Philosophy 109, Modern Logic, Queens College
Russell Marcus, Instructor
email: philosophy@thatmarcusfamily.org
website: thatmarcusfamily.org/philosophy
Office phone: (718) 997-5287

Propositional Logic Translation (§6.1)

I. The form of an argument.

Consider each of the following arguments:
1.
Either Gore won Florida or Bush did.
Gore didn't.
So, Bush did.

2.
You will get either rice or beans.
You don't get the rice.
So, you'll have the beans.

3.
The square root of two is either rational or irrational.
It's not rational.
So, it's irrational.

They have the same form:
Either p or q
not-p
So, q

This form is called 'Disjunctive Syllogism'
We'll study it later.
Just as an architect, when building a building, looks only at the essential structures, so a logician looks only at the form.

'p' and 'q', above, are like variables, standing for statements;
'either p or q' is a compound sentence, made of simple ones

The language of propositional logic uses capital letters to stand for simple, positive propositions.
Simple propositions are often of subject-predicate form, but not necessarily.
They are the shortest examples of statements; they can not be decomposed further in propositional logic.
In predicate logic, we go beneath the surface, at end of term.

II. Connectives.

From simple propositions, we can construct more complex ones using any of five connectives:
Negation: ~
Conjunction: ·
Disjunction: ∨
Material Implication, ⊃
Biconditional: ≡

What follows is a more detailed explication of each of the five connectives.

Negation.
Some English indicators of negation: Not, it is not the case that p, p is not true, it is false that p

Examples:
John will take the train: J
John won't take the train: ~J
It's not the case that John will take the train: ~J
John takes the train...not!: ~J

In symbols, all of the following are negations:
~R
~(P · Q)
~{[(A ∨ B) ⊃C] · ~D}

Conjunction.
Some English indicators of conjunction: and, but, also, however, yet, still, moreover, although, nevertheless, both.

Examples:
Marlo walks the dog and Phil cleans the floors: M · P
Although Marlo walks the dog, Phil cleans the floors: M · P
Bob and Ray are comedians: B · R
Carolyn is nice, but Emily is really nice: C · E

In symbols, all of the following are conjunctions:
P · ~Q
(A ⊃ B) · (B ⊃ A)
(P ∨ ~Q) · ~[P ≡ (Q · R)]

Disjunction.

Some English indicators of disjunction: or, either, unless

Examples:
Either Paco makes the Website, or Matt does: P ∨ M
Jared or Rene will go to the party: J ∨ R
Justin doesn't feed the kids unless Carolyn asks him to: J ∨ C

In symbols, all of the following are conjunctions:
~P ∨ Q
(A ⊃ B) ∨ (B ⊃ A)
(P ∨ ~Q) ∨ ~[P ≡ (Q · R)]

We'll discuss 'unless' in more detail after we are familiar with truth conditions.

The Conditional.

Some English indicators of a conditional: if, only if, only when, is a necessary condition for, is a sufficient condition for, implies, entails, provided that, given that, on the condition that, in case.

The conditional is also called 'material implication', or just 'implication'.
In 'A⊃B', A is called the antecedent, B is called the consequent.
Examples, using 'A' to stand for 'you join me' and 'B' to stand for 'I go to the movies'.

1. If you join me, then I go to the movies.
2. You join me if I go to the movies.
3. You join me only if (only when) I go to the movies.

4. Your joining me is a necessary condition for my going.

5. Your joining me is a sufficient condition for my going.

6. A necessary condition of your joining me is my going.
7. A sufficient condition for your joining me is my going.
8. Your joining me entails (implies) that I go to the movies.
9. You join me given (provided, on the condition) that I go.

1. If A then B	1. $A \supset B$
2. If B then A	2. $B \supset A$
3. A only if (only when) B	3. $A \supset B$
4. A is necessary for B	4. $B \supset A$
5. A is sufficient for B	5. $A \supset B$
6. B is necessary for A	6. $A \supset B$
7. B is sufficient for A	7. $B \supset A$
8. A entails (implies) B	8. $A \supset B$
9. A given B	9. $B \supset A$

Note that necessary conditions are consequents, while sufficient conditions are antecedents.
If A is necessary for B, then if B is true, we can logically infer that A must also be true.
We use the mnemonic ‘SUN’ to remember this, transforming the ‘U’ to a ‘$\supset$’ we get ‘$S \supset N$’

In symbols, all of the following are conditionals:
$\sim P \supset Q$
$(A \supset B) \supset (B \supset A)$
$(P \vee \sim Q) \supset \sim[P \equiv (Q \cdot R)]$

The Biconditional.

Some English indicators of a biconditional: if and only if, is a necessary and sufficient condition for, just in case.

The biconditional is short for ‘$(A \supset B) \cdot (B \supset A)$’, to which we will return, once we are familiar with truth conditions.
An example:
You’ll be successful just in case you work hard and are lucky: $S \equiv L$

In symbols, all of the following are biconditionals:
$\sim P \equiv Q$
$(A \supset B) \equiv (B \supset A)$
$(P \vee \sim Q) \equiv \sim[P \equiv (Q \cdot R)]$

III. Examples A. Translate to Propositional Logic, using obvious letters for the legend:
1. Alvin doesn’t like sports.
2. Bert and Ernie are muppets.
3. Claudia wants to surf or snorkel.
4. Dogs bite just in case they are startled.
5. Everyone loves logic, or not.
6. If Flora wants candy, Geronimo will get her some.
7. Harold is generous unless his wife is listening.
8. Toyota opens a new plant only if Honda initiates an ad campaign.

IV. Some Complications

Ambiguous cases.
Consider: ‘You may have salad or potatoes and carrots’
Do we translate this as ‘$(S \vee P) \vee C$’?
Or as ‘$S \vee (P \vee C)$’?
Look to commas and semicolons, and translate accordingly, using parentheses:
You may have salad, or potatoes and carrots: $S \vee (P \vee C)$
You may have salad or potatoes, and carrots: $(S \vee P) \vee C$
Commas are almost always located at the main connective.

Wffs (pronounced ‘woofs’, as if you are barking)
Compare:‘baker’ and ‘aebkr’
One is a word and the other isn’t.
We call statements of logic which are constructed properly ‘wffs’, for ‘well-formed formulas’.
Similarly, these are wffs:
P · Q
(~P v Q) ⊃ ~R
These are not wffs:
·P Q
PqvR~

Formation rules for wffs
1. a single capital English letter is a wff
2. If α is a wff, so is ~α
3. If α and β are wffs, then so are:
(α · β)
(α ∨ β)
(α ⊃ β)
(α ≡ β)
By convention, you may drop the outermost brackets
4. these are the only ways to make wffs

Main connectives
The last connective added according to the formation rules is called the main connective

Analyze: (~M ⊃ P) · (~N ⊃ Q)
‘M’, ‘P’, ‘N’, and ‘Q’ are all wffs, by rule 1
‘~M’ and ‘~N’ are wffs by rule 2
‘(~M ⊃ P)’ and ‘(~N ⊃ Q)’ are then wffs by rule 3
Finally, the whole formula is a wff also by rule 3, and the convention of dropping the outermost brackets

V. Exercises B. Are the following formulas wffs? If so, find the main connective.
1. (P ∨ Q) ⊃ ~R
2. ~X(Y ∨ Z)
3. (S ∨ T · U) ⊃ S
4. ~(G ⊃ H)
5. ~{(P ⊃ Q) · [P ≡ ~(Q ∨ R)]}
6. ~[A · (B ∨ C)] ≡ [(A · B) ∨ (A · C)]
7. [(D · E) ∨ F] · G

VI. Exercises C. Translate these into propositional logic, using obvious letters:
1. Ford introduces a new model and either Chrysler raises prices or General Motors changes colors.
2. Both Toyota does not open a new plant and Ford does not introduce a new model.
3. Honda initiates an ad campaign if and only if Chrysler raises prices.
4. Either Saab increases salaries and Toyota opens a new plant or Honda initiates an ad campaign and General Motors changes colors.
5. Toyota’s opening a new plant is a necessary condition for General Motors’ changing colors, and Ford’s introducing a new model is a sufficient condition for Chrysler’s raising prices.
6. If Saab increases salaries, then if Toyota opens a new plant, then Honda initiates an ad campaign.
7. Audi lays off workers; however, if Chrysler raises prices then either General Motors does not change colors or Ford does not introduce a new model.

VII. Solutions

Answers to Exercises A (Note that alternative letters are sometimes possible):

1. ~A
2. B · E
3. F ∨ L
4. B ≡ S
5. L ∨ ~L
6. F ⊃ G
7. G ∨ L
8. T ⊃ H

Answers to Exercises B

1. Yes, ⊃
2. No
3. No
4. Yes, ~
5. Yes, ~
6. Yes, ≡
7. Yes, ·

Answers to Exercises C

1. F · (C ∨ G)
2. ~T · ~F
3. H ≡ C
4. (S · T) ∨ (H · G)
5. (G ⊃ T) · (F ⊃ C)
6. S ⊃ (T ⊃ H)
7. A · [C ⊃ (~G ∨ ~F)]