

**Philosophy 240: Symbolic Logic**

Fall 2008

Mondays, Wednesdays, Fridays: 9am - 9:50am

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## Class 18: Practice with Proofs (§7.1 - §7.4)

1.     1.  $A \supset B$   
       2.  $(E \vee D) \supset \sim B$      / $A \supset (\sim E \cdot \sim D)$
2.     1.  $\sim(G \cdot H) \supset I$   
       2.  $\sim(I \vee J)$   
       3.  $\sim J \supset F$              / $(F \cdot G) \cdot H$
3.     1.  $P \vee O$   
       2.  $Q \supset \sim O$            / $\sim Q \vee P$
4.     1.  $T \equiv V$   
       2.  $\sim T \vee \sim V$        / $\sim T$
5.     1.  $(K \supset \sim M) \cdot [(L \cdot N) \supset \sim M]$   
       2.  $(K \vee L) \cdot (K \vee N)$      / $\sim M$
6.     1.  $(X \vee Y) \cdot (Z \supset W)$   
       2.  $\sim(Y \cdot W)$            / $\sim(\sim X \cdot Z)$
7.     1.  $X \vee Z$   
       2.  $Z \supset \sim Y$   
       3.  $X \supset Y$                / $\sim X \equiv Z$
8.     1.  $A \supset B$   
       2.  $B \supset \sim A$   
       3.  $(A \vee D) \vee E$   
       4.  $(D \vee E) \supset F$        / $F$
9.     1.  $G \supset (H \supset I)$   
       2.  $I \supset (J \cdot K)$          / $G \supset (H \supset J)$
10.    1.  $(\sim L \vee M) \cdot (L \vee N)$   
       2.  $\sim O \supset \sim N$        / $M \vee O$
11.    1.  $(P \supset Q) \cdot (R \supset S)$    / $(P \cdot R) \supset (Q \cdot S)$
12.    1.  $(T \supset W) \cdot (X \supset Y)$   
       2.  $T \vee X$   
       3.  $(T \supset \sim Y) \cdot (X \supset \sim W)$   
       4.  $(W \cdot \sim Y) \supset Z$   
       5.  $Y \supset (W \vee A)$        / $Z \vee A$

Sample Solutions

1.     1.  $A \supset B$   
        2.  $(E \vee D) \supset \sim B$      / $A \supset (\sim E \cdot \sim D)$   
        3.  $B \supset \sim(E \vee D)$      2, Trans, DN  
        4.  $A \supset \sim(E \vee D)$      1, 3, HS  
        5.  $A \supset (\sim E \cdot \sim D)$    4, DM

QED

2.     1.  $\sim(G \cdot H) \supset I$   
        2.  $\sim(I \vee J)$   
        3.  $\sim J \supset F$              / $(F \cdot G) \cdot H$   
        4.  $\sim I \cdot \sim J$          2, DM  
        5.  $\sim I$                  4, Simp  
        6.  $G \cdot H$                1, 5, MT, DN  
        7.  $\sim J$                  4, Com, Simp  
        8.  $F$                     3, 7, MP  
        9.  $F \cdot (G \cdot H)$        8, 6, Conj  
        10.  $(F \cdot G) \cdot H$        9, Assoc

QED

3.     1.  $P \vee O$   
        2.  $Q \supset \sim O$            / $\sim Q \vee P$   
        3.  $\sim O \supset P$            1, Com, DN, Impl  
        4.  $Q \supset P$              2, 3, HS  
        5.  $\sim Q \supset P$          4, Impl

QED

4.     1.  $T \equiv V$   
        2.  $\sim T \vee \sim V$        / $\sim T$   
        3.  $(T \cdot V) \vee (\sim T \cdot \sim V)$    1, Equiv  
        4.  $\sim(T \cdot V)$            2, DM  
        5.  $\sim T \cdot \sim V$          3, 4, DS  
        6.  $\sim T$                  5, Simp

QED

5.     1.  $(K \supset \sim M) \cdot [(L \cdot N) \supset \sim M]$   
        2.  $(K \vee L) \cdot (K \vee N)$      / $\sim M$   
        3.  $K \vee (L \cdot N)$          2, Dist  
        4.  $\sim M \vee \sim M$          1, 3, CD  
        5.  $\sim M$                  4, Taut

QED

6.    1.  $(X \vee Y) \cdot (Z \supset W)$   
       2.  $\sim(Y \cdot W)$                      $/\sim(\sim X \cdot Z)$   
       3.  $\sim Y \vee \sim W$                     2, DM  
       4.  $(Y \vee X) \cdot (Z \supset W)$             1, Com  
       5.  $(\sim Y \supset X) \cdot (Z \supset W)$         4, DN, Impl  
       6.  $(\sim Y \supset X) \cdot (\sim W \supset \sim Z)$     5, Trans  
       7.  $X \vee \sim Z$                         6, 3, CD  
       8.  $\sim\sim(X \vee \sim Z)$                 7, DN  
       9.  $\sim(\sim X \cdot Z)$                     8, DM, DN

QED

7.    1.  $X \vee Z$   
       2.  $Z \supset \sim Y$   
       3.  $X \supset Y$                              $/\sim X \equiv Z$   
       4.  $\sim Y \supset \sim X$                     3, Trans  
       5.  $Z \supset \sim X$                         2, 4, HS  
       6.  $\sim X \supset Z$                         1, DN, Impl  
       7.  $(\sim X \supset Z) \cdot (Z \supset \sim X)$     6, 5, Conj  
       8.  $\sim X \equiv Z$                         7, Equiv

QED

8.    1.  $A \supset B$   
       2.  $B \supset \sim A$   
       3.  $(A \vee D) \vee E$   
       4.  $(D \vee E) \supset F$                      $/F$   
       5.  $A \supset \sim A$                         1, 2, HS  
       6.  $\sim A \vee \sim A$                     5, Impl  
       7.  $\sim A$                                 6, Taut  
       8.  $A \vee (D \vee E)$                 3, Assoc  
       9.  $\sim A \supset (D \vee E)$                 8, Dn, Impl  
       10.  $D \vee E$                          9, 7, MP  
       11.  $F$                                 4, 10, MP

QED

9.    1.  $G \supset (H \supset I)$   
       2.  $I \supset (J \cdot K)$                      $/G \supset (H \supset J)$   
       3.  $(G \cdot H) \supset I$                     1, Exp  
       4.  $\sim I \vee (J \cdot K)$                 2, Impl  
       5.  $(\sim I \vee J) \cdot (\sim I \vee K)$     4, Dist  
       6.  $\sim I \vee J$                         5, Simp  
       7.  $I \supset J$                             6, Impl  
       8.  $(G \cdot H) \supset J$                 3, 7, HS  
       9.  $G \supset (H \supset J)$                 8, Impl

QED

10. 1.  $(\sim L \vee M) \cdot (L \vee N)$   
 2.  $\sim O \supset \sim N$  /  $M \vee O$   
 3.  $\sim L \vee M$  1, Simp  
 4.  $L \supset M$  3, Impl  
 5.  $\sim M \supset \sim L$  4, Trans  
 6.  $L \vee N$  1, Com, Simp  
 7.  $\sim L \supset N$  6, DN, Impl  
 8.  $\sim M \supset N$  5, 7, HS  
 9.  $N \supset O$  2, Trans  
 10.  $\sim M \supset O$  8, 9, HS  
 11.  $M \vee O$  10, Impl, DN

QED

11. 1.  $(P \supset Q) \cdot (R \supset S)$  /  $(P \cdot R) \supset (Q \cdot S)$   
 2.  $P \supset Q$  1, Simp  
 3.  $\sim P \vee Q$  2, Impl  
 4.  $(\sim P \vee Q) \vee \sim R$  3, Add  
 5.  $\sim P \vee (Q \vee \sim R)$  4, Assoc  
 6.  $R \supset S$  1, Com, Simp  
 7.  $\sim R \vee S$  6, Impl  
 8.  $\sim P \vee (\sim R \vee S)$  7, Add, Com  
 9.  $[\sim P \vee (Q \vee \sim R)] \cdot [\sim P \vee (\sim R \vee S)]$  5, 8, Conj  
 10.  $\sim P \vee [(Q \vee \sim R) \cdot (\sim R \vee S)]$  9, Dist  
 11.  $\sim P \vee [(\sim R \vee Q) \cdot (\sim R \vee S)]$  10, Com  
 12.  $\sim P \vee [\sim R \vee (Q \cdot S)]$  11, Dist  
 13.  $P \supset [R \supset (Q \cdot S)]$  12, Impl, Impl  
 14.  $(P \cdot R) \supset (Q \cdot S)$  13, Exp

QED

12. 1.  $(T \supset W) \cdot (X \supset Y)$   
 2.  $T \vee X$   
 3.  $(T \supset \sim Y) \cdot (X \supset \sim W)$   
 4.  $(W \cdot \sim Y) \supset Z$   
 5.  $Y \supset (W \vee A)$  /  $Z \vee A$   
 6.  $W \vee Y$  1, 2, CD  
 7.  $\sim Y \vee \sim W$  3, 2, CD  
 8.  $\sim W \supset Y$  6, DN, Impl  
 9.  $Y \supset \sim W$  7, Impl  
 10.  $(\sim W \supset Y) \cdot (Y \supset \sim W)$  8, 9, Conj  
 11.  $\sim W \equiv Y$  10, Equiv  
 12.  $(W \cdot \sim Y) \vee (\sim W \cdot Y)$  11, Equiv, DN  
 13.  $\sim Y \vee (W \vee A)$  5, Impl  
 14.  $(\sim Y \vee W) \vee A$  13, Assoc  
 15.  $\sim(\sim Y \vee W) \supset A$  14, Impl, DN  
 16.  $(\sim W \cdot Y) \supset A$  15, DM, DN, Com  
 17.  $[(W \cdot \sim Y) \supset Z] \cdot [(\sim W \cdot Y) \supset A]$  4, 16, Conj  
 18.  $Z \vee A$  17, 12, CD

QED