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Indirect Proof, §7.6

I. Indirect Proof: Another method for derivations.

Consider the following two proofs:

1)

1. A · ~A	/ B
2. A	1, Simp
3. A ∨ B	2, Add
4. ~A	1, Com, Simp
5. B	3, 4, DS

QED

The moral: Anything follows from a contradiction.

2)

1. B ⊃ (P · ~P) / ~B

    *2. B
    *3. P · ~P
    *4. P
    *5. P ∨ ~B
    *6. ~P
    *7. ~B

8. B ⊃ ~B
9. ~B ∨ ~B
10. ~B

QED

The moral: If a statement entails a contradiction, then its negation is true.

Indirect proof is based on these two morals.
It's also called the method of reductio ad absurdum.
Assume your desired conclusion is false.
Try to get a contradiction
If you get it, then you know the opposite of the assumption is true.

Procedure for Indirect Proof, (IP)
1. Indent, assuming the opposite of what you want to conclude (one more or one fewer '~')
2. Derive a contradiction, using any letter
3. Discharge the negation of your assumption

Sample Derivation:

1. A ⊃ B		
2. A ⊃ ~B	/~A	
*3. A	AIP	Let's see what happens if the opposite of the conclusion is true.
*4. B	1, 3, MP	
*5. ~B	2, 3, MP	
*6. B · ~B	4, 5, Conj	This is impossible - a contradiction
7. ~A	3-6, IP	So ~~A must be false, and so ~A is true

QED

This method is useful for proving disjunctions as well as simple statements and negations.

II. More sample derivations:

Plain indirect proof:

1. F ⊃ ~D	
2. D	
3. (D · ~E) ⊃ F	/ E
*4. ~E	AIP
*5. D · ~E	2, 4, Conj
*6. F	3, 5, MP
*7. ~D	1, 6, MP
*8. D · ~D	2, 7, Conj
9. ~~E	4-8, CP
10. E	9, DN

QED

Indirect proof with conditional proof

1. E ⊃ (A · D)		
2. B ⊃ E	/ (E ∨ B) ⊃ A	
*3. E ∨ B		ACP
* *4. ~A		AIP
* *5. ~A ∨ ~D		4, Add
* *6. ~(A · D)		5, DM
* *7. ~E		1, 6, MT
* *8. B		3, 7, DS
* *9. ~B		2, 7, MT
* *10. B · ~B		8, 9, Conj
*11. ~~A		4-10, IP
*12. A		11, DN
12. (E ∨ B) ⊃ A		3-12, CP

QED

Proving logical truths using indirect proof
Prove that '~[(X ≡ Y) · ~(X ∨ ~Y)]' is a logical truth.

*1. (X ≡ Y) · ~(X ∨ ~Y)	AIP
*2. X ≡ Y	1, Simp
*3. (X ⊃ Y) · (Y ⊃ X)	2, Equiv
*4. ~(X ∨ ~Y)	1, Com, Simp
*5. ~X · Y	4, DM DN
*6. Y ⊃ X	3, Com, Simp
*7. ~X	5, Simp
*8. ~Y	6, 7, MT
*9. Y	5, Com, Simp
*10. Y · ~Y	9, 8, Conj
11. ~[(X ≡ Y) · ~(X ∨ ~Y)]	1-10, IP

QED
Remember that the conclusion, here, is not part of the proof, and has no line number until the end.

III. Exercises. Derive the conclusions of the following arguments using the 18 rules, and either CP or IP.

1)
1. A ⊃ B
2. ~A ∨ ~B   / ~A

2)
1. F ⊃ (~E ∨ D)
2. F ⊃ ~D / F ⊃ ~E

3)
1. ~J ⊃ (G · H)
2. G ⊃ I
3. H ⊃ ~I / J

4)
1. S ⊃ (T ∨ U)
2. W ⊃ ~U / S ⊃ ~(W · ~T)

5)
1. (L ⊃ M) · (N ⊃ O)
2. (M ∨ O) ⊃ P
3. ~P / ~(L ∨ N)

6)
Prove that (A ⊃ B) ∨ (B ⊃ A) is a logical truth.