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Introducing and Eliminating Quantifiers, §8.2

There are four rules for deductions in predicate logic, in addition to the ones used in propositional logic.

I. Taking off the universal quantifier.

Recall the argument:

All philosophers are happy: $(x)(Px \supset Hx)$

Emily is a philosopher: Pe

\therefore Emily is happy: He

We need to add a rule that will generate this conclusion out of these premises

Rule #1: Universal Instantiation (UI)

Remove the leading universal quantifier.

Replace all occurrences of variables bound by that quantifier with either a variable (x, y, z) or a constant (a, b, c).

Use only when the main connective is a quantifier, i.e. on whole lines.

We write these as:

$(x)\mathcal{F}x / \mathcal{F}y$

$(x)\mathcal{F}x / \mathcal{F}a$

Binding Rule: When you use instantiate or generalize, you must change all the bound variables in the same way.

So $(x)[Sx \vee (Pa \cdot Tx)]$ can be instantiated as:

$Sa \vee (Pa \cdot Ta)$

$Sb \vee (Pa \cdot Tb)$

$Sx \vee (Pa \cdot Tx)$

$Sy \vee (Pa \cdot Ty)$

But it can not be instantiated as:

$Sa \vee (Pa \cdot Tb)$

$Sx \vee (Pa \cdot Ta)$

Returning to the argument above:

1. $(x)(Px \supset Hx)$

2. Pe / He

3. $Pe \supset He$ 1, UI

4. He 3, 2, MP

QED

II. Putting on the universal quantifier.

Consider:

1. Everything happy is content

2. No miser is content.

\therefore No miser is happy.

Regiment this as:

1. $(x)(Hx \supset Cx)$

2. $(x)(Mx \supset \sim Cx)$ / $(x)(Mx \supset \sim Hx)$

We need to remove the quantifiers and put them back on

But never: $(\exists x)\mathcal{F}x / \mathcal{F}y$

About new names. Consider what would happen without that restriction:

1. $(\exists x)(Fx \cdot Px)$ (Some fruits are pears)
2. $(\exists x)(Fx \cdot Ox)$ (Some fruits are oranges)
3. $Fa \cdot Pa$ 1, EI
4. $Fa \cdot Oa$ 2, EI: but wrong!
5. Pa 3, Com, Simp
6. Oa 4, Com, Simp
7. $Pa \cdot Oa$ 5, 6, Conj
8. $(\exists x)(Px \cdot Ox)$ 7, EG (Some pears are oranges. Uh-oh!)

Since we have this restriction on EI, but not on UI, always EI before you UI!

A sample derivation:

1. $(x)(Nx \supset Ax)$
 2. $(\exists x)(Nx \cdot Bx)$ / $(\exists x)(Ax \cdot Bx)$
 3. $Na \cdot Ba$ 2, EI
 4. $Na \supset Aa$ 1, UI
 5. Na 3, Simp
 6. Aa 4, 5, MP
 7. Ba 3, Com, Simp
 8. $Aa \cdot Ba$ 6, 7, Conj
 9. $(\exists x)(Ax \cdot Bx)$ 8, EG
- QED

V. Exercise A. Derive the conclusions of each of the following arguments:

- 1)
1. $(x)(Ax \supset Bx)$
2. $(\exists x)(Ax)$ / $(\exists x)(Bx)$

- 2)
1. $(\exists x)(\sim Fx \cdot \sim Gx)$
2. $(x)(\sim Gx \supset \sim Hx)$ / $(\exists x)\sim(Fx \vee Hx)$

- 3)
1. $(x)(Jx \vee \sim Kx)$
2. $(x)\sim Jx$ / $(x)\sim Kx$

VI. Examples of more complex derivations:

Sometimes terms can be unbound

1. $(x)(Dx \cdot Ex)$
 2. $(x)Dx \supset Fa$ / $(\exists x)Fx$
 3. $Dx \cdot Ex$ 1, UI
 4. Dx 3, Simp
 5. $(x)Dx$ 4, UG
 6. Fa 2, 5, MP
 7. $(\exists x)Fx$ 6, EG
- QED

Two quantifiers on a line

1. $(x)(Jx \vee Kx) \supset (\exists y)Ly$
2. $(x)(Jx \vee Lx)$
3. $(x)(\sim Lx \vee Kx)$ / $(\exists y)Ly$

- | | |
|-------------------------|-------------|
| 4. $Jx \vee Lx$ | 2, UI |
| 5. $\sim Jx \supset Lx$ | 4, DN, Impl |
| 6. $\sim Lx \vee Kx$ | 3, UI |
| 7. $Lx \supset Kx$ | 6, Impl |
| 8. $\sim Jx \supset Kx$ | 5, 7, HS |
| 9. $Jx \vee Kx$ | 8, Impl, DN |
| 10. $(x)(Jx \vee Kx)$ | 9, UG |
| 11. $(\exists y)Ly$ | 1, 10, MP |
- QED

Instantiating Twice from the same statement

- | | |
|-------------------------|-----------------|
| 1. $(x)(Mx \supset Nx)$ | |
| 2. $(x)(Nx \supset Ox)$ | |
| 3. $Ma \cdot Mb$ | / $Na \cdot Ob$ |
| 4. $Ma \supset Na$ | 1, UI |
| 5. Ma | 3, Simp |
| 6. Na | 4, 5, MP |
| 7. $Mb \supset Nb$ | 1, UI |
| 8. Mb | 3, Com, Simp |
| 9. Nb | 7, 8, MP |
| 10. $Nb \supset Ob$ | 2, UI |
| 11. Ob | 10, 9, MP |
| 12. $Na \cdot Ob$ | 6, 11, Conj |
- QED

VII. Exercise B. Derive the conclusions of the following arguments:

- 1)
- | | |
|-------------------------|-------------|
| 1. $(x)(Mx \supset Nx)$ | |
| 2. $\sim Na$ | / $\sim Ma$ |
- 2)
- | | |
|-------------------------------|-----------------------------------|
| 1. $(x)(Ox \supset \sim Px)$ | |
| 2. $(\exists x)(Rx \cdot Px)$ | / $(\exists x)(Rx \cdot \sim Ox)$ |
- 3)
- | | |
|--|------------------------------|
| 1. $(\exists x)Sx \supset (x)Tx$ | |
| 2. $(\exists x)Ux \supset (\exists x)Wx$ | |
| 3. $Sb \cdot Ub$ | / $(\exists x)(Wx \cdot Tx)$ |
- 4)
- | | |
|---|------------------------|
| 1. $(\exists x)Gx \supset (y)(\sim Hy \vee Iy)$ | |
| 2. Gc | |
| 3. $\sim If$ | / $(\exists x)\sim Hx$ |
- 5)
- | | |
|---|------------------------|
| 1. $(\exists x)Ax \supset (x)(Bx \supset Ex)$ | |
| 2. $(\exists x)Dx \supset (\exists x)\sim Ex$ | |
| 3. $(\exists x)(Ax \cdot Dx)$ | / $(\exists x)\sim Bx$ |