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Conditional and Indirect proof in Predicate Logic, §8.4

I. A problem arising from using CP and IP in Predicate Logic:  
 With Unrestricted CP we can construct the following derivation:

- |                          |          |
|--------------------------|----------|
| 1. $(x)Rx \supset (x)Bx$ | Premise  |
| *2. $Rx$                 | ACP      |
| *3. $(x)Rx$              | 2, UG    |
| *4. $(x)Bx$              | 1, 3, MP |
| *5. $Bx$                 | 4, UI    |
| 6. $Rx \supset Bx$       | 2-5, CP  |
| 7. $(x)(Rx \supset Bx)$  | 6, UG    |

This would mean that we could prove that everything red is blue (the conclusion) from ‘If everything is red, then everything is blue’ (the premise).

But that premise can be true while the conclusion is false.  
 So, the derivation should be invalid.

Moral of the story: we must restrict conditional proof.

The problem is in step 3

We may not generalize on  $x$  within the assumption.

The assumption just means that a random thing is  $R$ , not that everything is  $R$ .

We may generalize after we’ve discharged, as in line 7.

**The Restriction on (CP) and (IP):**

Never UG within an assumption on a variable that’s free in the first line of the assumption.

II. Examples of CP and IP in Predicate Logic

One of two typical uses of (CP)

- |                                   |                          |   |
|-----------------------------------|--------------------------|---|
| 1. $(x)[Ax \supset (Bx \vee Dx)]$ |                          |   |
| 2. $(x)\sim Bx$                   | $\neg(x)(Ax \supset Dx)$ |   |
| *3. $Ay$                          | ACP                      | pick a random object that has A                         |
| *4. $Ay \supset (By \vee Dy)$     | 1, UI                    |   |
| *5. $By \vee Dy$                  | 4, 3, MP                 |   |
| *6. $\sim By$                     | 2, UI                    |   |
| *7. $Dy$                          | 5, 6, DS                 |   |
| 8. $Ay \supset Dy$                | 3-7, CP                  | given any object, if it has A, it provably has D        |
| 9. $(x)(Ax \supset Dx)$           | 8, UG                    | no longer within the scope of the assumption, we may UG |
| QED                               |                          |   |

So, to prove statements of the form  $(x)(Px \supset Qx)$

Assume  $Px$

Derive  $Qx$

Discharge  $(Px \supset Qx)$

Then (UG)

Another typical use of CP:

- $(x)[Px \supset (Qx \cdot Rx)]$

2.  $(\forall x)(Rx \supset Sx) \quad / \quad (\exists x)Px \supset (\exists x)Sx$
- \*3.  $(\exists x)Px$                       ACP                      pick a random object that has A
  - \*4.  $Pa$                                 3, EI
  - \*5.  $Pa \supset (Qa \cdot Ra)$             1, UI
  - \*6.  $Qa \cdot Ra$                         5, 4, MP
  - \*7.  $Ra$                                 6, Com, Simp
  - \*8.  $Ra \supset Sa$                         2, UI
  - \*9.  $Sa$                                 8, 7, MP
  - \*10.  $(\exists x)Sx$                         9, EG
11.  $(\exists x)Px \supset (\exists x)Sx$             3-10, CP
- QED

Indirect Proof works basically in the same way as in propositional logic.  
 But the same restriction on CP holds for IP, too.  
 The restriction holds any time one makes an assumption.

Typical use of (IP):

1.  $(\forall x)[(Ax \vee Bx) \supset Ex]$
2.  $(\forall x)[(Ex \vee Dx) \supset \sim Ax] \quad / (\forall x)\sim Ax$
- \*3.  $\sim(\forall x)\sim Ax$                       AIP                      Remember, you're looking for a contradiction
  - \*4.  $(\exists x)Ax$                             3, CQ
  - \*5.  $Aa$                                 4, EI
  - \*6.  $(Ea \vee Da) \supset \sim Aa$             2, UI
  - \*7.  $\sim(Ea \vee Da)$                     6, 5, DN, MT
  - \*8.  $\sim Ea \cdot \sim Da$                     7, DM
  - \*9.  $\sim Ea$                               8, Simp
  - \*10.  $(Aa \vee Ba) \supset Ea$             1, UI
  - \*11.  $\sim(Aa \vee Ba)$                     10, 9, MT
  - \*12.  $\sim Aa \cdot \sim Ba$                     11, DM
  - \*13.  $\sim Aa$                             12, Simp
  - \*14.  $Aa \cdot \sim Aa$                       5, 13, Conj
15.  $(\forall x)\sim Ax$                         3-13, IP, DN
- QED

Note that with CP, sometimes you only assume part of a line, then generalize outside the assumption, but with IP, you almost always assume the negation of the whole conclusion!

### III. Exercises. Derive the conclusions of the following arguments:

- 1)
- 1.  $(\forall x)(Fx \supset Gx)$
  - 2.  $(\forall x)(Fx \supset Hx) \quad / \quad (\forall x)[Fx \supset (Gx \cdot Hx)]$
- 2)
- 1.  $(\forall x)(Jx \supset \sim Kx) \quad / \quad \sim(\exists x)(Jx \cdot Kx)$
- 3)
- 1.  $(\forall x)(Rx \supset Bx) \quad / \quad (\forall x)Rx \supset (\forall x)Bx$
- 4)
- 1.  $(\forall x)(Lx \supset Mx)$
  - 2.  $\sim(\exists x)Lx \supset (\exists x)Mx \quad / \quad \sim(\forall x)\sim Mx$