Philosophy 109, Modern Logic, Queens College
Russell Marcus, Instructor
email: philosophy@thatmarcusfamily.org
website: thatmarcusfamily.org/philosophy
Office phone: (718) 997-5287

Philosophical interlude on three-valued logic

1. Failure of presupposition
We've talked about a third truth value for propositions referring to future events.
Appeal to a four-dimensional framework, a god's-eye point of view, suffices to resolve this issue.
For another reason to try three truth values, consider:
'The present King of France is bald'
Compare with: 'The present King of France is not bald'.
Both of these may be deemed false, or neither true, since they contain a false presupposition.
Similarly, for 'The woman on the moon is six feet tall' and 'The woman on the moon is not six feet tall'.
We might just call such propositions neither true nor false, and introduce a new truth value for them.

2. Mathematical statements neither proven nor refuted
Another reason one might introduce a third-truth value. Consider Goldbach's conjecture.
This says that every even number greater than 4 can be written as the sum of two odd primes.
It has neither been proven, nor proven false, though it has been verified up to very large values.
Some philosophers of mathematics, those who believe that mathematics is constructed, rather than discovered, believe that this means that Goldbach's conjecture is neither true nor false.

3. Paradoxical sentences
A final example: 'This sentence is false'.
If it's true, then it's false and vice versa.
In other words, it seems to lack a definite truth value.

4. If we wanted to adopt a third truth value, we would have to revise all the truth tables.
For examples, three-valued truth tables for negation and disjunction:

P	**~P**
T	F
U	U
F	T

P	**Q**	**P • Q**
T	T	T
T	U	U
T	F	F
U	T	U
U	U	U
U	F	F
F	T	F
F	U	F
F	F	F

(There are other ways of doing these, fyi.)

5. But there are ways to deal with all of these examples that do not force us to introduce a third truth-value.

A. For failures of presupposition, we can use Bertrand Russell's analysis.
In brief, we analyze the sentence to make the assumption explicit.

Roughly, recast ‘The woman on the moon is six feet tall’ as:
There is a woman on the moon and she is six feet tall.
This has the form ‘P • Q’.
P is false, so ‘P • Q’ is false.
Similarly, ‘There is a woman on the moon and she is not six feet tall’ becomes ‘P • ~Q’
P is false, so ‘P • ~Q’ is false.
We thus do not have a situation in which the same proposition seems true and false.

B. The case of unproven and unrefuted mathematical statements can be assimilated to the case of the sea battle tomorrow.
We just don’t know, yet.
We can say that this statement is true or false, but we can’t say which, now.

C. And we can deny that paradoxical sentences even express propositions.
We may claim that just as some strings of letters don’t form words, and some strings of words don’t form sentences, some sentence types don’t express propositions.
This would be the same as to call them meaningless.
This solution is a bit awkward, since it does seem that ‘This sentence is false’ is perfectly meaningful.
But if it prevents us from adopting three-valued logics, it might be useful.