

Solutions to Proofs to Review for Test on Chapter 7

I. Do without CP or IP

- 1) 1.  $A \supset B$   
 2.  $(E \vee D) \supset \sim B$  /  $A \supset (\sim E \cdot \sim D)$   
 3.  $B \supset \sim(E \vee D)$  2, Trans, DN  
 4.  $A \supset \sim(E \vee D)$  1, 3, HS  
 5.  $A \supset (\sim E \cdot \sim D)$  4, DM

QED

- 2) 1.  $\sim(G \cdot H) \supset I$   
 2.  $\sim(I \vee J)$   
 3.  $\sim J \supset F$  /  $(F \cdot G) \cdot H$   
 4.  $\sim I \cdot \sim J$  2, DM  
 5.  $\sim I$  4, Simp  
 6.  $G \cdot H$  1, 5, MT, DN  
 7.  $\sim J$  4, Com, Simp  
 8.  $F$  3, 7, MP  
 9.  $F \cdot (G \cdot H)$  8, 6, Conj  
 10.  $(F \cdot G) \cdot H$  9, Assoc

QED

- 3) 1.  $P \vee O$   
 2.  $Q \supset \sim O$  /  $\sim Q \vee P$   
 3.  $\sim O \supset P$  1, Com, DN, Impl  
 4.  $Q \supset P$  2, 3, HS  
 5.  $\sim Q \supset P$  4, Impl

QED

- 4) 1.  $T \equiv V$   
 2.  $\sim T \vee \sim V$  /  $\sim T$   
 3.  $(T \cdot V) \vee (\sim T \cdot \sim V)$  1, Equiv  
 4.  $\sim(T \cdot V)$  2, DM  
 5.  $\sim T \cdot \sim V$  3, 4, DS  
 6.  $\sim T$  5, Simp

QED

- 5) 1.  $(K \supset \sim M) \cdot [(L \cdot N) \supset \sim M]$   
 2.  $(K \vee L) \cdot (K \vee N)$  /  $\sim M$   
 3.  $K \vee (L \cdot N)$  2, Dist  
 4.  $\sim M \vee \sim M$  1, 3, CD  
 5.  $\sim M$  4, Taut

QED

- 6) 1.  $(X \vee Y) \cdot (Z \supset W)$   
2.  $\sim(Y \cdot W)$   $\quad / \sim(\sim X \cdot Z)$   
3.  $\sim Y \vee \sim W$   $\quad$  2, DM  
4.  $(Y \vee X) \cdot (Z \supset W)$   $\quad$  1, Com  
5.  $(\sim Y \supset X) \cdot (Z \supset W)$   $\quad$  4, DN, Impl  
6.  $(\sim Y \supset X) \cdot (\sim W \supset \sim Z)$   $\quad$  5, Trans  
7.  $X \vee \sim Z$   $\quad$  6, 3, CD  
8.  $\sim\sim(X \vee \sim Z)$   $\quad$  7, DN  
9.  $\sim(\sim X \cdot Z)$   $\quad$  8, DM, DN

QED

- 7) 1.  $X \vee Z$   
2.  $Z \supset \sim Y$   
3.  $X \supset Y$   $\quad / \sim X \equiv Z$   
4.  $\sim Y \supset \sim X$   $\quad$  3, Trans  
5.  $Z \supset \sim X$   $\quad$  2, 4, HS  
6.  $\sim X \supset Z$   $\quad$  1, DN, Impl  
7.  $(\sim X \supset Z) \cdot (Z \supset \sim X)$   $\quad$  6, 5, Conj  
8.  $\sim X \equiv Z$   $\quad$  7, Equiv

QED

- 8) 1.  $A \supset B$   
2.  $B \supset \sim A$   
3.  $(A \vee D) \vee E$   
4.  $(D \vee E) \supset F$   $\quad / F$   
5.  $A \supset \sim A$   $\quad$  1, 2, HS  
6.  $\sim A \vee \sim A$   $\quad$  5, Impl  
7.  $\sim A$   $\quad$  6, Taut  
8.  $A \vee (D \vee E)$   $\quad$  3, Assoc  
9.  $\sim A \supset (D \vee E)$   $\quad$  8, Dn, Impl  
10.  $D \vee E$   $\quad$  9, 7, MP  
11.  $F$   $\quad$  4, 10, MP

QED

II. You may use CP or IP.

- 1) 1.  $G \supset H$   
2.  $G \supset I$   $\quad / G \supset (H \cdot I)$   
\*3.  $G$   $\quad$  ACP  
\*4.  $H$   $\quad$  1, 3, MP  
\*5.  $I$   $\quad$  2, 3, MP  
\*6.  $H \cdot I$   $\quad$  4, 5, Conj  
7.  $G \supset (H \cdot I)$   $\quad$  3-6, CP

QED

- 2) 1.  $J \equiv K$   
2.  $K \equiv L$   $\quad / J \equiv L$   
3.  $(J \supset K) \cdot (K \supset J)$   $\quad$  1, Equiv

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|---|--------------|
| 4. $(K \supset L) \cdot (L \supset K)$  | 2, Equiv     |
| 5. $J \supset K$                        | 3, Simp      |
| 6. $K \supset L$                        | 4, Simp      |
| 7. $J \supset L$                        | 5, 6, HS     |
| 8. $L \supset K$                        | 4, Com, Simp |
| 9. $K \supset J$                        | 3, Com, Simp |
| 10. $L \supset J$                       | 8, 9, HS     |
| 11. $(J \supset L) \cdot (L \supset J)$ | 7, 10, Conj  |
| 12. $J \equiv L$                        | 11, Equiv    |

QED

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|---|------------|
| 3) 1. $M \supset [N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}]$ |            |
| *2. $N$   | ACP        |
| *3. $O$   | ACP        |
| *4. $P$   | ACP        |
| *5. $M \cdot N$   | 1, 2, Conj |
| *6. $O \cdot P$   | 3, 4, Conj |
| *7. $(M \cdot N) \cdot (O \cdot P)$   | 5, 6, Conj |
| *8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$   | 4-7, CP    |
| *9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$                           | 3-8, CP    |
| 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$             | 2-9, CP    |

QED

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|--|----------|
| 4) $R \supset [(R \supset S) \supset S]$ |          |
| *1. $R$                                  | ACP      |
| *2. $R \supset S$                        | ACP      |
| *3. $S$                                  | 2, 1, MP |
| *4. $(R \supset S) \supset S$            | 2-3, CP  |
| 5. $R \supset [(R \supset S) \supset S]$ | 1-4, CP  |

QED

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|----------------------------|--------------|
| 5) 1. $D \supset E$        |              |
| 2. $E \supset (F \cdot G)$ |              |
| 3. $\sim F \vee \sim G$    | $\sim D$     |
| *4. $D$                    | AIP          |
| *5. $E$                    | 1, 4, MP     |
| *6. $F \cdot G$            | 2, 5, MP     |
| *7. $F$                    | 6, Simp      |
| *8. $\sim G$               | 3, 7, DN, DS |
| *9. $G$                    | 6, Com, Simp |
| *10. $G \sim G$            | 9, 8, Conj   |
| 11. $\sim D$               | 4-10, IP     |

QED

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|---|----------|
| 6) 1. $(P \vee Q) \vee (\sim R \vee S)$ |          |
| 2. $\sim Q \cdot (\sim S \cdot \sim P)$ | $\sim R$ |
| 3. $\sim Q$                             | 2, Simp  |
| 4. $(Q \vee P) \vee (\sim R \vee S)$    | 1, Com   |
| 5. $Q \vee [P \vee (\sim R \vee S)]$    | 4, Assoc |

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|-----------------------------|----------------|
| 6. $P \vee (\sim R \vee S)$ | 5, 3, DS       |
| 7. $\sim S \cdot \sim P$    | 2, Com, Simp   |
| 8. $\sim P$                 | 7, Com, Simp   |
| 9. $\sim R \vee S$          | 6, 8, DS       |
| 10. $\sim S$                | 7, Simp        |
| 11. $\sim R$                | 9, 10, Com, DS |

QED

7)  $\neg(A \vee \sim A)$

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|--------------------------------|-------------|
| *1. $\sim(A \vee \sim A)$      | AIP         |
| *2. $\sim A \cdot \sim \sim A$ | 1, DM       |
| *3. $A \cdot \sim A$           | 2, DN, Com  |
| 4. $A \vee \sim A$             | 1-3, IP, DN |

QED

8)  $\neg A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$

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|--|--------------|
| *1. $\sim\{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$   | AIP          |
| *2. $\sim A \cdot \sim[(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1, DM        |
| *3. $\sim A$   | 2, Simp      |
| *4. $\sim[(\sim A \vee B) \cdot (\sim A \vee C)]$              | 2, Com, Simp |
| *5. $\sim[\sim A \vee (B \cdot C)]$                            | 4, Dist      |
| *6. $A \cdot \sim(B \cdot C)$                                  | 5, DM, DN    |
| *7. $A$  | 6, Simp      |
| *8. $A \cdot \sim A$   | 7, 3, Conj   |
| 9. $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$            | 1-8, IP, DN  |

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