

Philosophy 109, Modern Logic, Queens College
Russell Marcus, Instructor
email: philosophy@thatmarcusfamily.org
website: thatmarcusfamily.org/philosophy
Office phone: (718) 997-5287

Changing Quantifiers, §8.3

I. The four sets of equivalences.

Note that the two statements in each of the following pairs are equivalent.

- 1a. Everything is made of atoms
- 1b. It's not the case that something is not made of atoms.

- 2a. Something is fishy
- 2b. It's wrong to say that nothing is fishy.

- 3a. Nothing is perfect.
- 3b. It's false that something is perfect.

- 4a. At least one thing isn't blue.
- 4b. Not everything is blue.

Now look at the predicate logic regimentations of each.

- 1a. $(x)Ax$
- 1b. $\sim(\exists x)\sim Ax$

- 2a. $(\exists x)Fx$
- 2b. $\sim(x)\sim Fx$

- 3a. $(x)\sim Px$
- 3b. $\sim(\exists x)Px$

- 4a. $(\exists x)\sim Bx$
- 4b. $\sim(x)Bx$

The rule of **Changing Quantifiers (CQ)**: Any place where you have an expression of one of the above forms, you may replace it with a statement of its logically equivalent form.

Like rules of replacement, (CQ) may be used on part of a line

Another way to look at these four rules.

There are three spaces around each quantifier:

Directly before the quantifier

The quantifier itself

Directly following the quantifier

To change a quantifier, change each space

Add or remove a negation directly before the quantifier.

Switch quantifiers: existential to universal or vice versa.

Add or remove a negation directly after the quantifier.

II. Some transformations permitted by CQ.

'It's not the case that every P is Q' is equivalent to 'something is P and not Q'

$\sim(x)(Px \supset Qx)$

$(\exists x)\sim(Px \supset Qx)$ CQ

$(\exists x)\sim(\sim Px \vee Qx)$ Impl
 $(\exists x)(Px \cdot \sim Qx)$ Dm, DN

'It's not the case that something is both P and Q' is equivalent to 'everything that's P is not Q'
 also, to 'everything that's Q is not P'

$\sim(\exists x)(Px \cdot Qx)$
 $(x)\sim(Px \cdot Qx)$ CQ
 $(x)(\sim Px \vee \sim Qx)$ DM
 $(x)(Px \supset \sim Qx)$ Impl
 $(x)(Qx \supset \sim Px)$ Trans, DN

III. Examples, using CQ

1)

1. $(\exists x)Lx \supset (\exists y)My$
2. $(y)\sim My$ / $\sim La$
3. $\sim(\exists y)My$ 2, CQ
4. $\sim(\exists x)Lx$ 1, 3, MT
5. $(x)\sim Lx$ 4, CQ
6. $\sim La$ 5, UI Note: You may not use EI to get this conclusion!

QED

2)

1. $(x)[(Ax \cdot Bx) \supset Ex]$
2. $\sim(x)(Ax \supset Ex)$ / $\sim(x)Bx$
3. $(\exists x)\sim(Ax \supset Ex)$ 2, CQ
4. $(\exists x)\sim(\sim Ax \vee Ex)$ 3, Impl
5. $(\exists x)(Ax \cdot \sim Ex)$ 4, DM, DN
6. $Aa \cdot \sim Ea$ 5, EI
7. $(Aa \cdot Ba) \supset Ea$ 1, UI
8. $\sim Ea$ 6, Com, Simp
9. $\sim(Aa \cdot Ba)$ 7, 8, MT
10. $\sim Aa \vee \sim Ba$ 9, DM
11. Aa 6, Simp
12. $\sim Ba$ 10, 11, DN, DS
13. $(\exists x)\sim Bx$ 12, EG
14. $\sim(x)Bx$ 13, CQ

QED

3)

1. $(x)\sim Dx \supset (x)Ex$
2. $(\exists x)\sim Ex$ / $(\exists x)Dx$
3. $\sim(x)Ex$ 2, CQ
4. $\sim(x)\sim Dx$ 1, 3, MT
5. $(\exists x)Dx$ 4, CQ

QED Note: No instantiation!

IV. Exercises. Derive the conclusions of each of the following arguments:

1)

1. $\sim(\exists x)Hx$
2. $(x)\sim Hx \supset (z)Iz$ / a

2)

1. $(\exists x)(Hx \cdot Gx) \supset (x)Ix$

2. $\sim \text{Ia}$ $\quad \quad \quad / (\forall x)(\text{Hx} \supset \sim \text{Gx})$

3)

1. $(\exists x)(\text{Ax} \vee \text{Bx}) \supset (\forall x)\text{Dx}$

2. $(\exists x)\sim \text{Dx} \quad \quad \quad / \sim (\exists x)\text{Ax}$

4)

1. $(\forall x)\sim \text{Fx} \supset (\forall x)\sim \text{Gx} \quad \quad \quad / (\exists x)\text{Gx} \supset (\exists x)\text{Fx}$

5)

1. $(\exists x)\sim \text{Ax} \supset (\forall x)\sim \text{Bx}$

2. $(\exists x)\sim \text{Ax} \supset (\exists x)\text{Bx}$

3. $(\forall x)(\text{Ax} \supset \text{Fx}) \quad \quad \quad / (\forall x)\text{Fx}$