Philosophy 240: Symbolic Logic
Fall 2008
Mondays, Wednesdays, Fridays: 9am - 9:50am

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Relational Predicates, Translation II Handout

1. Jen reads all books written by Asimov. (Bx: x is a book; Wxy: x writes y; Rxy: x reads y; j: Jen; a: Asimov)

2. Some people read all books written by Asimov.

3. Some people read all books written by some one.

4. Everyone buys something from some store. (Bxyz: x buys y from z)

5. There is a store from which everyone buys something.

6. No store has everyone for a customer.

7. $(x)[Px \supset (y)(Py \supset Lxy)]$
7'. $(x)(y)[(Px \cdot Py) \supset Lxy]$
7''. $(y)(x)[(Px \cdot Py) \supset Lxy]$

8. $(\exists x)[Px \cdot (\exists y)(Py \cdot Lxy)]$
8'. $(\exists x)(\exists y)[(Px \cdot Py) \cdot Lxy]$
8''. $(\exists y)(\exists x)[(Px \cdot Py) \cdot Lxy]$

9. $(x)(\exists y)[(Px \cdot Py) \cdot Lxy]$

9'. $(x)[Px \supset (\exists y)(Py \cdot Lxy)]$

10. $(\exists y)(x)[(Px \cdot Py) \supset Lxy]$

10'. $(\exists x)[Px \cdot (y)(Py \supset Lyx)]$

11. $(\exists x)(y)[(Px \cdot Py) \supset Lxy]$

11'. $(\exists x)[Px \cdot (y)(Px \supset Lxy)]$

12. $(y)(\exists x)[(Px \cdot Py) \cdot Lxy]$

12'. $(x)[Px \supset (\exists y)(Py \bullet Lyx)]$

13. $(x)[(\exists y)Lxy \supset Hx]$

13'. $(x)(\exists y)(Lxy \supset Hx)$

14. $(x)Fx \bullet (x)Gx \;::\; (x)(Fx \bullet Gx)$

15. $(x)Fx \vee (x)Gx \;\vdash\; (x)(Fx \vee Gx)$

16. $(x)(Fx \vee Gx) \;\sim\vdash\; (x)Fx \vee (x)Gx$

17. $(\exists x)(Fx \bullet Gx) \;\vdash\; (\exists x)Fx \bullet (\exists x)Gx$

18. $(\exists x)Fx \bullet (\exists x)Gx \;\; \sim\vdash \;\; (\exists x)(Fx \bullet Gx)$

19. $(x)(Fx \bullet \alpha) \;\; :: \;\; (x)Fx \bullet \alpha$
    e.g. $(x)[Px \bullet (\exists y)Qy] \;\; :: \;\; (x)Px \bullet (\exists y)Qy$

20. $(x)(Fx \bullet \alpha) \;\; :: \;\; (x)Fx \bullet \alpha$
    e.g. $(x)[Px \bullet (\exists y)Qy] \;\; :: \;\; (x)Px \bullet (\exists y)Qy$

21. $(x)(\alpha \supset Fx) \;\; :: \;\; \alpha \supset (x)Fx$
    e.g. $(x)[(\exists y)Py \supset Qx)] \;\; :: \;\; (\exists y)Py \supset (x)Qx$

22. $(\exists x)(\alpha \supset Fx) \;\; :: \;\; \alpha \supset (\exists x)Fx$
    e.g. $(\exists x)[(y)Py \supset Qx)] \;\; :: \;\; (y)Py \supset (\exists x)Qx$

23. $(x)(Fx \supset \alpha) \;\; \vdash \;\; (x)Fx \supset \alpha$
    e.g. $(x)[Px \supset (\exists y)Qy] \;\; \vdash \;\; (x)Px \supset (\exists y)Qy$

24. $(x)Fx \supset \alpha \;\; \sim\vdash \;\; (x)(Fx \supset \alpha)$
    e.g. $(x)Px \supset (\exists y)Qy \;\; \sim\vdash \;\; (x)[Px \supset (\exists y)Qy]$

25. $(x)(Fx \supset \alpha) \;\; :: \;\; (\exists x)Fx \supset \alpha$
    e.g. $(x)[Px \supset (\exists y)Qy] \;\; :: \;\; (\exists x)Px \supset (\exists y)Qy$

If α is true, then both formulas will turn out to be true.
‘$Fx \supset \alpha$’ will be true for every instance of x, since the consequent is true.
So, the universal generalization of each such formula (which is the formula on the left) will be true.
Similarly, the consequent of the formula on the right is just α, so if α is true, the whole formula will be true.
If α is false, then the truth value of each formula will depend.
If the formula on the left turns out to be true, it must be because ‘Fx’ is false, for every x.
But then, ‘$(\exists x)Fx$’ will be false, and so the formula on the right turns out to be true.
If the formula on the right turns out to be true, then it must be because ‘$(\exists x)Fx$’ is false.
And so, there will be no value of ‘x’ that makes ‘Fx’ true, and so the formula on the right will also turn out to be (vacuously) true.

26. $(\exists x)Dx \supset (x)(Px \supset Ux)$

27. $(x)[Dx \supset (y)(Py \supset Uy)]$

28. $(\exists x)Fx \supset \alpha \;\; \vdash \;\; (\exists x)(Fx \supset \alpha)$
    e.g. $(\exists x)Px \supset (\exists y)Qy \;\; \vdash \;\; (\exists x)[Px \supset (\exists y)Qy]$

29. $(\exists x)(Fx \supset \alpha) \;\; \sim\vdash \;\; (\exists x)Fx \supset \alpha$
    e.g. $(\exists x)[Px \supset (\exists y)Qy] \;\; \sim\vdash \;\; (\exists x)Px \supset (\exists y)Qy$

30. (∃x)(ℱx ⊃ α) :: (x)ℱx ⊃ α
e.g. (∃x)[Px ⊃ (∃y)Qy] ⊢ (x)Px ⊃ (∃y)Qy

31. (y)[(x)Fx ⊃ Fy]

32. (y)[Fy ⊃ (∃x)Fx]

33. (∃y)[Fy ⊃ (x)Fx]

34. (∃y)[(∃x)Fx ⊃ Fy]

35. (∃x)[Px • (y)(Qy ⊃ Rxy)]

36. (∃x)(y)[Px • (Qy ⊃ Rxy)]

37. (∃x)(y)[Px ⊃ (Qy ⊃ Rxy)]

37'. (y)[Pa ⊃ (Qy ⊃ Ray)] ∨ (y)[Pb ⊃ (Qy ⊃ Rby)]
37". {[Pa ⊃ (Qa ⊃ Raa)] • [Pa ⊃ (Qb ⊃ Rab)]} ∨ {[Pb ⊃ (Qa ⊃ Rba)] • [Pb ⊃ (Qb ⊃ Rbb)]}

35'. [Pa • (y)(Qy ⊃ Ray)] ∨ [Pb • (y)(Qy ⊃ Rby)]
35". {[Pa • (Qa ⊃ Raa) • (Qb ⊃ Rab)]} ∨ {[Pb • (Qa ⊃ Rba) • (Qb ⊃ Rbb)]}

Exercises. Translate each of the following sentences into predicate logic.

1. Everyone loves something. (Px, Lxy)
2. No one knows everything. (Px, Kxy)
3. No one knows everyone.
4. Every woman is stronger than some man. (Wx, Mx, Sxy: x is stronger than y)
5. No cat is smarter than any horse. (Cx, Hx, Sxy: x is smarter than y)
6. Dead men tell no tales. (Dx, Mx, Tx, Txy: x tells y)
7. There is a city between New York and Washington. (Cx, Bxyz: y is between x and z)
8. Everyone gives something to someone. (Px, Gxyz: y gives x to z)
9. A dead lion is more dangerous than a live dog. (Ax: x is alive, Lx, Dx, Dxy: x is more dangerous than y)
10. A lawyer who pleads his own case has a fool for a client. (Lx, Fx, Pxy: x pleads y's case; Cxy: y is a client of x)

Appendix

35 ⊢ 36

1. (∃x)[Px • (y)(Qy ⊃ Rxy)]	
2. Pa • (y)(Qy ⊃ Ray)	1, EI
| 3. Qy	ACP
| 4. (y)(Qy ⊃ Ray)	2, Com, Simp
| 5. Qy ⊃ Ray	4, UI
| 6. Ray	5, 3, MP
7. Qy ⊃ Ray	3-6, CP
8. Pa	2, Simp
9. Pa • (Qy ⊃ Ray)	8, 7, Conj
10. (y)[Pa • (Qy ⊃ Ray)]	9, UG
11. (∃x)(y)[Px • (Qy ⊃ Rxy)]	10, EG

QED

36 ⊢ 35

1. (∃x)(y)[Px • (Qy ⊃ Rxy)]	
2. (y)[Pa • (Qy ⊃ Ray)]	1, EI
3. Pa • (Qy ⊃ Ray)	2, UI
4. Qy ⊃ Ray	3, Com, Simp
5. (y)(Qy ⊃ Ray)	4, UG
6. Pa	3, Simp
7. Pa • (y)(Qy ⊃ Ray)	6, 5, Conj
8. (∃x)[Px • (y)(Qy ⊃ Rxy)	7, EG

QED

35 ⊢ 37

1. (∃x)[Px • (y)(Qy ⊃ Rxy)]	
| 2. ~(∃x)(y)[Px ⊃ (Qy ⊃ Rxy)]	AIP
| 3. (x)(∃y)~[Px ⊃ (Qy ⊃ Rxy)]	2, CQ
| 4. (x)(∃y)~[~Px ∨ ~Qy ∨ Rxy]	3, Impl, Impl
| 5. (x)(∃y)(Px • Qy • ~Rxy)	4, DM, DN
| 6. Pa • (y)(Qy ⊃ Ray)	1, EI
| 7. (∃y)(Pa • Qy • ~Ray)	5, UI
| 8. Pa • Qb • ~Rab	7, EI
| 9. (y)(Qy ⊃ Ray)	6, Com, Simp
| 10. Qb ⊃ Rab	9, UI
| 11. Qb	8, Com, Simp
| 12. Rab	10, 11, MP
| 13. ~Rab	8, Com, Simp
| 14. Rab • ~Rab	12, 13, Conj
15. (∃x)(y)[Px ⊃ (Qy ⊃ Rxy)]	2-14, IP, DN

QED