

Philosophy 109, Modern Logic, Queens College
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Predicate invalidity, §8.5

I. Recall how we proved an argument invalid in propositional logic:

1. $A \supset B$
2. $\sim(B \cdot A) \quad / A \equiv B$

Line up the premises and conclusion

$A \supset B \quad / \quad \sim(B \cdot A) \quad // \quad A \equiv B$

Assign truth values to the component sentences to form a counterexample which makes the premises true and the conclusion false

F T T T T F F F F T

So, the argument is shown invalid when A is false and B is true.

II. Proving an argument invalid in Predicate Logic.

In Predicate logic, we may use the method of finite universes.

If an argument is valid, then it is true, no matter what the interpretation of the constants, and no matter what exists in the world.

Logical truths are true in all possible universes.

They have no prejudices about what exists in the world.

If they did, then they wouldn't be logical truths, but empirical truths!

Even if the world has only one member, or two or three, valid arguments should be valid.

Consider the following invalid argument:

- $(x)(Wx \supset Hx)$
 $(x)(Ex \supset Hx) \quad / \quad (x)(Wx \supset Ex)$

Imagine there were only one object in the universe. Let's call it 'a'.

$(x)(Wx \supset Hx)$ would actually be $Wa \supset Ha$
 $(x)(Ex \supset Hx)$ would be $Ea \supset Ha$
 $\therefore (x)(Wx \supset Ex)$ would be $Wa \supset Ea$

Now, assign truth values, as in the propositional case to make the premises true and the conclusion false:

$Wa \supset Ha \quad / \quad Ea \supset Ha \quad // \quad Wa \supset Ea$
 T T F T T F

So, the argument is shown invalid in a one-member universe, where Wa is true, Ha is true, and Ea is false.

Not all invalid arguments are invalid in a one-member universe.

All we need is one universe in which they are invalid, to show them invalid.

Even if an argument works in a one-member universe, it might still be invalid!

III. Universes of more than one member

Consider the following invalid argument:

- $(x)(Wx \supset Hx)$
 $(\exists x)(Ex \cdot Hx) \quad / \quad (x)(Wx \supset Ex)$

In a one-object universe, we have

$Wa \supset Ha \quad / \quad Ea \cdot Ha \quad // \quad Wa \supset Ea$

Pa: T/F
Pb: T
Qa: F
Qb: T
Ra: F
Rb: T

III. Exercises. Show each of the following arguments invalid by generating a counter-example using the method of finite universes.

1)
1. $(x)(Ex \supset Fx)$
2. $(\exists x)(Gx \cdot \sim Fx)$ / $(\exists x)(Ex \cdot \sim Gx)$

2)
1. $(x)(Bx \supset \sim Dx)$
2. $\sim Bj$ / Dj

3)
1. $(x)(Hx \supset \sim Ix)$
2. $(\exists x)(Jx \cdot \sim Ix)$ / $(x)(Hx \supset Jx)$

4)
1. $(x)(Kx \supset \sim Lx)$
2. $(\exists x)(Mx \cdot Lx)$ / $(x)(Kx \supset \sim Mx)$

5)
1. $(\exists x)(Px \cdot Qx)$
2. $(x)(Qx \supset \sim Rx)$
3. Pa / $(x)\sim Rx$

6)
1. $(x)(Ax \supset Bx)$
2. $(\exists x)(Dx \cdot Bx)$
3. $(\exists x)(Dx \cdot \sim Bx)$ / $(x)(Ax \supset Dx)$

IV. Solutions to exercises (alternate solutions are possible):

1) Invalid in a one-member universe, where Ga: true; Ea: false; Fa: false

2) Invalid in a one-member universe, where Bj: false; Dj: false

3) Invalid in a two-member universe, where Ha: true; Ia: false; Ja: false; Hb: T/F; Ib: false; Jb: true

4) Invalid in a two-member universe, where Ka: false; La: true; Ma: true; Kb: true; Lb: false; Mb: true.

5) Invalid in a two-member universe, where Pa: true; Qa: false; Ra: true; Pb: true; Qb: true; Rb: false

6) Invalid in a three-member universe, where Aa: true; Ba: true; Da: false; Ab: T/F; Bb: true; Db: true; Ac: false; Bc: false; Dc: true