Philosophy 109, Modern Logic, Queens College
Russell Marcus, Instructor
email: philosophy@thatmarcusfamily.org
website: thatmarcusfamily.org/philosophy
Office phone: (718) 997-5287

Truth Functions (§6.2)

I. Introduction

Consider: Either Pacino or DeNiro starred in *Taxi Driver* but Pacino starred in *Taxi Driver* iff Scorcese directed it.
Translated into Propositional Logic: $(P \vee D) \cdot (P \equiv S)$
We know the values of the component propositions P, D, and S
P is False
D is true
S is true
But what's the value of the complex proposition?
The value of a complex proposition is the value of its main connective

II. The basic truth tables

We can derive the truth value of a complex proposition given the truth values of its component propositions using the basic truth tables for each connective.

Negation:
Note that while '2+2=4' is true, it's negation, ' 2+2≠4' is false.
Also, while '2+2=5' is false, it's negation, ' 2+2≠5' is true.
We summarize these results using a truth table.

~	p
F	T
T	F

The column under the 'p' represents all possible assignemnts of truth values to a single proposition.
The column under the '~' represents the values of the negation of that proposition in each row.
A truth table for a complex proposition containing one variable has two lines, since there are only two possible assignments of truth values.

Conjunction:
Consider: 'He likes logic and metaphysics.'
This statement is true if 'He likes logic' is true and 'He likes metaphysics' is true.
It is false otherwise.

p	·	q
T	T	T
T	F	F
F	F	T
F	F	F

Note that we need 4 lines to explore all the possibilities
When both are true (row 1)
When one is true and the other is false (rows 2 and 3)

When both are false (row 4)
With 3 variables, we need 8 lines, and with 4 variables, we need 16 lines.
How many rows would one need for 5 variables? or for n variables?

Disjunction:
Consider: ‘She can get an A in either history or physics’
We use an inclusive disjunction, on which this statement is false only when both component statements are false.

p	∨	q
T	T	T
T	T	F
F	T	T
F	F	F

Material Implication:
Consider when this will statement be falsified: ‘If you paint my house, then I will give you $500.'
It’s true if both the antecedednt and consequent are true.
It’s false if the antecedent is true and the consequent is false.
If the antecedent is false, we consider this statement as unfalsified, and, thus, true.

p	⊃	q
T	T	T
T	F	F
F	T	T
F	T	F

The Material Biconditional
Consider: Supplies rise iff demand falls
This is true if the component statements share the same truth value.
It is false if the components have different values.

p	≡	q
T	T	T
T	F	F
F	F	T
F	T	F

III. Determining the truth value of a complex proposition.
The basic truth tables can be used to evaluate the truth value of any proposition built using the formation rules.
1) Assign truth values to each simple term.
2) Evaluate any negations of those terms.
3) Evaluate any connectives for which both values are known.

4) Repeat 2) and 3) working inside out, until you reach the main operator.

So, consider:
(A ∨ X) · ~B, given that A and B are true and X is false

First, assign the values to A, B, and X

(A	∨	X)	·	~	B
T		F			T

Next, evaluate the negation of B

(A	∨	X)	·	~	B
T		F		F	T

Since you know the values of the disjuncts, you can next evaluate the disjunction:

(A	∨	X)	·	~	B
T	T	F		F	T

Finally, you can evaluate the main connective, the conjunction:

(A	∨	X)	·	~	B
T	T	F	F	F	T

So, the proposition is false.

Returning to the problem from the beginning of the lesson: (P ∨ D) · (P ≡ S)

(P	∨	D)	·	(P	≡	S)
F	T	T	F	F	F	T

Thus, the entire proposition is false.

Consider these further examples:

1. A ⊃ (~X · ~Y), given that A is true and X and Y are false

A	⊃	(~	X	·	~	Y)
T	T	T	F	T	T	F

Thus, the proposition is true

2. [(A · B) ⊃ Y] ⊃ [A ⊃ (C ⊃ Z)], given that A, B, and C are true, and Y and Z are false.

[(A	·	B)	⊃	Y]	⊃	[A	⊃	(C	⊃	Z)]
T	T	T	F	F	T	T	F	T	F	F

Thus, the proposition is true.

IV. Examples A. Assume A, B, C are true and X, Y, Z are false. Evaluate the truth values of each:
1) Z ⊃ ~B
2) (B ≡ C) ⊃ ~A
3) B ⊃ (A ∨ C)
4) X ∨ (A · Y)
5) A ∨ ~A
6) Y ∨ ~Y
7) A · ~A
8) (A ⊃ Z) ∨ (~X ⊃ B)
9) [X · (A ∨ C)] ∨ ~[(X ∨ A) · (X ∨ C)]

V. Examples B. Translate to propositional logic, and use your knowledge of the truth values of the component sentences to determine the truth values of the given complex propositions.

1) Mark Twain wrote *Huckleberry Finn* and Shakespeare wrote *Moby Dick*.
2) If Dickens was not American, then Proust was German.
3) It's not the case that Hemingway wrote both *The Old man and the Sea* and *The Great Gatsby*.
4) Steinbeck wrote *Of Mice and Men* if and only if Robert Frost didn't write 'The Wasteland'.
5) The assertion that neither Dostoevsky wrote both *Crime and Punishment* and *The Brothers Karamozov* nor Tolstoy wrote *War and Peace* is false.

VI. Determining the truth values of complex propositions, when one component is unknown.
Sometimes, you don't know truth values of one or more component variable.
(For the remainder of this section, suppose that P and Q are unkown, and the others are as in Examples A.)

Consider: P · A
If P is true, then we have:
T · T
which is true.
If P is false, then we have
F · T
which is false.
Since the truth value of the compound expression dependes on the truth value of P, it too is unknown.

But consider: P ∨ A
If P is true, then we have
T ∨ T
which is true.
If P is false, then we have
F ∨ T
which is also true.
Since the truth value of the complex proposition is true in both cases, the value of that statement is true.

Similarly, consider: Q · Y
If Q is true, then we have
T · F
which is false.

If Q is false, then we have
F · F
which is also false.
Since the truth value of the complex proposition is false in both cases, the value of that statement is false.

VII. Exercises C. Evaluate the truth value of each complex expression, using the same truth values as above.
1) ~(P · X) ⊃ Y
2) P ⊃ A
3) A ⊃ P
4) Q ∨ ~Z
5) P · ~P
6) Q ∨ ~Q
7) ~P ∨ (~X ∨ P)
8) [(P ⊃ X) ⊃ P] ⊃ P
9) (X ⊃ Q) ⊃ X

VIII. Determining the truth values of complex propositions, when more than one component is unknown.
Lastly, one can have more than one unknown in a statement. If there are two unknowns, we must consider four cases.

Consider: ~(P · Q) ∨ P
If P and Q are both true
~(T · T) ∨ T
which is true.
If P is true and Q is false
~(T · F) ∨ T
which is true.
If P is false and Q is true
~(F · T) ∨ F
which is true.
If P and Q are both false
~(F · F) ∨ F
which is again true.

So the statement is true.

IX. Exercises D. Evaluate the truth value of each complex expression, using the same truth values as above.

1) (P · Q) ∨ (~Q ∨ ~P)
2) (P ∨ Q) · (~B ∨ Y)
3) (P ⊃ Q) ⊃ {[P ⊃ (Q ⊃ A)] ⊃ (P ⊃ A)}

X. Solutions.

Answers to Exercises A

1) T
2) F
3) T
4) F
5) T
6) T
7) F
7) T
8) F

Answers to Exercises B:

1) False
2) False
3) True
4) True
5) True

Answers to Exercises C:

1) False
2) True
3) Unknown
4) True
5) False
6) True
7) True
8) True
9) False

Answers to Exercises D:

1) True
2) False
3) True