:

1. Alvin doesn't like sports.
2. Bert and Ernie are muppets.
3. Claudia wants to surf or snorkel.
4. Dogs bite just in case they are startled.
5. Everyone loves logic, or not.
6. If Flora wants candy, Geronimo will get her some.
7. Harold is generous unless his wife is listening.
8. Toyota opens a new plant only if Honda initiates an ad campaign.

VI. Solutions

Answers to Exercise A.

1. Valid, unsound
2. Valid, sound
3. Invalid
4. Valid, unsound
5. Invalid
6. Valid, unsound

Answers to Exercises B (Note that alternative letters are possible):

1. $\sim A$
2. $B \cdot E$
3. $F \vee L$
4. $B \equiv S$
5. $L \vee \sim L$
6. $F \supset G$
7. $G \vee L$
8. $T \supset H$