

Handout: Three-Valued Logics

I. Motivations for three-valued logics

- A. Every even number greater than four can be written as the sum of two odd primes.
- B. There will be a party tomorrow night on the Dunham Quad.
- C. The king of America is bald.
- D. The king of America is not bald.
- E. The woman on the moon is six feet tall.
- F. The rational square root of three is less than two.
- G. When did you stop beating your wife?
- H. 'H' is false.
- I. 'Yields falsehood when appended to its own quotation' yields falsehood when appended to its own quotation.
- J. 'Heterological' is heterological.
- K. Quadruplicity drinks procrastination. (From Bertrand Russell)
- L. Colorless green ideas sleep furiously. (From Noam Chomsky)
- M. My minivan is a car.
- N. It is a nice day.
- O: 'O' is untrue
- Y: There is a woman on the moon and she is six feet tall.
- Z: 'There is a woman on the moon and she is not six feet tall'

II. Exploring three-valued logics

1. The rules for each;
2. How the new rules affect the logical truths (tautologies); and
3. How the new rules affect the allowable inferences (valid arguments).

III. Some exercises:

Construct truth tables for each of the following propositions, under classical semantics and each of the three three-valued semantics (Bochvar, Kleene, Lukasiewicz). Compare the results. Note: these exercises might form some part of a paper on three-valued semantics.

1. $P \vee \sim P$
2. $P \supset P$
3. $(P \supset Q) \equiv (\sim P \vee Q)$

Note: you can construct the truth table for the biconditional by remembering that ' $P \equiv Q$ ' is logically equivalent to ' $(P \supset Q) \cdot (Q \supset P)$ '