Syllabus

Course Description and Overview:

      This course is divided into two parts. The first part, covering roughly the first nine weeks of the term, is an historical survey of the philosophical questions which arise from considering how to explain our knowledge of mathematics. Do we have a priori knowledge of necessary truths? Is our knowledge of mathematics empirical? Do we really have mathematical knowledge at all? The readings in the first part of the course, covered mainly chronologically, range from ancient philosophy through the twentieth century, with special attention paid to the fruitful period between Frege, in the late nineteenth century, and Gödel. We will devote the second part of the course, the last five weeks of the term, to recent work, including my own, on the indispensability argument.

      Mathematics has a long and prominent place in philosophy. Plato's students were implored to excel in mathematics; a sign over the door to his Academy said, "Let no one enter who is ignorant of geometry." Aristotle wrote, "Mathematics has come to be the whole of philosophy for modern thinkers" (*Metaphysics* I.9: 992a32).

      Some prominent philosophers in the early modern period were mathematicians, including Descartes, who developed analytic geometry, and Leibniz, who developed the calculus. In the late nineteenth and early twentieth centuries, philosophers including Frege and Russell made advances in the foundations of mathematics proper. In recent years, many philosophers have made contributions to set theory and mathematical logic, independently of their philosophical work.

      In the other direction, mathematicians from Euclid forward have contributed to philosophy. Cantor's work on transfinite numbers transformed the philosopher's concept of infinity, which had played a central role in philosophical debate about God and the origins of the universe for millennia. Other philosophical topics like necessity and contingency have received mathematical treatment which has changed the way philosophers argue about these concepts. Indeed some mathematicians, like Hilbert, Gödel, von Neumann, and Tarski, are central philosophical figures.

      Even philosophers who have not contributed to mathematics have made mathematical insights central to their work. Berkeley tried to debunk the calculus on philosophical grounds. Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty. Wittgenstein's *Remarks on the Foundations of Mathematics* contain core elements of his philosophical positions.

      Still, even philosophers who spend time with mathematics deny that the relationship of mathematics to philosophy is particularly close. Wittgenstein wrote that philosophy, "Leaves mathematics as it is, and no mathematical discovery can advance it." (*Philosophical Investigations*, §124) Kripke implored that, "There is no mathematical substitute for philosophy."

      In this course, in addition to examining the philosophical questions which arise from considerations of our knowledge of mathematics, we will try to see what makes mathematics so interesting to philosophers, and also what contributions mathematics can make to philosophy.

Texts:

James Robert Brown, *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*, New York: Routledge, 2000.
Stewart Shapiro, *Thinking About Mathematics: The Philosophy of Mathematics*, New York: Oxford, 2000.
Various readings, available on the course website

On-Line Resources:

The website for this course is:

http://www.thatmarcusfamily.org/philosophy/Course_Websites/Math_F10/Course_Home.html

Limited material will be available on the Blackboard course pages. The Blackboard page will include a link to the course website. The course website includes an html syllabus, course schedule, course bibliography, class notes, assignments, other readings and handouts, and links to websites specifically selected for this course.

Assignments and Grading

Your responsibilities for this course include the following, with their contributions to your grade calculation in parentheses:

1. All the primary readings listed below, including seminar papers.
2. Twenty reading prècises (10%)
3. Two seminar papers/presentations (2-4 pages; 5-10 minutes) (40%; 20% each)
4. Term paper (8-12 pages) (30%)
5. Final exam (20%)

Readings are to be completed before the class indicated. The Primary Readings are required; the secondary readings are optional. Some secondary readings, notably the readings from the Brown and Shapiro texts, are introductory elucidations of the primary readings. Some secondary readings are further scholarly articles on a given topic, critical commentaries on the primary readings, or extended studies of a point we will study only briefly. All of the readings on the syllabus that are not from either the Brown or Shapiro texts will be accessible from the course website. The course bibliography includes further readings, many of which are also accessible from the course website.

Reading prècises are 100- to 150- word summaries, or distillations, of some portion of an assigned reading. In preparing for most classes, you should write one prècis before class. You may choose to write about an entire reading, or to focus on a small portion of one reading. If there is more than one reading, you may choose one reading on which to focus. You need not complete prècises for the two classes in which you are presenting a seminar paper. In lieu of up to five prècises, you can write a list of 6-8 detailed questions on the reading. Your twenty prècises are due on **Friday, December 10, at 4pm**. You will mainly be graded on the completion of twenty prècises, rather than their quality. I expect that the prècises will be useful to you in preparing both for classes and for the final exam.

Many classes will run as extended discussions of a 750- to 1500-word **seminar paper**. Seminar papers should assimilate the assigned readings and summarize the main arguments. I also encourage you to include some critical analysis. You are instigating class discussion, focusing our thoughts on the

central theses, and raising questions. It is good practice to end a seminar paper with a few questions you believe will be useful for the class to consider. Each seminar paper is **due at noon by email to all seminar participants the day before the class in which it will be discussed** (i.e. Sunday or Tuesday). This deadline is necessary for all participants in the seminar to be able to read the paper and prepare comments and questions for class.

You will lead the class on the day we discuss your seminar paper. You may be creative with your presentation. You may focus on the content of your paper. You may also discuss any particular difficulties in the material, or topics that you were unable to cover in the paper. Your grade for the seminar paper will depend on both the paper and your presentation of it. Each student in the course will write and present two seminar papers.

Your **term papers** will be completed in three stages. A one-paragraph abstract of you paper is due on **Wednesday, October 13**. A full draft of your term paper is due on **Monday, November 15**. The final draft is due on **Monday, December 6**. See the Paper Assignment handout for various options for paper topics. I will be happy to meet with you to discuss your topic, in advance. Failure to hand in a draft, or handing in an insufficient draft, will reduce your final paper grade by two steps (e.g. from B+ to B-).

The **final exam** will be on **Wednesday, December 15**, from 9am to noon. Preparatory questions will be posted on the course website.

Both the Writing Center and the Oral Communications Center have an astoundingly wonderful set of resources to help you write and speak more effectively.

The Hamilton College Honor Code will be enforced.

Contacting Me

My office hours for the Fall 2010, term are 10:30am - noon, Monday through Friday. My office is room 201 of 210 College Hill Road, which is at the northwest corner of CHR and Griffin Road. My email address is rmarcus1@hamilton.edu.

Schedule:

	Date	Topic	Primary Readings	Secondary Readings
1	Monday, August 30	What is Mathematics? What is Philosophy of Mathematics?	Brown, Chapter 1 Shapiro, pp 21-29	
2	Wednesday, September 1	Pythagoras and the Pythagoreans	Kline, "The Creation of Classical Greek Mathematics" Kline, "The Greek Rationalization of Nature"	
3	Monday, September 6	Plato's Platonism	Selections from Plato on Mathematics Aristotle, *Metaphysics* I.9	Shapiro, pp 49-63 Brown, Chapter 2
4	Wednesday, September 8	Aristotle	Aristotle, *Metaphysics* XIII.1-3 Aristotle, *Physics* II.2 Lear, "Aristotle's Philosophy of Mathematics"	Shapiro, pp 63-71 Aristotle, *Metaphysics* XIII-XIV
5	Monday, September 13	Modern Rationalism I	Descartes, Third and Fifth Meditations Descartes, Synthetic Presentation from Second Replies Leibniz, "Meditations on Knowledge, Truth, and Ideas"	Kline, "Coordinate Geometry" Kline, "The Mathematization of Science"
6	Wednesday, September 15	Modern Rationalism II	Locke, *Essay*, Bk 1, Ch. 1 Leibniz, Selections from *New Essays*	Kline, "The Creation of the Calculus"
7	Monday, September 20	Modern Empiricism	Locke, *Selections on Mathematics* Selections from Berkeley's *Principles* Selections from Hume on Mathematics	
8	Wednesday, September 22	The Synthetic *A Priori* I	Kant, *Prolegomena*, §§1-2 Selections from Kant's *Critique*	Shapiro, pp 76-91
9	Monday, September 27	The Synthetic *A Priori* II		
10	Wednesday, September 29	Radical Empiricism	Mill, *System of Logic*, Book II, §V and §VI Frege, from *The Foundations of Arithmetic*, I	Shapiro, pp 91-102

	Date	Topic	Primary Readings	Secondary Readings
11	Monday, October 4	Cantor's Paradise	Tiles, "Cantor's Transfinite Paradise"	Dauben, "Cantor's Philosophy of the Infinite" Tiles, "Numbering the Continuum"
12	Wednesday, October 6	Logicism	Frege, from *The Foundations of Arithmetic*, II Russell, "Letter to Frege" Frege, "Letter to Russell"	Shapiro, pp 107-115 Russell, "On Our Knowledge of General Principles" Russell, "How *A Priori* Knowledge is Possible"
13	Monday, October 11	Formalism and Incompleteness	Hilbert, "On the Infinite" Johann von Neumann, "The Formalist Foundations of Mathematics"	Brown, Chapter 5 Shapiro, pp 140-168 Smullyan, "The General Idea Behind Gödel's Proof"
14	Wednesday, October 13 **Abstracts due**	Gödel Platonism	"What is Cantor's Continuum Problem? (1964)"	Shapiro, pp 201-212 Brown, Chapter 11 Feferman, et al., "Introductory Note..." Gödel, "What is Cantor's Continuum Problem? (1947)"
15	Monday, October 18	Intuitionism	Heyting, "Disputation" Brouwer, "Intuitionism and Formalism" Brouwer, "Consciousness, Philosophy, and Mathematics"	Brown, Chapter 8 Shapiro, pp 172-189
16	Wednesday, October 20	Conventionalism	Carnap, "Empiricism, Semantics and Ontology" Ayer, "The A Priori"	Shapiro, pp 124-133 Brown, Chapter 9
17	Monday, October 25	Two Dogmas of Empiricism	Quine, "Two Dogmas of Empiricism"	Shapiro, pp 212-220 Grice and Strawson, "In Defense of a Dogma"
18	Wednesday, October 27	The Problem	Benacerraf, "Mathematical Truth" Field, "Knowledge of Mathematical Entities"	Shapiro, pp 29-39
19	Monday, November 1	The Indispensability Argument	Quine, "Existence and Quantification" Quine, "On What There Is" Quine on Recreation	Azzouni, "On 'On What There Is'" Marcus, "Quine's Indispensability Argument"
20	Wednesday, November 3	Dispensabilism I	Field, from *Science without Numbers*	Shapiro, pp 226-237 Brown, Chapter 4

	Date	Topic	Primary Readings	Secondary Readings
21	Monday, November 8	Dispensabilism II	Field, "Introduction: Fictionalism, Epistemology, and Modality" MacBride, "Listening to Fictions: A Study of Fieldian Nominalism"	Melia, "Field's Programme: Some Interference"
22	Wednesday, November 10	The Weasel	Melia, "Weaseling Away the Indispensability Argument"	Colyvan, "Mathematics and Aesthetic Considerations in Science? Melia, "Response to Colyvan"
23	Monday, November 15 **Draft due**	The Eleatic and the Indispensabilist	Colyvan, "The Quinean Backdrop" Colyvan, "The Eleatic Principle"	Marcus, "The Eleatic and the Indispensabilist"
24	Wednesday, November 17	Mathematical Recreation	Leng, "What's Wrong with Indispensability? (Or, the Case for Recreational Mathematics)" Colyvan, "Mathematical Recreation versus Mathematical Knowledge"	Marcus, "Why the Indispensability Argument Does Not Justify Belief in Mathematical Objects" Maddy, "Indispensability and Practice" Sober, "Mathematics and Indispensability"
25	Monday, November 29	The Explanatory Argument	Baker, "Are There Genuine Mathematical Explanations of Physical Phenomena?" Mancosu, "Mathematical Explanation: Problems and Prospects," §3	Lyon and Colyvan, "The Explanatory Power of Phase Spaces"
26	Wednesday, December 1	The Nominalist Against the Explanatory Argument	Bangu, "Inference to the Best Explanation and Mathematical Realism"	
27	Monday, December 6 **Paper due**	The Platonist Against the Explanatory Argument	Marcus, "Explanation and Indispensability"	Brown, Chapter 3
28	Wednesday, December 8	Contemporary Platonism	Katz, "Conclusion: The Problems of Philosophy" Katz, "The Epistemic Challenge to Realism" Katz, "Toward a Realistic Rationalism"	Marcus, "Toward Autonomy Realism"

Prècises due: Friday, December 10
Final Exam: Wednesday, December 15, 9am.