Philosophy 240: Symbolic Logic Hamilton College
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Class 23: Handout Modal Logic

A. It is not the case that the sun is shining.
B. It is possible that the sun is shining.
C. It is necessary that this sun is shining.
D. Formation rules for propositional modal logic

1. A single capital English letter is a wff.
2. If α is a wff, so is $\sim\alpha$.
3. If α and β are wffs, then so are $(\alpha \cdot \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$, and $(\alpha \equiv \beta)$.
4. If α is a wff, then so is $\Diamond\alpha$.
5. These are the only ways to make wffs.

E. $\Box\alpha :: \sim\Diamond\sim\alpha$
F. $\Diamond\alpha :: \sim\Box\sim\alpha$

G. Leibniz's argument that this is the best of all possible worlds

1. God is omnipotent so he can create the best possible world.
2. God is omni-benevolent, so he wants to create the best possible world.
3. The world exists.

So, this is the best of all possible worlds.
Corollary: All of the evil in this world is necessary.

H. Actual World semantics

$V(\sim\alpha) = \top$ if $V(\alpha) = \bot$; otherwise $V(\sim\alpha) = \bot$
$V(\alpha \bullet \beta) = \top$ if $V(\alpha) = \top$ and $V(\beta) = \top$; otherwise $V(\alpha \bullet \beta) = \bot$
$V(\alpha \supset \beta) = \top$ if $V(\alpha) = \bot$ or $V(\beta) = \top$; otherwise $V(\alpha \supset \beta) = \bot$

F. Some sample propositions

P: The penguin is on the TV.
Q: The cat is on the mat.
R: The rat is in the hat.
S: The seal is in the sea.

In w_1, we'll take P, Q, R, and S all to be true.

G. Possible World semantics (Leibnizian)

$V(\sim\alpha, w_n) = \top$ if $V(\alpha, w_n) = \bot$; otherwise $V(\sim\alpha, w_n) = \bot$
$V(\alpha \bullet \beta, w_n) = \top$ if $V(\alpha, w_n) = \top$ and $V(\beta, w_n) = \top$; otherwise $V(\alpha \bullet \beta, w_n) = \bot$
$V(\alpha \supset \beta, w_n) = \top$ if $V(\alpha, w_n) = \bot$ or $V(\beta, w_n) = \top$; otherwise $V(\alpha \supset \beta, w_n) = \bot$

$V(\Box\alpha, w_n) = \top$ if $V(\alpha, w_n) = \top$ for all w_n in U
$V(\Box\alpha, w_n) = \bot$ if $V(\alpha, w_n) = \bot$ for any w_n in U
$V(\Diamond\alpha, w_n) = \top$ if $V(\alpha, w_n) = \top$ for any w_n in U
$V(\Diamond\alpha, w_n) = \bot$ if $V(\alpha, w_n) = \bot$ for all w_n in U

H. Translate the following claims. Determine their truth values.

1. $\Diamond P$
2. $\Box q$
3. $\Box(Q \supset P)$
4. $\Diamond P \supset [Q \supset \Box(R \bullet S)]$

I. Three possible worlds

w_1 will be just like w_a, above, except we will assume that these are all the truths at w_1.
At w_2, P and Q are true, but R and S are false.
At w_3, P is true, and Q, R, and S are false.

J. Indexing by world

$P_1 \supset P_3$
$\sim(Q_2 \bullet Q_3)$

K. Two types of questions

Metaphysical
- What is the nature of a possible world?
- Do they exist?
- Are they abstract objects?
- Are they other states of this world, or are they independent of us?

Epistemological
- How do we know about possible worlds?
- Do we stipulate them?
- Do we discover them, or facts about them?
- Do we learn about them by looking at our world?
- Do we learn about them by pure thought?

L. Possible world semantics (Kripkean)

$\mathsf{R} = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_1, w_3\rangle, \langle w_2, w_2\rangle, \langle w_2, w_3\rangle, \langle w_3, w_3\rangle\}$

$\mathsf{V}(\Box\alpha, w_n) = \top$ if $\mathsf{V}(\alpha, w_m) = \top$ for all w_m in U such that $\langle w_n, w_m\rangle$ is in R
$\mathsf{V}(\Box\alpha, w_n) = \bot$ if $\mathsf{V}(\alpha, w_m) = \bot$ for any w_m in U such that $\langle w_n, w_m\rangle$ is in R
$\mathsf{V}(\Diamond\alpha, w_n) = \top$ if $\mathsf{V}(\alpha, w_m) = \top$ for any w_m in U such that $\langle w_n, w_m\rangle$ is in R
$\mathsf{V}(\Diamond\alpha, w_n) = \bot$ if $\mathsf{V}(\alpha, w_m) = \bot$ for all w_m in U such that $\langle w_n, w_m\rangle$ is in R

M. System K

K: $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$
(Nec) α / $\Box\alpha$
(Reg) $\alpha \supset \beta$ / $\Box\alpha \supset \Box\beta$

N. System D, deontic logic

D: $\Box\alpha \supset \Diamond\alpha$
Not provable in D: $\Box\alpha \supset \alpha$

O. Epistemic logic, and S4

Hintkka's epistemic logic takes three axioms:
K: $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$
T: $\Box\alpha \supset \alpha$
4: $\Box\alpha \supset \Box\Box\alpha$
Any logic with the T axiom will have a reflexive accessibility relation.
Any logic with the 4 axiom will also have a transitive accessibility relation.
A system with all three of these axioms is called S4.

P. S5

S5, which takes K, T, 4, and:
B: $\alpha \supset \Box\Diamond\alpha$