Philosophy 109, Modern Logic, Queens College
Russell Marcus, Instructor
email: philosophy@thatmarcusfamily.org
website: thatmarcusfamily.org/philosophy
Office phone: (718) 997-5287

Solutions to Problems to Review for Final

WARNING: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

I. Translations (no relational predicates or identity theory)

1. (x)[(Sx · Cx) ⊃ Ex)]
2. (x)[(Mx ∨ Wx) ⊃ Ax]
3. (x)[Lx ⊃ (Ix · Sx)]
4. ~(x)(Dx ⊃ Gx) or (∃x)(Dx · ~Gx)
5. (x)[(Dx · Ex) ⊃ ~Gx] or ~(∃x)[(Ex · Dx) · Gx]
6. (x)[(Px · Ax) ⊃ ~Ix] or ~(∃x)[(Px · Ax) · Ix]
7. (x)[Ex ⊃ (Gx · Px)]
8. (x)(Px ⊃ ~Ex) ∨ (∃x)(Px · Nx)
9. (∃x)(Px · Lx) · (∃x)(Px · Cx)
10. (x)(Px ⊃ Mx) ∨ (x)(Px ⊃ ~Mx)

II. Derivations

1)
1.	Ab ∨ Bc	
2.	(x)~Bx	/ (∃x)Ax
3.	~Bc	2, UI
4.	Ab	1,3 Com, DS
5.	(∃x)Ax	4, EG

QED

2)
1.	(x)(Rx ⊃ Ox)	
2.	(∃x)~Ox	
3.	(x)(~Rx ⊃ Px)	/ (∃x)Px
4.	~Oa	2, EI
5.	Ra ⊃ Oa	1, UI
6.	~Ra	5, 4, MT
7.	~Ra ⊃ Pa	3, UI
8.	Pa	7, 6, MP
9.	(∃x)Px	8, EG

QED

3)
1.	(x)(Fx ⊃ Gx)	
2.	(y)(Gy ⊃ Hy)	/ (z)(~Hz ⊃ ~Fz)
3.	Fx ⊃ Gx	1, UI
4.	Gx ⊃ Hx	2, UI
5.	Fx ⊃ Hx	3, 4, HS
6.	~Hx ⊃ ~Fx	5, Trans
7.	(z)(~Hz ⊃ ~Fz)	6, UG

QED

4)
1.	(∃x)(Ax · Bx) ⊃ (x) Dx	
2.	~Da	/ (x)(Ax ⊃ ~Bx)
3.	(∃x)~Dx	2, EG

4. ~(x)Dx 3, CQ
5. ~(∃x)(Ax · Bx) 1, 4, MT
6. (x)~(Ax · Bx) 5, CQ
7. (x)(~Ax ∨ ~Bx) 6, DM
8. (x)(Ax ⊃ ~Bx) 7, Impl

QED

5) 1. (y)[Ay ⊃ (~By ⊃ Dy)]
2. ~Ba / Aa ⊃ Da
3. Aa ACP
4. Aa ⊃ (~Ba ⊃ Da) 1, UI
5. ~Ba ⊃ Da 4, 3, MP
6. Da 5, 2, MP
7. Aa ⊃ Da 3-6, CP

QED

6) 1. (x)(Qx ⊃ ~Px) /(∃x)Px ⊃ ~(x)Qx
2. (∃x)Px ACP
3. Pa 2, EI
4. Qa ⊃ ~Pa 1, UI
5. ~Qa 4, 3, DN, MT
6. (∃x)~Qx 5, EG
7. ~(x)Qx 6, CQ
8. (∃x)Px ⊃ ~(x)Qx 2-7, CP

QED

7) 1. (x)[Ax ⊃ (Bx · Dx)]
2. (x)[(Ax · Dx) ⊃ Ex]
3. (x)(Ex ⊃ ~Dx) / ~Aa
4. Aa AIP
5. Aa ⊃ (Ba · Da) 1, UI
6. Ba · Da 5, 4, MP
7. Da 6, Com, Simp
8. Aa · Da 4, 7, Conj
9. (Aa · Da) ⊃ Ea 2, UI
10. Ea 9, 8, MP
11. Ea ⊃ ~Da 3, UI
12. ~Da 11, 10, MP
13. Da · ~Da 7, 12, Conj
14. ~Aa 4-13, CP

QED

8) 1. (x)(Ax ⊃ Bx)
2. (x)[Bx ⊃ (Ax ⊃ ~Fx)]
3. (x)[(~Cx · Dx) ⊃ Fx] / (x)[Ax ⊃ (Cx ∨ ~Dx)]
4. Ax ACP
5. Ax ⊃ Bx 1, UI
6. Bx 5, 4, MP
7. Bx ⊃ (Ax ⊃ ~Fx) 2, UI
8. Ax ⊃ ~Fx 7, 6, MP
9. ~Fx 8, 4, MP
10. (~Cx · Dx) ⊃ Fx 3, UI
11. ~(~Cx · Dx) 10, 9, MT
12. Cx ∨ ~Dx 11, DM, DN
13. Ax ⊃ (Cx ∨ ~Dx) 4-12, CP
14. (x)[Ax ⊃ (Cx ∨ ~Dx)] 13, UG

QED

9) 1. (∃x)Gx ⊃ (x)(Fx ⊃ Dx)
2. (∃x)(Gx · ~Dx) / ~(x)Fx
3. Ga · ~Da 2, EI
4. Ga 3, Simp
5. (∃x)Gx 4, EG
6. (x)(Fx ⊃ Dx) 1, 5, MP
7. Fa ⊃ Da 6, UI
8. ~Da 3, Com, Simp
9. ~Fa 7, 8, MT
10. (∃x)~Fx 9, EG
11. ~(x)Fx 10, CQ

QED

10) 1. (∃x)Qx ⊃ (x)(Rx ⊃ Sx)
2. (x)~Qx ⊃ (∃x)Sx
3. (x) Rx / (∃x)Sx
4. ~(∃x)Sx AIP
5. ~(x)~Qx 2, 4, MT
6. (∃x)Qx 5, CQ
7. (x)(Rx ⊃ Sx) 1, 6, MP
8. Rx ⊃ Sx 7, UI
9. Rx 3, UI
10. Sx 8, 9, MP
11. (∃x)Sx 10, EG
12. (∃x)Sx · ~(∃x)Sx 11, 4, Conj
13. (∃x)Sx 4-12, IP, DN

QED

III. Translations (including relational predicates and identity theory)

1. (x)(Fx ⊃ Px)
2. (x)[(Gx · Fx) ⊃ Px]
3. (x)(Fx ⊃ Mnx)
4. (x)(Wpx ⊃ Mnx)
5. (x)(Sxn ⊃ Mnx)
6. (x)(~Mxx ⊃ Mnx)
7. (x)(y)(Sxy ⊃ ~Syx)
8. (x){(Fx ·Px) ⊃ (∃y)[(Gy · Py) · Rxy]} ⊃ (∃x)[Gx · (y)(Fy · Sxy)]
9. Bac · (x)(Bxc ⊃ x=a)
(∃x)(Nx · Wex)
~(∃x)(Nx · Wax) / ~Bec
10. (x)(y)(Rxy ⊃ ~x=y) / (x)~Rxx
11. (x)(y)(z)(x=y ∨ y=z ∨ x=z)
(∃x)(~x=c · Hx) / (∃x)(∃y)[~x=y · (z)(z=x ∨ z=y)]
12. (∃x)[Bxc · (y)(Byc ⊃ y=x) · Hx] / (∃x)(Bxc)
13. (x)[(~x=c · ~x=b) ⊃ Hx]
~Ha / a=c ∨ a=b

IV. Derivations

1) 1. (x)(∃y)(~Fx ∨ Gy) / (x)Fx ⊃ (∃y)Gy
2. (x)Fx ACP
3. (∃y)(~Fx ∨ Gy) 1, UI
4. ~Fx ∨ Ga 3, EI
5. Fx 2, UI
6. Ga 4, 5, DN, DS

7. (∃y)Gy 6, EG
8. (x)Fx ⊃ (∃y)Gy 2-7 CP

QED

2) 1. (x)(∃y)Fxy ⊃ (x)(∃y)Gxy
2. (∃x)(y)~Gxy / (∃x)(y)~Fxy
3. ~(x)(∃y)Gxy 2, CQ, CQ
4. ~(x)(∃y)Fxy 1, 3, MT
5. (∃x)(y)~Fxy 4, CQ, CQ

QED

3) 1. (x)[(Fx ∨ Gx) ⊃ (Hx · Kx)]
2. (x){(Hx ∨ Lx) ⊃ [(Hx· Nx) ⊃ Px]} / (x)[Fx ⊃ (Nx ⊃ Px)]
3. Fx ACP
4. Fx ∨ Gx 3, Add
5. (Fx ∨ Gx) ⊃ (Hx · Kx) 1, UI
6. Hx · Kx 5, 4, MP
7. Hx 6, Simp
8. Hx ∨ Lx 7, Add
9. (Hx ∨ Lx) ⊃ [(Hx · Nx) ⊃ Px] 2, UI
10. (Hx · Nx) ⊃ Px 9, 8, MP
11. Nx ACP
12. Hx · Nx 7, 11, Conj
13. Px 10, 12, MP
14. Nx ⊃ Px 11-13, CP
15. Fx ⊃ (Nx ⊃ Px) 3-14, CP
16. (x)[Fx ⊃ (Nx ⊃ Px)] 15, UG

4) 1. ~(∃x)(Axa · ~Bxb)
2. ~(∃x)(Dxd · Dbx)
3. (x)(Bex ⊃ Dxg) / ~(Aea · Dgd)
4. Aea · Dgd AIP
5. (x)~(Axa · ~Bxb) 1, CQ
6. (x)(~Axa ∨ Bxb) 5, DM, DN
7. ~Aea ∨ Beb 6, UI
8 Aea 4, Simp
9. Beb 7, 8, DN, DS
10. (x)~(Dxd · Dbx) 2, CQ
11. (x)(~Dxd ∨ ~Dbx) 10, DM
12. ~Dgd ∨ ~Dbg 11, UI
13. Dgd 4, Com, Simp
14. ~Dbg 12, 13, DN, DS
15. Beb ⊃ Dbg 3, UI
16. Dbg 15, 9, MP
17. Dbg · ~Dbg 16, 14, Conj
18. ~(Aea · Dgd) 4-17, IP

QED

5) 1. (x)(Ax ⊃ Bx) / (x)[(∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz)]
2. (∃y)(Ay · Cxy) ACP
3. Aa · Cxa 2, EI
4. Aa 3, Simp
5. Aa ⊃ Ba 1, UI
6. Ba 5, 4, MP
7. Cxa 3, Com, Simp
8. Ba · Cxa 6, 7, Conj

9. (∃z)(Bz · Cxz) 8, EG
10. (∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz) 2-9, CP
11. (x)[(∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz)] 10, UG

QED

6)
1. (∃x)(Nx · Wjx · Ix)
2. Nc · Wjc · (x)[(Nx · Wjx) ⊃ x=c] / Ic
3. Na · Wja · Ia 1, EI
4. (x)[(Nx · Wjx) ⊃ x=c] 2, Com, Simp
5. (Na · Wja) ⊃ a=c 4, UI
6. Na · Wja 3, Simp
7. a=c 5, 6, MP
8. Ia 3, Com, Simp
9. Ic 8, 7, ID

QED

7)
1. Pa · Oa · (y)[(Py · Oy) ⊃ y=a]
2. Pw · Sw · (y)[(Py · Sy) ⊃ y=w]
3. (∃x)(Px · Sx · Ox) / a=w
4. Pb · Sb · Ob 3, EI
5. (y)[(Py · Oy) ⊃ y=a] 1, Com, Simp
6. Pb · Ob 4, Simp
7. (Pb · Ob) ⊃ b=a 5, UI
8. b=a 7, 6, MP
9. (y)[(Py · Sy) ⊃ y=w] 2, Com, Simp
10. Pb · Sb 4, Simp
11. (Pb · Sb) ⊃ b=w 9, UI
12. b=w 11, 10, MP
13. a=w 12, 8, ID

QED

8)
1. (∃x){Mx · Tx · (y)[(My · y≠x) ⊃ Hxy] / (∃x){Mx · Tx · (y)[(My · ~Ty) ⊃Hxy]}
2. Ma · Ta · (y)[(My · ~y=a) ⊃ Hay] 1, EI
3. My · ~Ty ACP
4. (y)[(My · ~y=a) ⊃ Hay] 2, Com, Simp
5. (My · ~y=a) ⊃ Hay 4, UI
6. y=a AIP
7. Ta 2, Simp
8. ~Ty 3, Com, Simp
9. Ty 7, 6, ID
10. Ty · ~Ty 9, 8, Conj
11. ~y=a 6-10, IP
12. My 3, Simp
13. My · ~y=a 12, 11, Conj
14. Hay 5, 13, MP
15. (My · ~Ty) ⊃ Hay 3-14, CP
16. (y)[(My · ~Ty) ⊃ Hay] 15, UG
17. Ma · Ta 2, Simp
18. Ma · Ta · (y)[(My · ~Ty) ⊃ Hay] 17, 16, Conj
19. (∃x){Mx · Tx · (y)[(My · ~Ty) ⊃Hxy]} 18, EG

QED

9)
1. (x)(y)(z)[(Sx · Lx · Sy · Ly · Sz · Lz) ⊃ (x=y ∨ y=z ∨ x=z)]
2. (∃x)(∃y)(Sx · Lx · Sy · Ly · Rx · Ry · x≠y)

3. (x)(Rx ⊃ ~Cx)	/ (Sa · Ca) ⊃ ~La	
4. Sa · Ca		ACP
5. La		AIP
6. (∃y)(Sb · Lb · Sy · Ly · Rb · Ry · b≠y)		3, EI
7. Sb · Lb · Sc · Lc · Rb · Rc · b≠c		6, EI
8. Sb · Lb · Sc · Lc		7, Simp
9. Sa		4, Simp
10. Sa · La		9, 5, Conj
11. Sa · La · Sb · Lb · Sc · Lc		10, 8, Conj
12. (y)(z)[(Sa · La · Sy · Ly · Sz · Lz) ⊃ (a=y ∨ y=z ∨ a=z)]		1, UI
13. (z)[(Sa · La · Sb · Lb · Sz · Lz) ⊃ (a=b ∨ b=z ∨ a=z)]		12, UI
14. (Sa · La · Sb · Lb · Sc · Lc) ⊃ (a=b ∨ b=c ∨ a=c)		13, UI
15. a=b ∨ b=c ∨ a=c		14, 11, MP
16. ~b=c		7, Simp
17. a=b ∨ a=c		17, 16, Com, DS
18. Ra ⊃ ~Ca		3, UI
19. Ca		4, Com, Simp
20. ~Ra		18, 19, DN, MT
21. Rb		7, Simp
22. a=b		AIP
23. ~Rb		20, 22, ID
24. Rb ·~Rb		21, 24, Conj
25. ~a=b		22-24, IP
26. a=c		17, 25, DS
27. Rc		7, Simp
28. Rc ⊃ ~Cc		3, UI
29. ~Cc		28, 27, MP
30. Cc		19, 26, ID
31. Cc ·~Cc		30, 29, Conj
33. ~La		5-31, IP
34. (Sa · Ca) ⊃ ~La		4-33, CP

QED

V. Invalidity

1) Invalid in a 1-member universe, where: Aa: True; Ba: False; Ca: False.

2) Invalid in a 2-member universe, where Ea: True; Fa: False; Ga: False; Eb: True or False; Fb: True; Gb: True

3) Invalid in a 3-member universe, where Pa: True; Qa: False; Ra: False
Pb: False; Qb: True; Rb: False
Pc: False; Qc: False; Rc: True or False